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COMPARISON OF PREPARATORY SIGNAL ANALYSIS TECHNIQUES FOR CONSIDERATION IN THE (POST-)KYOTO POLICY PROCESS

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Mathematical Background and Numerical Tables

to

Comparison of Preparatory Signal Analysis Techniques for Consideration in the (Post-) Kyoto Policy Process

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To avoid duplication, this mathematical background also serves as appendix to a special journal version (under preparation) which builds on the aforementioned short paper.

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Appendix A: Critical Relative Uncertainty (CRU) Concept

With x_i denoting the net emissions (best estimates) and ε_i their absolute uncertainty at t_i (i = 1, 2), we can write for the relative uncertainty:

$$\rho = \frac{\varepsilon_1}{x_1} = \frac{\varepsilon_2}{x_2} = \text{const}^{1} \qquad \text{(A-1a,b,c)}$$

and for the ratio of emissions:

$$1 - \delta_{KP} = \frac{x_2}{x_1}, \qquad \text{(A-2)}$$

where δ_{KP} is the normalized emissions change committed under the Kyoto Protocol (KP) between t_1 and t_2 ($\delta_{KP} > 0$: emission reduction; $\delta_{KP} \leq 0$: emission limitation). Requiring that the absolute change in emissions outstrips uncertainty at t_2,

$$|x_1 - x_2| > \varepsilon_2 \qquad \text{(A-3)}$$

(see Fig. 1)[2], which is equivalent to

$$|x_1 - x_2| > x_2 \rho, \qquad \text{(A-4)}$$

we find with the help of Equation (A-2)

$$\frac{|\delta_{KP}|}{(1 - \delta_{KP})} > \rho, \qquad \text{(A-5)}$$

where

$$\rho_{crit} := \frac{|\delta_{KP}|}{(1 - \delta_{KP})} \qquad \text{(A-6)}$$

is called critical relative uncertainty (CRU; Gusti and Jęda, 2006: Section 3.2).

[1] The CRU concept only considers uncertainty in the commitment year/period, not in the base year (i.e., formally $\varepsilon_1 = 0$). However, for reasons of comparability, we continue to abide by the condition of constant relative uncertainty.

[2] If not preceded by a letter, figure numbers in this document refer to figures in the short paper of Jonas *et al.* (2007) and, likewise, its special journal version (under development).

Tab. A-1: The CRU concept (Eq. A-6) applied to Annex B countries. In the last column, we assess the hypothetical situation that the CRU concept had been applied prior to/in negotiating the Kyoto Protocol. Note the dissimilarity between countries committed to emission reduction ($\delta_{KP} > 0$) and emission limitation ($\delta_{KP} \leq 0$) with the introduction of more lenient or stricter Kyoto emission targets.

Country Group	KP Commitment δ_{KP}[a] %	CRU ρ_{crit} %	If the CRU Concept had been applied
1a 1b 1c 1d	8.0	8.7	a) Compliance with the Kyoto emission target: It must be expected that Annex B countries exhibit relative uncertainties in the range of 5–10% and above rather than below (excluding emissions/removals due to LULUCF and Kyoto mechanisms). Thus, it is impossible for a number of countries in groups 1–4 to meet the condition that their overall relative uncertainty is smaller than their CRU ($\rho < \rho_{crit}$). b) Towards more lenient Kyoto emission targets: To unambiguously attest a decrease in emissions, Annex B countries have to fulfill increasingly smaller CRUs. c) Towards stricter Kyoto emission targets: CRUs increase and can be met more easily.
2	7.0	7.5	
3a 3b 3c	6.0	6.4	
4	5.0	5.3	
--	4.0	4.2	
--	3.0	3.1	
--	2.0	2.0	
--	1.0	1.0	
5	0.0	0.0	a) Compliance with the Kyoto emission target: Same conclusion for countries in groups 5–8 as for countries committed to emission reduction (see a) above. b) Towards more lenient Kyoto emission targets: CRUs increase and can be met more easily. c) Towards stricter Kyoto emission targets: To unambiguously attest a decrease in emissions Annex B countries have to fulfill increasingly smaller CRUs.
6	-1.0	1.0	
--	-2.0	2.0	
--	-3.0	2.9	
--	-4.0	3.8	
--	-5.0	4.8	
--	-6.0	5.7	
--	-7.0	6.5	
7	-8.0	7.4	
--	-9.0	8.3	
8	-10.0	9.1	

[a] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP}, the normalized change in emissions between t_1 and t_2: $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Appendix B: Verification Time (VT) Concept

Requiring that the absolute change in net emissions, $\left|\Delta x(t)\right|$, outstrips the absolute uncertainty of emissions, $\varepsilon(t)$, at time t, we can write

$$\left|\Delta x(t)\right| > \varepsilon(t) \; ; \tag{B-1}$$

and after making use of linear approximations on both sides (in line with preparatory signal analysis):

$$\left|\frac{dx}{dt}\right|_{t_1} \Delta t > \varepsilon(t_1) + \left(\frac{d\varepsilon}{dt}\right)_{t_1} \Delta t \; . \tag{B-2}$$

Rearranging Ineq. B-2, we can solve for the VT, the minimal time Δt required for the emission signal to outstrip the absolute uncertainty of emissions (Jonas *et al.*, 1999: Section 3):

$$\Delta t > \frac{\varepsilon(t_1)}{\left|\frac{dx}{dt}\right|_{t_1} - \left(\frac{d\varepsilon}{dt}\right)_{t_1}} \; , \tag{B-3}$$

where $\left|\frac{dx}{dt}\right|_{t_1} > \left(\frac{d\varepsilon}{dt}\right)_{t_1}$. With the help of δ_{KP}, the committed (normalized) change in emissions under the Kyoto Protocol (see App. A), we can write for the two terms in the denominator on the right side of Ineq. B-3:

$$\left|\frac{dx}{dt}\right|_{t_1} = \frac{\left|\delta_{KP}\right|}{t_2 - t_1} x(t_1) \tag{B-4}$$

$$\left(\frac{d\varepsilon}{dt}\right)_{t_1} = -\frac{\delta_{KP}}{t_2 - t_1} \varepsilon(t_1) \; . \tag{B-5}$$

Thus, Ineq. B-3 reads:

$$\Delta t > \frac{\varepsilon(t_1)}{\left|\delta_{KP}\right| x(t_1) + \delta_{KP} \varepsilon(t_1)} (t_2 - t_1) \; ; \tag{B-6}$$

or, if the VT is normalized and expressed with the help of the relative uncertainty ρ (see App. A):

$$\frac{\Delta t}{t_2 - t_1} > \frac{\rho}{\left|\delta_{KP}\right| + \delta_{KP}\rho} = \frac{\rho}{\left|\delta_{KP}\right| \left\{1 + \mathrm{sgn}(\delta_{KP})\rho\right\}} \; . \tag{B-7a,b}$$

The right side of Ineq. B-7 becomes 1 for $\rho = \rho_{crit}$, the CRU (see Eq. A-6).

Tab. B-1: The VT concept (Ineq. B-7a) applied to Annex B countries. The table has to be read as follows: The maximal allowable VT ($t_2 - t_1$) for an Annex 1 country is given for $\rho = \rho_{crit}$ (see second column). For a country of group 1a the maximal allowable VT is 20 years or 1, if normalized. Normalized VTs equal to or smaller than 1 (see green fields for emission reduction and orange fields for emission limitation) are compatible with the Kyoto Protocol, i.e., countries report with $\rho \leq \rho_{crit}$; normalized VTs greater than 1 (see red fields) are not, i.e., countries report with $\rho > \rho_{crit}$. In the last column, we assess the hypothetical situation that the VT concept had been applied prior to/in negotiating the Kyoto Protocol. Note the dissimilarity between countries committed to emission reduction ($\delta_{KP} > 0$) and emission limitation ($\delta_{KP} \leq 0$) with the introduction of more lenient or stricter Kyoto emission targets.

Country Group	Max. Allow. VT[a] $t_2 - t_1$ yr	KP Commit. δ_{KP}[b] %	Normalized VTs if countries report with ρ = 2.5 %	7.5 %	15 %	30 %	If the VT Concept had been applied
1a	20	8.0	0.3	0.9	1.6	2.9	<u>a) Compliance with the Kyoto emissions target:</u> It must be expected that Annex B countries exhibit relative uncertainties in the range of 5–10% and above rather than below (excluding emissions/ removals due to LULUCF and Kyoto mechanisms). Thus, it is impossible for a number of countries in groups 1–4 to meet the condition ρ < ρ_{crit} or, equivalently, achieve normalized VTs ≤ 1. <u>b) Towards more lenient Kyoto emission targets:</u> To unambiguously attest a decrease in emissions, Annex B countries have to fulfill increasingly smaller CRUs or, equivalently, find it more difficult to comply with normalized VTs ≤ 1. <u>c) Towards stricter Kyoto emission targets:</u> CRUs increase and can be met more easily or, equivalently, compliance with VTs ≤ 1 becomes less difficult.
1b	22						
1c	21						
1d	24						
2	20	7.0	0.3	< 1.0	1.9	3.3	
3a	20	6.0	0.4	1.2	2.7	3.8	
3b	24						
3c	22						
4	20	5.0	0.5	1.4	2.6	4.6	
--	--	4.0	0.6	1.7	3.1	5.8	
--	--	3.0	0.8	2.3	4.3	7.7	
--	--	2.0	1.2	3.5	6.5	11.5	
--	--	1.0	2.3	7.0	13.0	23.1	
5	20	0.0	infinite				<u>a) Compliance with the Kyoto emissions target:</u> Same conclusion for countries in groups 5–8 as for countries committed to emission reduction (see a) above). <u>b) Towards more lenient Kyoto emission targets:</u> CRUs increase and can be met more easily or, equivalently, compliance with VTs ≤ 1 becomes less difficult. <u>c) Towards stricter Kyoto emission targets:</u> To unambiguously attest a decrease in emissions, Annex B countries have to fulfill increasingly smaller CRUs or, equivalently, find it more difficult to comply with normalized VTs ≤ 1.
6	20	-1.0	2.6	8.1	17.6	42.9	
--	--	-2.0	1.3	4.1	8.8	21.4	
--	--	-3.0	0.9	2.7	5.9	14.3	
--	--	-4.0	0.6	2.0	4.4	10.7	
--	--	-5.0	0.5	1.6	3.4	8.6	
--	--	-6.0	0.4	1.4	2.9	7.1	
--	--	-7.0	0.4	1.2	2.5	6.1	
7	20	-8.0	0.3	> 1.0	2.2	5.4	
--	--	-9.0	0.3	0.9	2.0	4.8	
8	20	-10.0	0.3	0.8	1.8	4.3	

[a] The maximal allowable VT is calculated for each country group as the difference between 2010 (as the temporal mean over the commitment period 2008–2012) and its base year, or mean base year, for CO_2, CH_4 and N_2O. (The 'CO_2-CH_4-N_2O system of gases' dominates over the 'HFC-PFC-SF_6 system of gases'.)

[b] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP} , the normalized change in emissions between t_1 and t_2 : $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Appendix C: Undershooting (Und) Concept

Starting from the true (t) but unknown net emissions $x_{t,i}$ at t_i (i = 1, 2), compliance under the Kyoto Protocol requires satisfying

$$x_{t,2}-(1-\delta_{KP})x_{t,1}\leq 0 \tag{C-1}$$

at t_2. The difference between the unknown true emissions and their best estimates (x_i) is captured with the help of ε_i, an uncertainty that (ideally) considers both accuracy and precision:

$$\left|x_{t,1}-x_1\right|\leq\varepsilon_1\ ,\ \left|x_{t,2}-x_2\right|\leq\varepsilon_2\ . \qquad \text{(C-2), (C-3)}$$

With Ineq. C-2 and (C-3) in the form of

$$x_{t,1}\in\left[x_1-\varepsilon_1,x_1+\varepsilon_1\right]\ ,\ x_{t,2}\in\left[x_2-\varepsilon_2,x_2+\varepsilon_2\right] \qquad \text{(C-2a), (C-3a)}$$

and applying interval calculus, the left side of Ineq. C-1 can be delimited (Nahorski *et al.*, 2003: Section 2.2; 2007: Section 3):

$$\begin{aligned}
&x_{t,2}-(1-\delta_{KP})x_{t,1}\in\left[x_2-\varepsilon_2,x_2+\varepsilon_2\right]-(1-\delta_{KP})\left[x_1-\varepsilon_1,x_1+\varepsilon_1\right]\\
&=\left[x_2,x_2\right]+\left[-\varepsilon_2,\varepsilon_2\right]-\left[(1-\delta_{KP})x_1,(1-\delta_{KP})x_1\right]-\left[-(1-\delta_{KP})\varepsilon_1,(1-\delta_{KP})\varepsilon_1\right]\\
&=\left[x_2-(1-\delta_{KP})x_1,x_2-(1-\delta_{KP})x_1\right]+\left[-(1-\delta_{KP})\varepsilon_1-\varepsilon_2,(1-\delta_{KP})\varepsilon_1+\varepsilon_2\right]\\
&=\left[Dx-\varepsilon'_{12},Dx+\varepsilon'_{12}\right]
\end{aligned} \qquad \text{(C-4a–d)}$$

where

$$Dx:=x_2-(1-\delta_{KP})x_1\ ,\ \varepsilon'_{12}:=(1-\delta_{KP})\varepsilon_1+\varepsilon_2\ . \qquad \text{(C-5), (C-6)}$$

Nahorski *et al.* (2007) suggest expanding ε'_{12} to account for correlation (corr) between ε_1 and ε_2 according to (first-order approach):

$$\varepsilon_{12}=(1-\delta_{KP})\varepsilon_1+\varepsilon_2-\varepsilon_{corr}=(1-\nu)\left\{(1-\delta_{KP})\varepsilon_1+\varepsilon_2\right\}\ , \qquad \text{(C-7a,b)}$$

where

$$\varepsilon_{corr}=\nu\varepsilon'_{12}\ . \tag{C-8}$$

The risk that $x_{t,2}$ is equal to, or greater than, $(1-\delta_{KP})x_{t,1}$ can be captured with the help of α:

$$Dx+\varepsilon_{12}\leq 2\alpha\varepsilon_{12}\ , \tag{C-9}$$

where $0\leq\alpha\leq 0.5$ (see Fig. C-1). The risk $\alpha=0.5$ corresponds to the situation $Dx=0\Leftrightarrow x_2=(1-\delta_{KP})x_1$, when we can judge with equal confidence that $x_{t,2}$ is $\leq$ or $\geq(1-\delta_{KP})x_{t,1}$ (case of ignoring uncertainty). With Dx decreasing ($Dx<0$), the risk α

also decreases that $x_{t,2} \geq (1-\delta_{KP})x_{t,1}$. The case $Dx = -\varepsilon_{12}$ corresponds to $\alpha = 0$ or, alternatively, $x_2 + (1-\nu)\varepsilon_2 = (1-\delta_{KP})\{x_1 - (1-\nu)\varepsilon_1\}$, the case of perfect credibility.

Rewriting Ineq. C-9, we find:

$$\frac{x_2}{x_1} \leq (1-\delta_{KP}) - (1-2\alpha)\frac{\varepsilon_{12}}{x_1} \; . \tag{C-10}$$

Using Eq. A-1a,b in combination with Eq. C-7b, $\frac{\varepsilon_{12}}{x_1}$ can be expressed as:

$$\frac{\varepsilon_{12}}{x_1} = (1-\nu)\left\{(1-\delta_{KP})\rho + \frac{x_2}{x_1}\rho\right\} \tag{C-11}$$

and inserted into Eq. C-10:

$$\frac{x_2}{x_1} \leq (1-\delta_{KP}) - (1-2\alpha)(1-\nu)\left\{(1-\delta_{KP})\rho + \frac{x_2}{x_1}\rho\right\} \; . \tag{C-12}$$

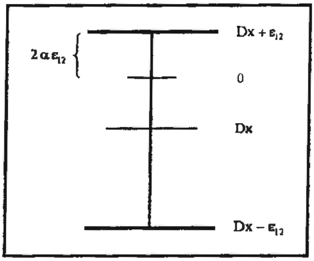

Fig. C-1: Illustration of the risk α ($0 \leq \alpha \leq 0.5$) to capture the situation $x_{t,2} \geq (1-\delta_{KP})x_{t,1}$. Source: Nahorski *et al.* (2007: Fig. 1); modified.

After rearrangement:

$$\frac{x_2}{x_1} \leq (1-\delta_{KP})\frac{1-(1-2\alpha)(1-\nu)\rho}{1+(1-2\alpha)(1-\nu)\rho} = (1-\delta_{KP})\frac{\{1-(1-2\alpha)(1-\nu)\rho\}^2}{1-\{(1-2\alpha)(1-\nu)\rho\}^2} \; . \tag{C-13a,b}$$

For educational purposes, it is useful to initially study the approximation of Ineq. C-13b for the ranges of α and ρ values of interest here (see Tab. C-1 below) and a correlation of $\nu = 0.75$ typical for currently reported uncertainties (see EEA 2007: Tab. 1.13)

$$\approx (1-\delta_{KP})\{1-2(1-2\alpha)(1-\nu)\rho\} \tag{C-14a}$$

$$= 1-\delta_{KP} - 2(1-2\alpha)(1-\delta_{KP})(1-\nu)\rho \tag{C-14b}$$

$$=1-\left\{\delta_{KP}+2(1-2\alpha)(1-\delta_{KP})(1-\nu)\rho\right\} \ . \tag{C-14c}$$

The term in parentheses on the right of Ineq. C-14c is called the modified (mod) emission limitation or reduction targets for all Annex B countries,

$$\delta_{mod} := \delta_{KP} + U \tag{C-15}$$

and

$$U := 2(1-2\alpha)(1-\delta_{KP})(1-\nu)\rho \tag{C-16}$$

the undershooting, which is required for decreasing the '$x_{t,2}$-greater-than-$(1-\delta_{KP})x_{t,1}$' risk that one is willing to tolerate in coping with the combined (correlated) uncertainty ε_{12}.

To avoid the aforementioned approximation, we write Ineq. C-13a in the form

$$\frac{x_2}{x_1} \le (1-\delta_{KP})\frac{1-(1-2\alpha)(1-\nu)\rho}{1+(1-2\alpha)(1-\nu)\rho} = 1-\delta_{mod} \ . \tag{C-13a,c}$$

Thus, the modified emission limitation or reduction target δ_{mod} is given by

$$\delta_{mod} = 1-(1-\delta_{KP})\frac{1-(1-2\alpha)(1-\nu)\rho}{1+(1-2\alpha)(1-\nu)\rho} = \delta_{KP} + U \qquad \text{(C-17), (C-15)}$$

and still by Eq. C-15.

Resolving for U:

$$U = 2(1-\delta_{KP})\frac{(1-2\alpha)(1-\nu)\rho}{1+(1-2\alpha)(1-\nu)\rho} \ . \tag{C-18}$$

Treating δ_{KP}, ρ and α as parameters and setting $\nu = 0.75$ (typical for currently reported uncertainties; see EEA, 2007: Tab. 1.13), Eq. C-15 in combination with Eq. C-18 allows calculating the modified emission limitation or reduction targets δ_{mod}, and the undershooting U contained in δ_{mod}, for all Annex B countries.

Tab. C-1: The Und concept (Eq. C-15 in combination with Eq. C-18 and a correlation of $\nu = 0.75$ typical for currently reported uncertainties) applied to Annex B countries. The table lists modified emission limitation or reduction targets δ_{mod} for all Annex B countries, where the '$x_{t,2}$-greater-than-$(1-\delta_{KP})x_{t,1}$' risk α is specified to be 0, 0.1, 0.3 and 0.5. If an Annex B country complies with its emission limitation or reduction commitment ($x_2 = (1-\delta_{KP})x_1$), the risk that its true, but unknown, emissions $x_{t,2}$ are equal to or greater than its true, but unknown, target $(1-\delta_{KP})x_{t,1}$ is 50%. Undershooting decreases this risk. For instance, a country of group 1 has committed itself to reduce its net emissions by 8%. Reporting with a 7.5% relative uncertainty, it needs to reduce emissions by 11.4% to decrease the risk from 50% to 0%. In the last column, we assess the hypothetical situation that the Und concept had been applied prior to/in negotiating the Kyoto Protocol. Note the unfavorable situation, which arises when δ_{KP} varies while ρ and α are kept constant.

Country Group	KP Commit. δ_{KP}[a] %	Modified Emission Limitation or Reduction Target δ_{mod} in % for α = 1	ρ = 2.5 %	7.5 %	15 %	30 %
1a–d	8.0	0.0	9.1	11.4	14.7	20.8
		0.1	8.9	10.7	13.4	18.4
		0.3	8.5	9.4	10.7	13.4
		0.5	8.0	8.0	8.0	8.0
2	7.0	0.0	8.2	10.4	13.7	20.0
		0.1	7.9	9.7	12.4	17.5
		0.3	7.5	8.4	9.7	12.4
		0.5	7.0	7.0	7.0	7.0
3a–c	6.0	0.0	7.2	9.5	12.8	19.1
		0.1	6.9	8.8	11.5	16.6
		0.3	6.5	7.4	8.8	11.5
		0.5	6.0	6.0	6.0	6.0
4	5.0	0.0	6.2	8.5	11.9	18.3
		0.1	5.9	7.8	10.5	15.8
		0.3	5.5	6.4	7.8	10.5
		0.5	5.0	5.0	5.0	5.0
---	4.0	0.0	5.2	7.5	10.9	17.4
		0.1	5.0	6.8	9.6	14.9
		0.3	4.5	5.4	6.8	9.6
		0.5	4.0	4.0	4.0	4.0
---	3.0	0.0	4.2	6.6	10.0	16.5
		0.1	4.0	5.9	8.7	14.0
		0.3	3.5	4.4	5.9	8.7
		0.5	3.0	3.0	3.0	3.0
---	2.0	0.0	3.2	5.6	9.1	15.7
		0.1	3.0	4.9	7.7	13.1
		0.3	2.5	3.5	4.9	7.7
		0.5	2.0	2.0	2.0	2.0
---	1.0	0.0	2.2	4.6	8.2	14.8
		0.1	2.0	3.9	6.8	12.2
		0.3	1.5	2.5	3.9	6.8
		0.5	1.0	1.0	1.0	1.0

If the Und Concept had been applied

a) For given δ_{KP} and α: The greater ρ, the greater the modified emission reduction target δ_{mod} must be to keep the '$x_{t,2}$-greater-than-$(1-\delta_{KP})x_{t,1}$' risk α at a constant level (see, e.g., country group 1: third line: δ_{mod} values for $\alpha = 0.3$).

b) For given ρ and α: The smaller δ_{KP}, the smaller the modified emission reduction target δ_{mod} can be to keep the '$x_{t,2}$-greater-than-$(1-\delta_{KP})x_{t,1}$' risk α at a constant level (see, e.g., δ_{mod} values for $\rho = 7.5\%$ and $\alpha = 0.3$). As a consequence, countries that must comply with a small δ_{KP} (they exhibit a small δ_{mod}) are better off than countries that must comply with a great δ_{KP} (they exhibit a great δ_{mod}).

[a] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP}, the normalized change in emissions between t_1 and t_2: $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Table C-1 continued:

5	0.0	0.0	1.2	3.7	7.2	14.0	a) For given δ_{KP} and α:
		0.1	1.0	3.0	5.8	11.3	Same conclusion for country groups 5–8 as for countries committed to emission reduction (see a) above).
		0.3	0.5	1.5	3.0	5.8	b) For given ρ and α:
		0.5	0.0	0.0	0.0	0.0	Same conclusion for country groups 5–8 as for countries committed to emission reduction (see b) above).
6	-1.0	0.0	0.3	2.7	6.3	13.1	
		0.1	0.0	2.0	4.9	10.4	
		0.3	-0.5	0.5	2.0	4.9	
		0.5	-1.0	-1.0	-1.0	-1.0	
---	-2.0	0.0	-0.7	1.8	5.4	12.2	
		0.1	-1.0	1.0	3.9	9.5	
		0.3	-1.5	-0.5	1.0	3.9	
		0.5	-2.0	-2.0	-2.0	-2.0	
---	-3.0	0.0	-1.7	0.8	4.4	11.4	
		0.1	-2.0	0.0	3.0	8.7	
		0.3	-2.5	-1.5	0.0	3.0	
		0.5	-3.0	-3.0	-3.0	-3.0	
---	-4.0	0.0	-2.7	-0.2	3.5	10.5	
		0.1	-3.0	-0.9	2.1	7.8	
		0.3	-3.5	-2.5	-0.9	2.1	
		0.5	-4.0	-4.0	-4.0	-4.0	
---	-5.0	0.0	-3.7	-1.1	2.6	9.7	
		0.1	-4.0	-1.9	1.1	6.9	
		0.3	-4.5	-3.4	-1.9	1.1	
		0.5	-5.0	-5.0	-5.0	-5.0	
---	-6.0	0.0	-4.7	-2.1	1.7	8.8	
		0.1	-4.9	-2.9	0.2	6.0	
		0.3	-5.5	-4.4	-2.9	0.2	
		0.5	-6.0	-6.0	-6.0	-6.0	
---	-7.0	0.0	-5.7	-3.1	0.7	7.9	
		0.1	-5.9	-3.8	-0.8	5.1	
		0.3	-6.5	-5.4	-3.8	-0.8	
		0.5	-7.0	-7.0	-7.0	-7.0	
7	-8.0	0.0	-6.7	-4.0	-0.2	7.1	
		0.1	-6.9	-4.8	-1.7	4.2	
		0.3	-7.5	-6.4	-4.8	-1.7	
		0.5	-8.0	-8.0	-8.0	-8.0	
---	-9.0	0.0	-7.6	-5.0	-1.1	6.2	
		0.1	-7.9	-5.8	-2.7	3.3	
		0.3	-8.5	-7.4	-5.8	-2.7	
		0.5	-9.0	-9.0	-9.0	-9.0	
8	-10.0	0.0	-8.6	-6.0	-2.0	5.3	
		0.1	-8.9	-6.7	-3.6	2.5	
		0.3	-9.5	-8.4	-6.7	-3.6	
		0.5	-10.0	-10.0	-10.0	-10.0	

Appendix D: Undershooting and Verification Time (Und&VT) Concepts Combined

To distinguish the four cases shown in Figure 4, we introduce δ_{crit}, the critical emission limitation or reduction,

$$\delta_{crit} = \begin{cases} \dfrac{\rho}{1+\rho} & x_2 < x_1 \ \left(\delta_{KP} > 0\right) \\ & \text{for} \\ -\dfrac{\rho}{1-\rho} & x_2 \geq x_1 \ \left(\delta_{KP} \leq 0\right) \end{cases} \tag{D-1}$$

by replacing in Equation (A-6) δ_{KP} by δ_{crit} and ρ_{crit} by ρ (which is now arbitrary).[3] Table D-1 lists δ_{crit} values for selected ρ values that we use to cover a wide range of relative uncertainties.

Tab. D-1: Critical emission limitations or reductions (δ_{crit}) for selected relative uncertainties (ρ) derived via Eq. D-1.

ρ %	$\delta_{KP} >$ δ_{crit} %	δ δ %
0.0		0
2.5	2.44	-
5.0	4.76	-
7.5	6.98	-

[3] Note that we proceeded the other way around in Appendix A, where we determined ρ_{crit} for a given δ_{KP}. Jonas *et al.* (2004: Section 3.4) derived δ_{crit} alternatively via the maximal allowable VT ($t_2 - t_1$), which is given for $x_2/x_1 = 1 - \delta_{crit}$ with $\delta_{crit} = \varepsilon_2/x_1$ in the case $x_2 < x_1$ ($\delta_{KP} > 0$) and $\delta_{crit} = -\varepsilon_2/x_1$ in the case $x_2 \geq x_1$ ($\delta_{KP} \leq 0$). This formulation of δ_{crit} and Eq. (D-1) become equivalent after introducing $\rho = \varepsilon_2/x_2$ to Eq. (D-1).

10.0	9.09	-

Depending on how δ_{crit} and δ_{KP} relate to each other, four cases can be distinguished (see Fig. 4). These are distinguished further in Tab. D-2 in terms of detectability (Cases 1 and 4) versus nondetectability (Cases 2 and 3) and an initial obligatory undershooting U_{Gap} which is introduced (Cases 2–4) to ensure that detectability is given before Annex B countries are permitted to make economic use of potential excess emission reductions. Initial obligatory undershooting according to both Case 3 and Case 4 is special as we demand that emission reductions, not increases, become detectable.[4] This can result in considerable *a priori* emission cuts that countries which succeeded in negotiating emission limitations under the Kyoto Protocol must fulfill. However, to minimize their initial emission reduction burden (U_{Gap}), these countries are subjected to δ_{crit} conditions as under $\delta_{KP} > 0$, not to δ_{crit} conditions as under $\delta_{KP} \leq 0$; δ_{crit} is smaller in absolute terms for emission reduction than for emission limitation (see Tab. D-1). Adjusted δ_{crit} values for $\delta_{KP} \leq 0$ can be achieved via

$$\delta_{crit,adj} = \delta_{crit}\left(\frac{1-\rho}{1+\rho}\right). \tag{D-2}$$

Tab. D-2: The four cases that are distinguished in applying the Und&VT concept according to Fig. 4.

nission Reduction:	**:ase 1**	$\delta_{crit} \leq \delta_{KP}$	Detectable Kyoto target
$_P > 0$	**:ase 2**	$\delta_{crit} > \delta_{KP}$	Nondetectable Kyoto target: An initial obligatory undershooting is applied so that the Annex B countries' emission signals become detectable (before they are permitted to make economic use of excess emission reductions)
nission Limitation: $_P \leq 0$	**:ase 3**	$\delta_{crit} < \delta_{KP}$	s in Case 2, an initial obligatory undershooting is applied unconditionally so that the emission reductions, not increases, of Annex B countries subject to

[4] This measure helps to overcome the dissimilarity of both the VT concept and the CRU concept between countries committed to emission reduction ($\delta_{KP} > 0$) and emission limitation ($\delta_{KP} \leq 0$), which arises if more lenient or stricter Kyoto emission targets would be introduced (see Tab. A-1 and B-1).

			Cases 3 or 4 become detectable
Case 4	$\delta_{crit} \geq \delta_{KP}$		

[a] Detectability in Cases 1 and 4 differ: detectability in Case 1 is also given for any stricter Kyoto emissions target, detectability in Case 4 for any more lenient Kyoto emissions target.

In the following we use Ineq. C-13a as our starting point, which must be re-derived as a consequence of limiting ε_{12} by ε_2 (i.e., formally $x_{t,1} = x_1$ and $\nu = 0$), to obtain the modified emission limitation or reduction targets δ_{mod}, and the undershooting contained in δ_{mod}, for Cases 1–4 as well as the required initial obligatory undershooting U_{Gap} for Cases 2–4:

Case 1: $\delta_{KP} > 0$: $\delta_{crit} \leq \delta_{KP}$. Here, re-deriving Ineq. C-13a results in

$$\frac{x_2}{x_1} \leq (1-\delta_{KP}) \frac{1}{1+(1-2\alpha)\rho} = 1 - \delta_{mod} \;, \qquad \text{(D-3), (C-13c)}$$

where

$$\delta_{\text{mod}} = 1 - (1 - \delta_{\text{KP}}) \frac{1}{1 + (1 - 2\alpha)\rho} = \delta_{\text{KP}} + \text{U} \qquad \text{(D-4), (C-15)}$$

$$\text{U} = (1 - \delta_{\text{KP}}) \frac{(1 - 2\alpha)\rho}{1 + (1 - 2\alpha)\rho} \; . \qquad \text{(D-5)}$$

Case 2: $\delta_{KP} > 0$: $\delta_{crit} > \delta_{KP}$. Here, re-deriving Ineq. C-13a results in

$$\frac{x_2}{x_1} \leq \left(1-\delta_{crit}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = 1-\delta_{mod} \ , \qquad \text{(D-6), (C-13c)}$$

where

$$\delta_{mod} = 1-\left(1-\delta_{crit}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = \delta_{KP} + U \qquad \text{(D-7), (C-15)}$$

$$U = U_{Gap} + \left(1-\delta_{crit}\right)\frac{\left(1-2\alpha\right)\rho}{1+\left(1-2\alpha\right)\rho} \qquad \text{(D-8)}$$

with

$$U_{Gap} = \delta_{crit} - \delta_{KP} \ . \qquad \text{(D-9)}$$

Case 3: $\delta_{KP} \leq 0$: $\delta_{crit,adj} < \delta_{KP}$. Here, re-deriving Ineq. C-13a results in

$$\frac{x_2}{x_1} \leq \left(1+\delta_{crit,adj}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = 1-\delta_{mod} \ , \qquad \text{(D-10), (C-13c)}$$

where

$$\delta_{mod} = 1-\left(1+\delta_{crit,adj}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = \delta_{KP} + U \qquad \text{(D-11), (C-15)}$$

$$U = U_{Gap} + \left(1+\delta_{crit,adj}\right)\frac{\left(1-2\alpha\right)\rho}{1+\left(1-2\alpha\right)\rho} \qquad \text{(D-12)}$$

with

$$U_{Gap} = -\left(\delta_{KP} + \delta_{crit,adj}\right) \ . \qquad \text{(D-13)}$$

Tab. D-3: The Und&VT concept (Eq. C-15 in combination with: Eq. D-5 [Case 1: green fields], Eq. D-8 to D-9 [Case 2: red fields], Eq. D-12 to D-13 [Case 3: red fields], and Eq. D-16 to D-18 [Case 4: orange fields]) applied to Annex B countries. The table lists modified emission limitation or reduction targets δ_{mod} for all Annex B countries, where the '$x_{t,2}$-greater-than-$(1-\delta_{KP})x_{t,1}$' risk α (Case 1), the '$x_{t,2}$-greater-than-$(1-\delta_{crit})x_{t,1}$' risk α (Case 2), the '$x_{t,2}$-greater-than-$(1+\delta_{crit,adj})x_{t,1}$' risk α (Cases 3), and the '$x_{t,2}$-greater-than-$\left(1-\left(\delta_{KP}-2\delta_{crit,adj}\right)\right)x_{t,1}$' risk α (Case 4), respectively, are specified to be 0, 0.1, 0.3 and 0.5. In the last column, we assess the hypothetical situation that the Und&VT concept had been applied prior to/in negotiating the Kyoto Protocol. The Und&VT concept rectifies (notably, Cases 2 and 3, the cases of nondetectability before correction) the unfavourable situation under the Und concept, under which countries complying with a small δ_{KP} exhibit a small δ_{mod} while countries complying with a great δ_{KP} exhibit a great δ_{mod} (cf. Tab. C-1).

Country Group	KP Commit. δ_{KP}[a] %	α = 1	Modified Emission Limitation or Reduction Target δ_{mod} in % for ρ = 2.5 %	7.5 %	15 %	30 %
1a–d	8.0	0.0	10.2	14.4	24.4	40.8
		0.1	9.8	13.2	22.4	38.0
		0.3	8.9	10.7	18.0	31.3
		0.5	8.0	8.0	13.0	23.1
2	7.0	0.0	9.3	13.5	24.4	40.8
		0.1	8.8	12.3	22.4	38.0
		0.3	7.9	9.7	18.0	31.3
		0.5	7.0	7.0	13.0	23.1
3a–c	6.0	0.0	8.3	13.5	24.4	40.8
		0.1	7.8	12.2	22.4	38.0
		0.3	6.9	9.7	18.0	31.3
		0.5	6.0	7.0	13.0	23.1
4	5.0	0.0	7.3	13.5	24.4	40.8
		0.1	6.9	12.2	22.4	38.0
		0.3	5.9	9.7	18.0	31.3
		0.5	5.0	7.0	13.0	23.1
---	4.0	0.0	6.3	13.5	24.4	40.8
		0.1	5.9	12.2	22.4	38.0
		0.3	5.0	9.7	18.0	31.3
		0.5	4.0	7.0	13.0	23.1
---	3.0	0.0	5.4	13.5	24.4	40.8
		0.1	4.9	12.2	22.4	38.0
		0.3	4.0	9.7	18.0	31.3
		0.5	3.0	7.0	13.0	23.1
---	2.0	0.0	4.8	13.5	24.4	40.8
		0.1	4.4	12.2	22.4	38.0
		0.3	3.4	9.7	18.0	31.3
		0.5	2.4	7.0	13.0	23.1
---	1.0	0.0	4.8	13.5	24.4	40.8
		0.1	4.4	12.2	22.4	38.0
		0.3	3.4	9.7	18.0	31.3
		0.5	2.4	7.0	13.0	23.1

If the Und&VT Concept had been applied

Case 1 (green-colored area): $\delta_{crit} \leq \delta_{KP}$: No necessity to introduce U_{Gap}; the δ_{mod} values from Tab. C-1 are still valid.

Case 2 (red-colored area): $\delta_{crit} > \delta_{KP}$: Increase of δ_{KP} by U_{Gap} to reach δ_{crit}, the relevant reference for undershooting. Undershooting only depends on ρ and α and not anymore on δ_{KP} (see Eq. D-8 to D-9 in combination with Eq. C-15). This explains why δ_{mod} appears uniform for a given ρ and α. Thus, the Und&VT concept rectifies the Und concept (cf. Tab. C-1), under which countries complying with a small δ_{KP} exhibit a small δ_{mod} while countries complying with a great δ_{KP} exhibit a great δ_{mod}.

[a] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP}, the normalized change in emissions between t_1 and t_2: $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Table D-3 continued:

5	**0.0**	**0.0**	4.8	13.5	24.4	40.8
		0.1	4.4	12.2	22.4	38.0
		0.3	3.4	9.7	18.0	31.3
		0.5	2.4	7.0	13.0	23.1
6	**-1.0**	**0.0**	4.8	13.5	24.4	40.8
		0.1	4.4	12.2	22.4	38.0
		0.3	3.4	9.7	18.0	31.3
		0.5	2.4	7.0	13.0	23.1
---	-2.0	0.0	4.8	13.5	24.4	40.8
		0.1	4.4	12.2	22.4	38.0
		0.3	3.4	9.7	18.0	31.3
		0.5	2.4	7.0	13.0	23.1
---	-3.0	0.0	4.3	13.5	24.4	40.8
		0.1	3.8	12.2	22.4	38.0
		0.3	2.8	9.7	18.0	31.3
		0.5	1.9	7.0	13.0	23.1
---	-4.0	0.0	3.3	13.5	24.4	40.8
		0.1	2.8	12.2	22.4	38.0
		0.3	1.9	9.7	18.0	31.3
		0.5	0.9	7.0	13.0	23.1
---	-5.0	0.0	2.3	13.5	24.4	40.8
		0.1	1.8	12.2	22.4	38.0
		0.3	0.9	9.7	18.0	31.3
		0.5	-0.1	7.0	13.0	23.1
---	-6.0	0.0	1.3	13.5	24.4	40.8
		0.1	0.9	12.2	22.4	38.0
		0.3	-0.1	9.7	18.0	31.3
		0.5	-1.1	7.0	13.0	23.1
---	-7.0	0.0	0.4	13.4	24.4	40.8
		0.1	-0.1	12.2	22.4	38.0
		0.3	-1.1	9.7	18.0	31.3
		0.5	-2.1	7.0	13.0	23.1
7	**-8.0**	**0.0**	**-0.6**	**12.5**	24.4	40.8
		0.1	**-1.1**	**11.3**	22.4	38.0
		0.3	**-2.1**	**8.7**	18.0	31.3
		0.5	**-3.1**	**6.0**	13.0	23.1
---	-9.0	0.0	-1.6	11.6	24.4	40.8
		0.1	-2.1	10.3	22.4	38.0
		0.3	-3.1	7.7	18.0	31.3
		0.5	-4.1	5.0	13.0	23.1
8	**-10.0**	**0.0**	**-2.6**	**10.7**	24.4	40.8
		0.1	**-3.1**	**9.4**	22.4	38.0
		0.3	**-4.1**	**6.8**	18.0	31.3
		0.5	**-5.1**	**4.0**	13.0	23.1

<u>Case 3 (red-colored area): $\delta_{crit} < \delta_{KP}$:</u> Increase of δ_{KP} by U_{Gap} to reach $-\delta_{crit}$, the relevant reference for undershooting. Undershooting only depends on ρ and α and not anymore on δ_{KP} (see Eq. D-12 to D-13 in combination with Eq. C-15). This explains why δ_{mod} appears uniform for a given ρ and α. Thus, the Und&VT concept rectifies the Und concept (cf. Tab. C-1), under which countries complying with a small δ_{KP} exhibit a small δ_{mod} while countries complying with a great δ_{KP} exhibit a great δ_{mod}.

<u>Case 4 (orange-colored area): $\delta_{crit} \geq \delta_{KP}$:</u> Increase of δ_{KP} by U_{Gap} to reach $\delta_{KP} - 2\delta_{crit}$, the relevant reference for undershooting. In contrast to Case 3 ($\delta_{crit} < \delta_{KP}$) above, undershooting still depends on δ_{KP} (see Eq. D-16 to D-18 in combination with Eq. C-15). This is a consequence of how the undershooting is realized: detectability on the emissions limitation side is used to decrease the reference for undershooting ($\delta_{KP} - 2\delta_{crit}$) on the emissions reduction side.

Case 4: $\delta_{KP} \leq 0$: $\delta_{crit,adj} \geq \delta_{KP}$. Here, re-deriving Ineq. C-13a results in

$$\frac{x_2}{x_1} \leq \left(1+\delta'_{crit}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = 1-\delta_{mod} \ , \qquad \text{(D-14), (C-13c)}$$

where

$$\delta_{mod} = 1-\left(1+\delta'_{crit}\right)\frac{1}{1+\left(1-2\alpha\right)\rho} = \delta_{KP} + U \qquad \text{(D-15), (C-15)}$$

$$U = U_{Gap} + \left(1+\delta'_{crit}\right)\frac{\left(1-2\alpha\right)\rho}{1+\left(1-2\alpha\right)\rho} \qquad \text{(D-16)}$$

with

$$U_{Gap} = -2\delta_{crit,adj} \qquad \text{(D-17)}$$

$$-\delta'_{crit} = \delta_{KP} - 2\delta_{crit,adj} \ . \qquad \text{(D-18)}$$

Treating δ_{KP}, ρ and α as parameters, Eq. C-15 in combination with: Eq. D-5 (Case 1), Eq. D-8 to D-9 (Case 2), Eq. D-12 to D-13 (Case 3), and Eq. D-16 to D-18 (Case 4) allow calculating the modified emission limitation or reduction targets δ_{mod}, and the undershooting U contained in δ_{mod}, for all Annex B countries.

Appendix E: Adjustment of Emissions (GSC #1) Concept

Starting from the 95% confidence interval as the uncertainty range for the best estimate x_2 at t_2 (which complies with the target emissions commitment under the Kyoto Protocol) and assuming a normal distribution, the standard deviation of the distribution equals

$$\rho_{sd} = \frac{\rho}{1.96} x_2 \ . \qquad \text{(E-1)}$$

To follow the approach of Gillenwater, Sussman and Cohen (GSC #1; Gillenwater *et al.*, 2007: Section 2.1), we initially look at the probability (P) that true emissions ($x_{t,2}$) do not exceed (overshoot) estimated emissions by more than the fractional amount p or percentage amount p%. P can be estimated via the standardized cumulative normal distribution F_N:

$$P\left(X_2 \leq x_{u,2}\right) = P\left(Z_2 \leq z_{u,2}\right) = F_N\left(z_{u,2}\right) \ , \qquad \text{(E-2a,b)}$$

where the best estimate x_2 is the mean value to the random variable X_2, z_2 and Z_2 are the standardized equivalents, and $x_{u,2}$ and $z_{u,2}$ are the accepted upper (u) limits. $z_{u,2}$ and $x_{u,2}$ are linked via

$$z_{u,2} = \frac{x_{u,2} - x_2}{\frac{\rho}{1.96} x_2} = \frac{(1+p)x_2 - x_2}{\frac{\rho}{1.96} x_2} . \tag{E-3a,b}$$

For $p = 0.1$ and (e.g.) $\rho = 0.5$ we find $z_{u,2} = 0.3920$ and (with the help of a normal error integral table or using standard statistical software) $F_N(0.3920) = 0.6525$; that is, we can be 65% confident that $x_{t,2} \leq 1.1 x_2$.

Eq. E-3b can also be used to specify $(1+p)$, i.e., the upper limit below which true emissions must fall to satisfy a given probability or required confidence. For instance, for $F_N = 0.9$ we find $z_{u,2} = 1.2816$, thus resulting in

$$1 + p = 1 + z_{u,2}(F_N = 0.9)\frac{\rho}{1.96} \tag{E-4}$$

$(= 1.3269$ for $\rho = 0.5)$.

Let us now consider it acceptable *a priori* that true emissions can exceed (overshoot) the target emissions commitment by some fractional or percentage amount (GSC considered 10% in their study). The relative difference (RelDiff) between an accepted upper emissions limit and the accepted excess of $1.1 x_2$ is then given by

$$\text{RelDiff} = \frac{\left(1 + z_{u,2}(F_N)\frac{\rho}{1.96}\right) - 1.1}{1.1} . \tag{E-5}$$

Reordering Eq. E-5

$$\text{Adj} = 1 + \text{RelDiff} = \frac{1 + z_{u,2}(F_N)\frac{\rho}{1.96}}{1.1} \tag{E-6a,b}$$

allows calculating the adjustment (Adj), which is required to provide a margin of safety to make sure that countries remain in compliance with their commitments. For instance, if a country's emissions estimate is 50% uncertain and we want to be 90% certain its true emissions do not exceed its emissions target by more than 10%, its emissions inventory estimate has to be adjusted upward by 21%. That is, the country would effectively need to reduce its emissions by more than its commitment under the Kyoto Protocol to remain in compliance with commitments.

For the purposes of this study, we now modify the GSC #1 concept. We demand that the accepted emissions excess does not lead to an emissions increase. For instance, in the case that the emissions reduction target of a country is 8%, accepting an emissions excess of 10% could mean that its emissions can even increase. We therefore limit the fractional excess, denoted by p, individually for each Annex B country by ρ_{crit}, its CRU introduced in Appendix A. In the sequel, we distinguish, depending on ρ, between $\text{Adj} \leq$ or > 1 ($\Leftrightarrow$ $1 + z_{u,2}(F_N)\frac{\rho}{1.96} \leq$ or $> 1 + \rho_{crit}$). In contrast to $\text{Adj} > 1$, $\text{Adj} \leq 1$ already reflects favorable compliance conditions that do not require adjusting emissions (the accepted upper emissions limit $1+z_{u,2}(F_N)\frac{\rho}{1.96}$ falls below $1+\rho_{crit}=x_1/x_2$).[5] In the case of emission limitation, we unconditionally set p to 0; we do not accept any excess emissions, i.e., an additional emissions increase. Thus, we always have $\mathrm{Adj}\geq 1$ for $\rho\geq 0$. To summarize, we thus distinguish three cases:

Cases 1 and 2: $\delta_{KP}>0$: $p=\delta_{crit}$. The adjustment Adj is given by

$$\mathrm{Adj}=\begin{cases} 1 & \text{for } 1+z_{u,2}(F_N)\frac{\rho}{1.96}\leq 1+\rho_{crit} \\ \dfrac{1+z_{u,2}(F_N)\frac{\rho}{1.96}}{1+\rho_{crit}} & \phantom{\text{for }} 1+z_{u,2}(F_N)\frac{\rho}{1.96}>1+\rho_{crit} \end{cases} \qquad \text{(E-7,8)}$$

Case 3: $\delta_{KP}\leq 0$: $p=0$. The adjustment Adj is given by

$$\mathrm{Adj}=1+z_{u,2}(F_N)\frac{\rho}{1.96}\ . \qquad \text{(E-9)}$$

Distinguishing between $\delta_{KP}>0$ (emission reduction) and $\delta_{KP}\leq 0$ (emission limitation), and treating F_N and ρ as parameters allows calculating the required Adj for all Annex B countries.

Tab. E-1: The GSC #1 concept (Eq. E-7 [Case 1: green fields; here, the $\mathrm{Adj}<1$ values have not been set to 1], Eq. E-8 [Case 2: orange fields], and Eq. E-9 [Case 3: red fields]) applied to Annex B countries. The table lists the required adjustments Adj for all Annex B countries, where the confidence $(1-\alpha)$ that true emissions do not exceed (overshoot) target emissions by more than $p=\delta_{crit}$ (Cases 1 and 2) and $p=0$ (Case 3) is specified to be 0.9, 0.7 and 0.5. In the last column, we assess the hypothetical situation that the GSC #1 concept had been applied prior to/in negotiating the Kyoto Protocol. Note the potentially unfavorable situation in Case 2, which arises when δ_{KP} varies while ρ and $(1-\alpha)$ are kept constant.

[5] Insert Eq. A-2 into Eq. A-6 to show that $1+\rho_{crit}$ equals x_1/x_2.

Country Group	KP Commit. δ_{KP}[a] %	CRU ρ_{crit} %	Adjustment Factor Adj (absolute) for $1-\alpha=$ 1	$\rho=$ 2.5 %	7.5 %	15 %	30 %	If the GSC #1 Concept had been applied
1a–d	8.0	8.7	1.0					
			0.9	0.935	0.965	1.010	1.100	
			0.7	0.926	0.938	0.957	0.994	
			0.5	0.920	0.920	0.920	0.920	
2	7.0	7.5	1.0					
			0.9	0.945	0.976	1.021	1.112	
			0.7	0.936	0.949	0.967	1.005	
			0.5	0.930	0.930	0.930	0.930	
3a–c	6.0	6.4	1.0					
			0.9	0.955	0.986	1.032	1.124	
			0.7	0.946	0.959	0.978	1.015	
			0.5	0.940	0.940	0.940	0.940	
4	5.0	5.3	1.0					
			0.9	0.966	0.997	1.043	1.136	
			0.7	0.956	0.969	0.988	1.026	
			0.5	0.950	0.950	0.950	0.950	
---	4.0	4.2	1.0					
			0.9	0.976	1.007	1.054	1.148	
			0.7	0.966	0.979	0.999	1.037	
			0.5	0.960	0.960	0.960	0.960	
---	3.0	3.1	1.0					
			0.9	0.986	1.018	1.065	1.160	
			0.7	0.976	0.989	1.009	1.048	
			0.5	0.970	0.970	0.970	0.970	
---	2.0	2.0	1.0					
			0.9	0.996	1.028	1.076	1.172	
			0.7	0.987	1.000	1.019	1.059	
			0.5	0.980	0.980	0.980	0.980	
---	1.0	1.0	1.0					
			0.9	1.006	1.039	1.087	1.184	
			0.7	0.997	1.010	1.030	1.069	
			0.5	0.990	0.990	0.990	0.990	

If the GSC #1 Concept had been applied

Case 1 (green-colored area): $\rho = \delta_{crit}$, Adj ≤ 1:
Favorable compliance conditions; no need for an adjustment (Adj can be set to 1).

Case 2 (orange-colored area): $\rho = \delta_{crit}$, Adj > 1:
The greater ρ, the uncertainty surrounding the emissions inventory estimate, or the greater $(1-\alpha)$, the degree of confidence that is required, the greater the adjustment Adj. However, the smaller δ_{KP}, the greater the adjustment Adj to keep the confidence $(1-\alpha)$ at a constant level (see, e.g., Adj values for $\rho = 15\%$ and $1-\alpha = 0.9$). As a consequence, countries that must comply with a great δ_{KP} (they exhibit a small Adj) are better off than countries that must comply with a small δ_{KP} (they exhibit a great Adj). This is only true if adjustments must be compensated for by additional emission reductions (undershooting mode). However, the opposite is true if this compensation is not compulsory and adjustments are only used to establish a country comparison in terms of confidence (confidence mode). Then countries that must comply with a small δ_{KP} (they exhibit a great Adj) are better off than countries that must comply with a great δ_{KP} (they exhibit a small Adj).

[a] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP}, the normalized change in emissions between t_1 and t_2: $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Table E-1 continued:

5	**0.0**	**0.0**	**1.0**				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
6	**-1.0**	**1.0**	**1.0**				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-2.0	2.0	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-3.0	2.9	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-4.0	3.8	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-5.0	4.8	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-6.0	5.7	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-7.0	6.5	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
7	**-8.0**	**7.4**	**1.0**				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
---	-9.0	8.3	1.0				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000
8	**-10.0**	**9.1**	**1.0**				
			0.9	1.016	1.049	1.098	1.196
			0.7	1.007	1.020	1.040	1.080
			0.5	1.000	1.000	1.000	1.000

<u>Case 3 (red-colored area): p = 0, Adj ≥ 1:</u> The fractional excess emissions factor p is unconditionally set to 0. No excess emissions, i.e., additional emission increases are accepted. As a consequence, all countries exhibit identical adjustments Adj.

Appendix F: Adjustment of Emission Reductions (GSC #2) Concept

The GSC #2 concept (Gillenwater *et al.*, 2007: Section 2.1) is similar to the GSC #1 concept described in Appendix E, but this time the authors begin by looking at uncertainty (95% confidence) in emission reductions, ρ_{12}.[6] Hence, it is considered acceptable *a priori* that true emission reductions fall below (are smaller than) the committed level of reductions by some fractional amount p or percentage amount p%, respectively. The authors considered the case that true emission reductions equal at least 90% of estimated reductions (i.e., $p = 0.1$.)[7] RelDiff, the relative difference between an accepted lower reduction limit and the accepted diminishment in reduction (equal to 0.9 times the estimated reduction), is then given by

$$\mathrm{RelDiff} = \frac{\left(1 - z_{u,2}\left(F_N\right)\frac{\rho_{12}}{1.96}\right) - \left(1 - 0.1\right)}{\left(1 - 0.1\right)} \tag{F-1}$$

(cf. Eq. E-5). However, to facilitate easy comparability with the results of their GSC #1 concept, the authors ask how the emissions estimate for the commitment year/period would have to be adjusted upward to ensure that, given a reasonable level of confidence, true emission reductions do not fall below committed reductions by more than a specified amount ($p = 0.1$). Thus,

$$\mathrm{Adj} = \frac{1 - \left(1 - z_{u,2}\left(F_N\right)\frac{\rho_{12}}{1.96}\right)\delta_{KP}}{1 - \left(1 - 0.1\right)\delta_{KP}} \tag{F-2}$$

provides the adjustment that should be made to the emissions estimates for the commitment year/period in order to compensate for the uncertainty of emission reductions.[8]

For the purposes of this study, we now generalize the GSC #2 concept. First, we use Eq. F-2 also for the case of emission limitation ($\delta_{KP} \le 0$). However, we then set p unconditionally to 0 (i.e., we do not accept an additional emissions increase). Second, we express ρ_{12} with the help of the relative uncertainty ρ, which is assumed symmetrical and constant over time, i.e., $\rho_1 = \rho_2 \, (:= \rho)$. Making use of Eq. C-7b:

$$\rho_{12}\left|x_1 - x_2\right| = \varepsilon_{12} = \left(1 - v\right)\left\{\left(1 - \delta_{KP}\right)\varepsilon_1 + \varepsilon_2\right\} \tag{F-3a,b}$$

with v approximating (first-order approach) the net (effective) correlation between ε_1 and ε_2. Thus:

---

[6] The authors did not explicitly mention the 95% confidence interval which we apply here.

[7] We recall that it is assumed that Annex B countries comply with their emission limitation or reduction commitments under the Kyoto Protocol. Thus, 'estimated' and 'committed reduction' mean the same; they are exchangeable.

[8] Eq. F-2 reproduces Tab. 2 of Gillenwater *et al.* (2007) for ρ_{12} up to 50% (85% confidence interval) with an accuracy of ± 0.01, which is considered sufficient for the purposes of this study.

$$\frac{\varepsilon_{12}}{x_1} = \rho_{12}\frac{|x_1 - x_2|}{x_1} = (1-v)\left\{(1-\delta_{KP})\rho + \frac{x_2}{x_1}\rho\right\} . \qquad \text{(F-4a,b)}$$

Rewriting Eq. F-4 with the help of Eq. A-2:

$$\rho_{12}|\delta_{KP}| = (1-v)\{(1-\delta_{KP})\rho + (1-\delta_{KP})\rho\} . \qquad \text{(F-5)}$$

Thus, with the help of Eq. A-6:

$$\rho_{12} = 2(1-v)\frac{(1-\delta_{KP})}{|\delta_{KP}|}\rho = 2(1-v)\frac{\rho}{\rho_{crit}} \quad (\rho_{crit} \neq 0). \qquad \text{(F-6)}$$

To conclude, we distinguish four cases:

<u>*Cases 1 and 2: $\delta_{KP} > 0$: $p = 0.1$.*</u> The adjustment Adj is given by

$$\mathrm{Adj} = \begin{cases} 1 & \text{for} \quad 2(1-v)\dfrac{z_{u,2}(F_N)\rho}{1.96\rho_{crit}} \leq 0.1 \\[2ex] \dfrac{1-\left(1-2(1-v)\dfrac{z_{u,2}(F_N)\rho}{1.96\rho_{crit}}\right)\delta_{KP}}{1-(1-0.1)\delta_{KP}} & \qquad 2(1-v)\dfrac{z_{u,2}(F_N)\rho}{1.96\rho_{crit}} > 0.1 \end{cases} \qquad \text{(F-7,8)}$$

<u>*Case 3: $\delta_{KP} = 0$: $p = 0$.*</u> The adjustment Adj is given by

$$\mathrm{Adj} = 1 \qquad \text{(F-9)}$$

<u>*Case 4: $\delta_{KP} < 0$: $p = 0$.*</u> The adjustment Adj is given by

$$\mathrm{Adj} = \frac{1-\left(1+2(1-v)\dfrac{z_{u,2}(F_N)\rho}{1.96\rho_{crit}}\right)\delta_{KP}}{1-\delta_{KP}} . \qquad \text{(F-10)}$$

Distinguishing between $\delta_{KP} > 0$ (emission reduction) and $\delta_{KP} < 0$ (emission limitation), and treating F_N, ρ and v as parameters allows calculating the required Adj for all Annex B countries.

Tab. F-1: The GSC #2 concept (Eq. F-7 [Case 1: green fields; here, the $\mathrm{Adj} < 1$ values have not been set to 1], Eq. F-8 [Case 2: orange fields], and Eq. F-9 and F-10 [Cases 3 and 4: red fields]) applied to Annex B countries. The table lists the required adjustments Adj for all Annex B countries, where the confidence $1-\alpha$ that true emission reductions (increases) will not fall below (above) the committed level of reductions (increases) by more than $p = 0.1$ (Cases 1 and 2) and $p = 0$ (Cases 3 and 4) is specified to be 0.9, 0.7 and 0.5. The correlation v is 0.75 (as in App. C). In the last column, we assess the hypothetical situation that the GSC #2 concept had been applied prior to/in negotiating the Kyoto Protocol. Note the potentially unfavorable situation in Case 2, which arises when δ_{KP} varies while ρ and $(1-\alpha)$ are kept constant. However, for the given set of parameters (notably, $p = 0.1$ and $v = 0.75$) the span between the smallest and greatest Adj values is negligible.

Country Group	KP Commit. δ_{KP}[a] %	CRU δ_{cru} %	Adjustment Factor Adj (absolute) for $1-\alpha=$ 1	$\rho=$ 2.5 %	7.5 %	15 %	30 %
1a–d	8.0	8.7	1.0				
			0.9	0.999	1.016	1.040	1.089
			0.7	0.995	1.001	1.011	1.031
			0.5	0.991	0.991	0.991	0.991
2	7.0	7.5	1.0				
			0.9	1.001	1.017	1.041	1.090
			0.7	0.996	1.002	1.012	1.032
			0.5	0.993	0.993	0.993	0.993
3a–c	6.0	6.4	1.0				
			0.9	1.002	1.018	1.042	1.091
			0.7	0.997	1.004	1.014	1.034
			0.5	0.994	0.994	0.994	0.994
4	5.0	5.3	1.0				
			0.9	1.003	1.019	1.044	1.092
			0.7	0.998	1.005	1.015	1.035
			0.5	0.995	0.995	0.995	0.995
---	4.0	4.2	1.0				
			0.9	1.004	1.020	1.045	1.094
			0.7	0.999	1.006	1.016	1.036
			0.5	0.996	0.996	0.996	0.996
---	3.0	3.1	1.0				
			0.9	1.005	1.021	1.046	1.095
			0.7	1.000	1.007	1.017	1.037
			0.5	0.997	0.997	0.997	0.997
---	2.0	2.0	1.0				
			0.9	1.006	1.022	1.047	1.096
			0.7	1.001	1.008	1.018	1.038
			0.5	0.998	0.998	0.998	0.998
---	1.0	1.0	1.0				
			0.9	1.007	1.023	1.048	1.097
			0.7	1.002	1.009	1.019	1.039
			0.5	0.999	0.999	0.999	0.999

If the GSC #2 Concept had been applied

Case 1 (green-colored area): $p = 0.1$, $Adj \leq 1$:
Favorable compliance conditions; no need for an adjustment (Adj can be set to 1).

Case 2 (orange-colored area): $p = \delta_{crit}$, $Adj > 1$:
The higher ρ, the level of uncertainty surrounding the emissions inventory estimate, or the greater $(1-\alpha)$, the degree of confidence that is required, the greater the adjustment Adj. However, the smaller δ_{KP}, the greater the adjustment Adj to keep the confidence $(1-\alpha)$ at a constant level (see, e.g., Adj values for $\rho = 15\%$ and $1-\alpha = 0.9$). As a consequence, countries that must comply with a great δ_{KP} (they exhibit a small Adj) are better off than countries that must comply with a small δ_{KP} (they exhibit a great Adj). This is only true if adjustments must be compensated for by additional emission reductions (undershooting mode). But it must be mentioned that, for the given set of parameters (notably, $p = 0.1$ and $v = 0.75$), the span between smallest and greatest Adj values is negligible. However, the opposite is true if this compensation is not compulsory and adjustments are only used to establish a country comparison in terms of confidence (confidence mode). Then countries that must comply with a small δ_{KP} (they exhibit a great Adj) are better off than countries that must comply with a great δ_{KP} (they exhibit a small Adj).

[a] The countries' emission limitation and reduction commitments under the Kyoto Protocol are expressed with the help of δ_{KP}, the normalized change in emissions between t_1 and t_2: $\delta_{KP} > 0$ — emission reduction; $\delta_{KP} \leq 0$ — emission limitation.

Table F-1 continued:

5	**0.0**	**0.0**	**1.0**				
			0.9	1.000	1.000	1.000	1.000
			0.7	1.000	1.000	1.000	1.000
			0.5	1.000	1.000	1.000	1.000
6	**-1.0**	1.0	**1.0**				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-2.0	2.0	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-3.0	2.9	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-4.0	3.8	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-5.0	4.8	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-6.0	5.7	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-7.0	6.5	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
7	-8.0	7.4	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
---	-9.0	8.3	1.0				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000
8	**-10.0**	**9.1**	**1.0**				
			0.9	1.008	1.025	1.049	1.098
			0.7	1.003	1.010	1.020	1.040
			0.5	1.000	1.000	1.000	1.000

<u>Cases 3 and 4 (red-colored area): p = 0, Adj ≥ 1:</u>
The fractional factor p to allow additional diminishment in emission reductions is unconditionally set to 0. No excess emissions, i.e., additional emission increases, are accepted. As a consequence, all countries exhibit identical adjustments Adj.

References

EEA (2007): Annual European Community Greenhouse Gas Inventory 1990–2005 and Inventory Report 2007. Technical Report No. 7, European Environment Agency (EEA), Copenhagen, Denmark. Available at: http://reports.eea.europa.eu/technical_report_2007_7/en/.

Gillenwater, M., F. Sussman and J. Cohen, 2007: Practical policy applications of uncertainty analysis for national greenhouse gas inventories. In: *Accounting for Climate Change. Uncertainty in Greenhouse Gas Inventories – Verification, Compliance, and Trading* [D. Lieberman, M. Jonas, Z. Nahorski and S. Nilsson (eds.)]. Springer, Dordrecht, The Netherlands, pp. 159; Hardbound edition of *Water, Air & Soil Pollution: Focus*, 7(4–5), 31–54, doi: 10.1007/s11267-006-9118-2.

Gusti, M. and W. Jęda, 2002: Carbon Management: A New Dimension of Future Carbon Research. Interim Report IR-02-006, International Institute for Applied Systems Analysis, Laxenburg, Austria, pp. 100. Available at: http://www.iiasa.ac.at/Publications/Documents/IR-02-006.pdf.

Jonas, M., S. Nilsson, M. Obersteiner, M. Gluck and Y. Ermoliev: 1999: Verification Times Underlying the Kyoto Protocol: Global Benchmark Calculations. Interim Report IR-99-062, International Institute for Applied Systems Analysis, Laxenburg, Austria, 43 pp. Available at: http://www.iiasa.ac.at/Publications/Documents/IR-99-062.pdf.

Jonas, M., S. Nilsson, R. Bun, V. Dachuk, M. Gusti, J. Horabik, W. Jęda and Z. Nahorski, 2004: Preparatory Signal Detection for Annex I Countries under the Kyoto Protocol—A Lesson for the Post-Kyoto Policy Process. Interim Report IR-04-024, International Institute for Applied Systems Analysis, Laxenburg, Austria, pp. 91. Available at: http://www.iiasa.ac.at/Publications/Documents/IR-04-024.pdf.

Jonas, M., M. Gusti, W. Jęda, Z. Nahorski and S. Nilsson, 2007: Comparison of preparatory signal detection techniques for consideration in the (post-) Kyoto policy process. In: *Proceedings*. 2nd International Workshop on Uncertainty in Greenhouse Gas Inventories, 27–28 September, Laxenburg, Austria, 107–134. Available at: http://www.ibspan.waw.pl/ghg2007/GHG-total.pdf.

Nahorski, Z., W. Jęda and M. Jonas, 2003: Coping with Uncertainty in Verification of the Kyoto Obligations. In: J. Studzinski, L. Drelichowski and O. Hryniewicz (eds.) *Zastosowanai Informatyki i Analizy Systemowej w Zarzadzaniu (Application of Computer Science and System Analysis in Management)*, Systems Research Institute of the Polish Academy of Sciences, Warsaw, Poland, 305–317.

Nahorski, Z., J. Horabik and M. Jonas, 2007: Compliance and emissions trading under the Kyoto Protocol: Rules for uncertain inventories. In: *Accounting for Climate Change. Uncertainty in Greenhouse Gas Inventories – Verification, Compliance, and Trading* [D. Lieberman, M. Jonas, Z. Nahorski and S. Nilsson (eds.)]. Springer, Dordrecht, The Netherlands, pp. 159; Hardbound edition of *Water, Air & Soil Pollution: Focus*, 7(4–5), 119–138, doi: 10.1007/s11267-006-9112-8.

Acronyms and Nomenclature

Adj	adjustment
CRU	critical relative uncertainty
Eq.	equation
GSC	Gillenwater, Sussman and Cohen
IIASA	International Institute for Applied Systems Analysis
Ineq.	inequality
KP	Kyoto Protocol
LULUCF	land use, land-use change, and forestry
P	probability
RelDiff	relative difference
Und	undershooting
Und&VT	undershooting and verification time
VT	verification time
adj	adjusted
corr	correlation
crit	critical
mod	modified
t	true
u	upper