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Yield surface of material subjected to combined cyclic loadings

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IN THIS PAPER the results of investigations on the mechanical properties of the material subjected to combined multiple loadings are presented. Experiments are performed on the plane specimens cut off from the sheet of aluminium alloy for plastic forming PA2N (ALMg2) in a soft state. Initial combined cyclic loadings are realized by the uniaxial extension in two mutually proportional directions. Experiments are performed for three different values of loading. Analysis of the experimental data is carried out by comparing yield surfaces obtained after successive half-cycles of loadings. It was confirmed that with the increase of the number of half-cycles of loading, the differences between compared surfaces disappear. The growth of loading has a small influence on the number of cycles necessary for the secondary yield surfaces to stabilize. Theoretical analysis of the results was performed on the basis of the kinematic work-hardening hypothesis.

W pracy przedstawiono wyniki badań własności mechanicznych materiału poddanego złożonym wielokrotnym obciążeniom. Doświadczenia przeprowadzono na płaskich próbkach wycinanych z blachy, ze stopu aluminium do obróbki plastycznej PA2N (ALMg2) w stanie miękkim. Wstępne złożone obciążenia cykliczne realizowano metodą jednoosiowego rozciągania w dwóch wzajemnie prostopadłych kierunkach. Doświadczenia przeprowadzono dla trzech różnych wartości obciążenia. Analizę wyników doświadczalnych przeprowadzono przez porównanie powierzchni płynięcia, otrzymanych po kolejnych półcyklach obciążenia. Stwierdzono, że w miarę wzrostu liczby półcykli obciążenia, różnice między porównywanymi powierzchniami zacierają się. Wzrost wielkości obciążenia ma niewielki wpływ na liczbę cykli koniecznych do ustabilizowania wtórnych powierzchni plastyczności. Teoretyczną analizę wyników badań przeprowadzono na podstawie hipotezy kinematycznego wzmocnienia.

В работе представлены результаты исследований механических свойств материала подвергнутого сложным многократным нагрузкам. Эксперименты проведены на плоских образцах, вырезанных из жести из сплава алюминия, для пластической обработки PA2N (AlMg2) в мягком состоянии. Предварительные сложные циклические нагрузки реализованы методом одноосного растяжения в двух взаимно перпендикулярных направлениях. Эксперименты проведены для трех разных значений нагрузки. Анализ экспериментальных результатов проведен путем сравнения поверхностей течения полученных после очередных полуциклах нагрузки. Констатировано, что по мере роста количества полуциклов нагрузки разницы между сравниваемыми поверхностями исчезают. Рост величины нагрузки имеет небольшое влияние на количество циклов необходимых для стабилизации вторичных поверхностей пластичности. Теоретический анализ результатов исследований проведен опираясь на гипотезу кинематического упрочнения.

1. Introduction

THE LOADING exceeding elastic limit repeating cyclicly with small frequency is one of the characteristic features of the work of many modern structures. Therefore, apart from the classic investigations of the fatique characteristics, much attention is paid to problems of material resistance at the so called low-cyclic fatique. In this kind of fatique the structure element, generally under the static kind of loading, gets fracture after some thousands, hundreds or a significantly smaller number of loading cycles.

In the analysis of the work of structure the consideration of changes of the mechanical properties occurring in the material after successive cycles of loading plays an essential role. As MOSKVITIN [1] has shown, the consideration of these changes may give the correction in the magnitude of stresses in the limits 50%–100% in comparison with the value determined for a single loading at the inhomogeneous state of stress.

Consideration of the changes in mechanical properties of the material after successive cyclically repetitive loading has also an essential significance for the determination of the shake down loadings of statically indetermined elastic-plastic systems, when the bearing capacity of the structure is determined by the magnitude of the variable loading at which, after a small number of cycles, the plastic deformations become evanescent.

Studies on the elastic-plastic deformation at cyclic loadings were carried out for several years. The first experiments were performed with the aid of relatively simple equipment recently applied for small investigation programs only. Expansion of the range of studies and growth of the requirements concerning the exactness of the results has caused continuous change in the technique of the experiments performed. Contemporary methods are characterized by full automation of the loading processes and strain registration [13].

The majority of studies of elastic-plastic deformations under cyclic loadings is performed in conditions of uniaxial loading. In these examinations the repetitive cyclic loadings realized in extension and compression or oscillatory torsion of the sample are typical

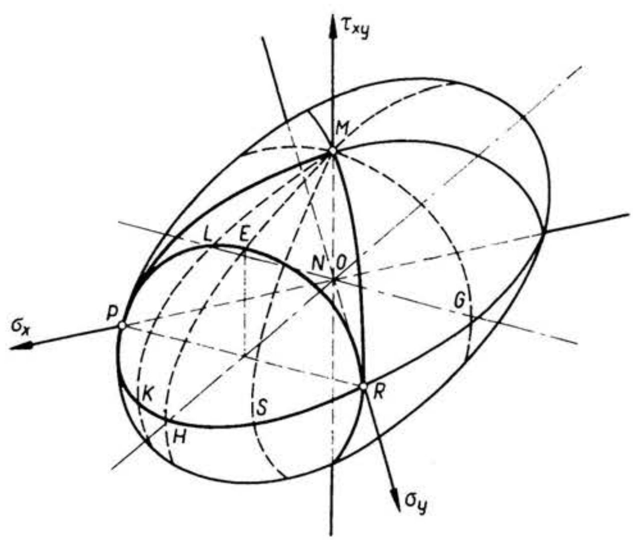


FIG. 1.

schemes of experimentes [1, 9, 10, 14, 18]. In the papers [1, 9, 10] certain analytical relations describing the curves of the cyclic loading in uniaxial conditions were presented.

Studies concerning the influence of the plastic strains at cyclic loadings on the yield surface still make up a small group [2, 5–8, 15, 16]. In the paper [5] the results of the influence of the cyclic torsion on the secondary yield surface of brass are given, while in the papers [6, 7] this surface was investigated after initial cyclic loading, excited in the brass tube samples through the combination of the axial force and internal pressure. In the other two works [8, 15] the yield surfaces of the steel samples were evaluated after previous cyclic torsion of samples. Finally, in the paper [16] the influence of the two-axial cyclic loadings on the shape of the yield surface of the aluminium alloy was examined. Theoretical analysis for this kind of loadings may be found in the papers [11, 17].

In the presented paper the results of the study of changes of the yield surface in a plane state of stress under the influence of the multiple combined cyclic loadings are presented. This is the first work in which the change of the *PLENR* ellipse on the three-axial ellipsoid (Fig. 1) subjected to the experimental verification at cyclic loadings, was investigated.

2. Program of examinations

Examination of the yield surface of the material was performed on the basis of the experimental verification of the PLENR ellipse lying on the three-axial ellipsoid (Fig. 1). This ellipsoid, as it is well known, constitutes the geometrical image of the HUBER-MISES-HENCKY yield condition for the plane state of stress [4]. The particular points of the *PLENR* ellipse are determined from the uniaxial extension tests of plane samples cut out from the sheet in different directions with respect to the x-axis [2, 3]. The table in Fig. 2 shows the

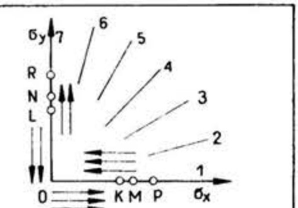
Loading path	Number of series and manner of initial loading									
OKOLO	00	01 OKO	02 OKOLO	03 OKOLO-OKO	04 OKOLO-OKOLO	05 OKOLO-OKOLO-OKO	 			
OMONO	—	11 OMO	12 OMONO	13 OMONO-OMO	14 OMONO-OMONO-	15 OMONO-OMONO-OMO	—	17 OMONO-OMONO-OMONO-OMO	$\sigma_K = \sigma_L = 10$ [kp mm^{-2}] $\sigma_M = \sigma_N = 12$ — " — $\sigma_P = \sigma_R = 15$ — " —	
OPORO	—	21 OPO	—	23 OPORO-OPO	—	25 OPORO-OPORO-OPO	—	27 OPORO-OPORO-OPORO-OPO	—	29 OPORO-OPORO-OPORO-OPORO-OPO

FIG. 2.

schemes of cyclic loadings in two mutually perpendicular directions making it possible to examine the influence of these loadings on the behaviour of the yield surfaces of the material in the space of stresses. Each of the big samples was initially loaded differently according to the programs described in the particular windows of the table.

Experiments were performed for three different values of loading. They allowed for an analysis of the influence of the magnitude of the cyclically repeating loadings on the behaviour of the yield surface of the material.

Before loading on the surfaces of the big samples the scratches were traced in two mutually perpendicular directions [2]. From the measurement on the microscope of thedis tance between scratches before and after successive loading the strain components of the material are evaluated in two directions. The third component of strain was determined from the measurement of the depth of the samples. After initial preparation, from each big sample the seven small samples are cut out under the angles: $\alpha = 0°, 15°, 30°, 45°, 60°, 75°$ and $90°$ with respect to the fixed x direction. From the uniaxial extension test of these small samples the point of secondary yield surfaces, corresponding to the points of the *PLENR* ellipse for the material in the initial state [3], were evaluated. The strains of the small specimens were measured by means of the electric resistance gauges sticked to the surface of the specimens.

3. Experimental results

On the basis of the uniaxial extension tests performed on small specimens the stress-strain diagrams were evaluated. As an example, in Fig. 3 the set of the extension curves for the small specimens cut out from the deformed material are shown. The series of points

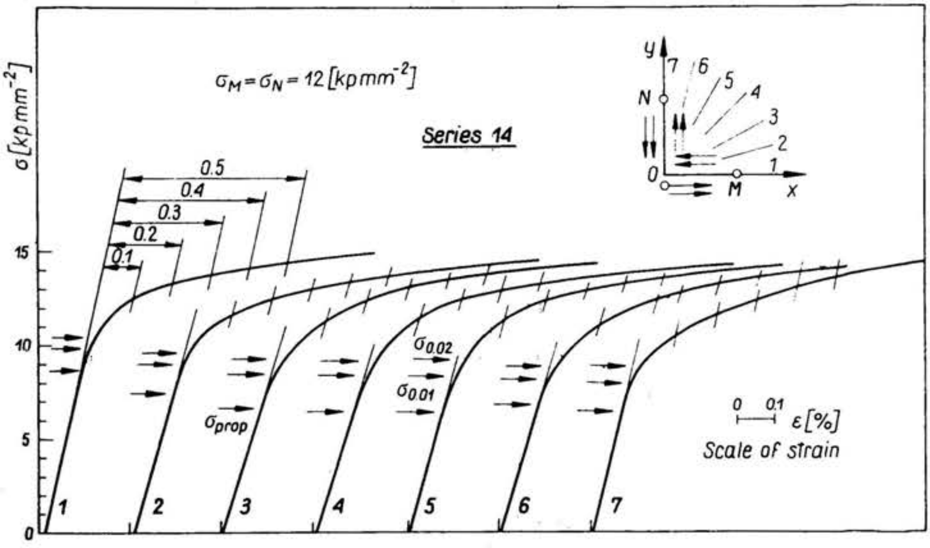


FIG. 3.

corresponding to the conventional plasticity limits are demonstrated on the diagrams. σ_{prop} denotes the proportionality limit corresponding to the origin of the inclination of the diagram from the rectilinear segment of the elastic region. The limits denoted by $\sigma_{0,01}, \ldots, \sigma_{0,5}$ correspond to the stresses responsible for the plastic strains $\varepsilon_p = 0{,}01\%, \ldots, \ldots, 0{,}5\%$ in the material. Knowing the values of the conventional plasticity limits the stress state components σ_x, σ_y and τ_{xy} were computed. These components determine the situation of the experimental points lying on the correspondingly defined yield surfaces of the deformed material in the plane state of stress. In Figs. 4–7 the diagrams of the yield surface for one of the three series of experiments are shown. They are presented in two projections.

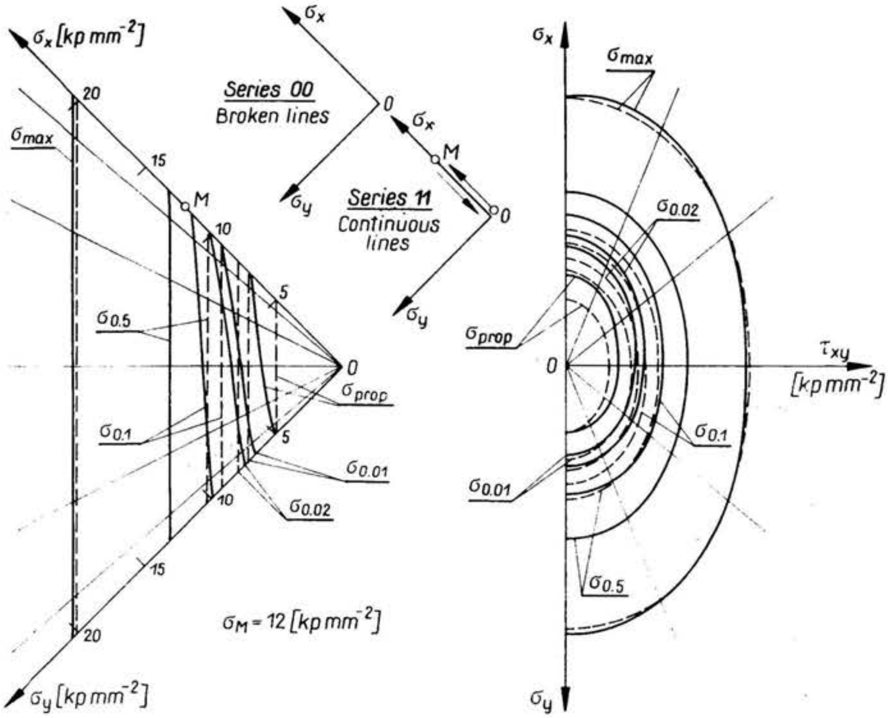

σx [kp mm-2]
σmax
20
15
M
10
5
0
σ0.5
σ0.1
σprop
σ0.01
σ0.02
σy [kp mm-2]
Series 00
Broken lines
Series 11
Continuous lines
σx
σy
τxy [kp mm-2]
σM = 12 [kp mm-2]

FIG. 4.

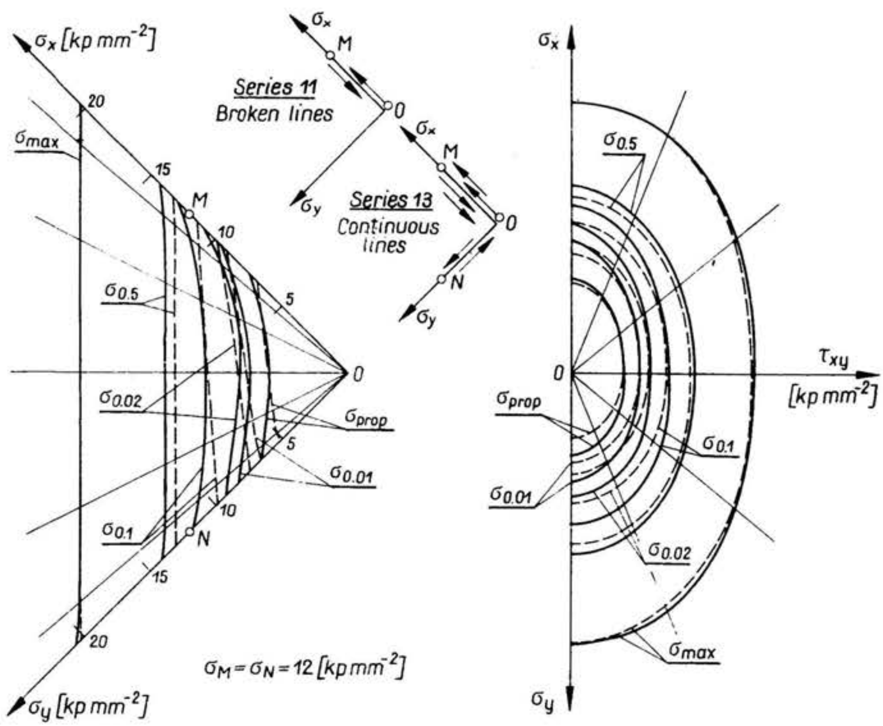

σx [kp mm-2]
σmax
20
15
M
10
5
0
N
σ0.5
σ0.02
σprop
σ0.01
σ0.1
σy [kp mm-2]
Series 11
Broken lines
Series 13
Continuous lines
σx
σy
τxy [kp mm-2]
σM = σN = 12 [kp mm-2]

FIG. 5.

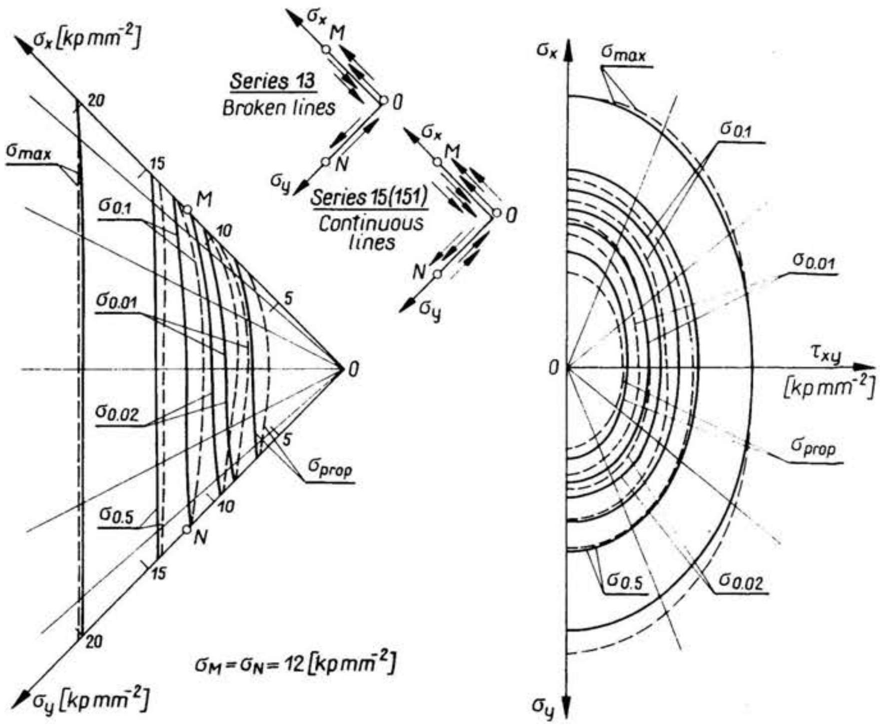

σx [kp mm-2]
Series 13
Broken lines
Series 15(151)
Continuous lines
σmax
σ0.1
σ0.01
σ0.02
σ0.5
σprop
σM = σN = 12 [kp mm-2]
σy [kp mm-2]
τxy [kp mm-2]
M
N
0
5
10
15
20

FIG. 6.

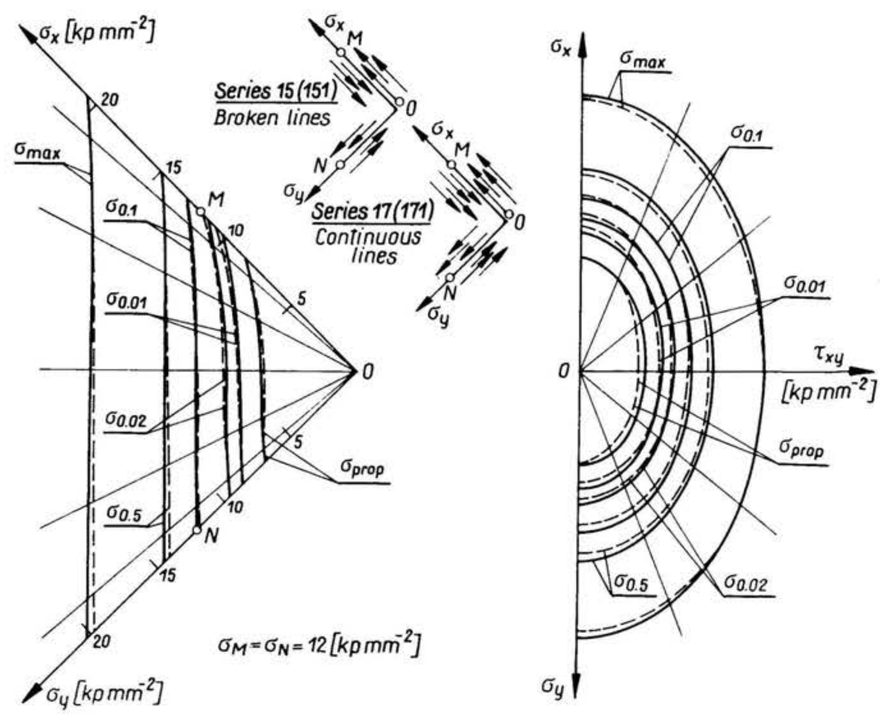

σx [kp mm-2]
Series 15 (151)
Broken lines
Series 17 (171)
Continuous lines
σmax
σ0.1
σ0.01
σ0.02
σ0.5
σprop
σM = σN = 12 [kp mm-2]
σy [kp mm-2]
τxy [kp mm-2]
M
N
0
5
10
15
20

FIG. 7.

On the left-hand side of the figures the projections are placed on the $\sigma_x \sigma_y$ plane, on the right-hand side the projections on the *MOG* plane going through the axis τ_{xy} perpendicularly to the longer axis of the ellipsoid are presented (Fig. 1).

3.1. Influence of combined cyclic loadings on the yield surface of material

Analysis of the experimental investigations was carried out by comparing the yield surfaces obtained after successive cycles of loading. In Fig. 4 the yield surfaces of the initial material are compared with the yield surfaces of the material loaded along the x-axis. In Fig. 5 the surfaces after single loading of the material along the x-axis and the surfaces of the material after triple loading along the path *OMONOMO* are shown. In Fig. 6 the surfaces after three and five initial loadings are given. Finally, in Fig. 7 the surfaces after five and seven half-cycles of loading of the material are compared.

From the figures above it is clearly seen that with an increase of the number of cycles of the initial loadings of the material the differences between the surfaces compared vanish. One may assume that for the case considered the yield surfaces become fixed after seven half-cycles of loading; further repetition of loadings has no essential influence on their course.

3.2. Influence of the magnitude of cyclic loading on the yield surfaces of the material

The experimental examinations discussed above were performed for three different values of loading (Fig. 2). In Figs. 8 and 9 the yield surfaces for two remaining loading

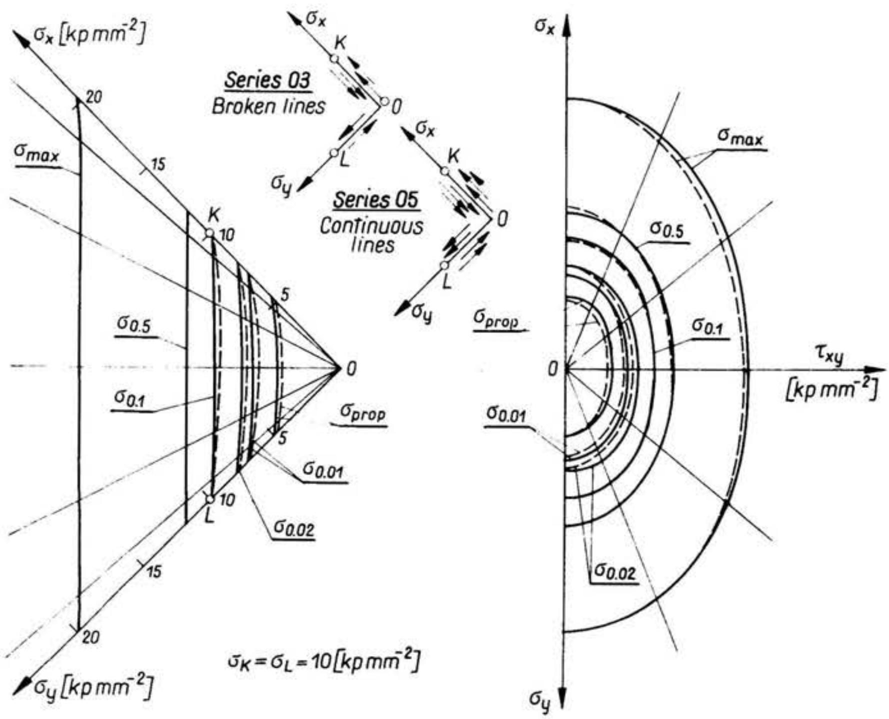


FIG. 8.

parameters are shown. Similarly, as in the first case here also the differences between compared surfaces evanesce with an increase of the number of cycles. For $\sigma_K = \sigma_L = 10\ \mathrm{kp/mm^2}$ it happened after 5 half-cycles of loading, for $\sigma_P = \sigma_R = 15\ \mathrm{kp/mm^2}$ — after seven.

On the basis of the results obtained it was confirmed that the growth of the magnitude of loading causes an increase of the number of half-cycles necessary for the evanescence

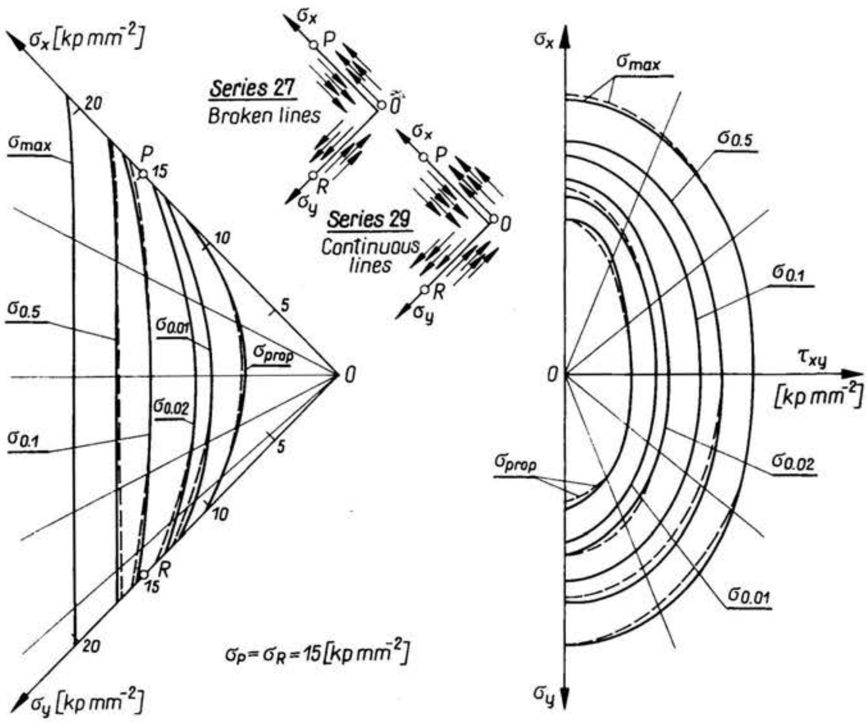


FIG. 9.

of the differences between compared yield surfaces. It should be stressed out, however, that the growth is small and is not in a form of the linear relation.

4. Analysis of the state of strain

In Fig. 10 a diagram of the extensional stresses versus the strain intensity function ε_i during initial loading was shown schematically. It can be seen that the strain has a maximal value during the first loading. At the successive loadings the strain gradually decreases so that after some half-cycles it stabilizes for a certain determined magnitude. Stabilization of the strain magnitudes occurs following the same number of the loading half-cycles after which the yield surfaces almost do not change (Fig. 7). Similar results were also obtained for the two remaining values of the cyclic loading.

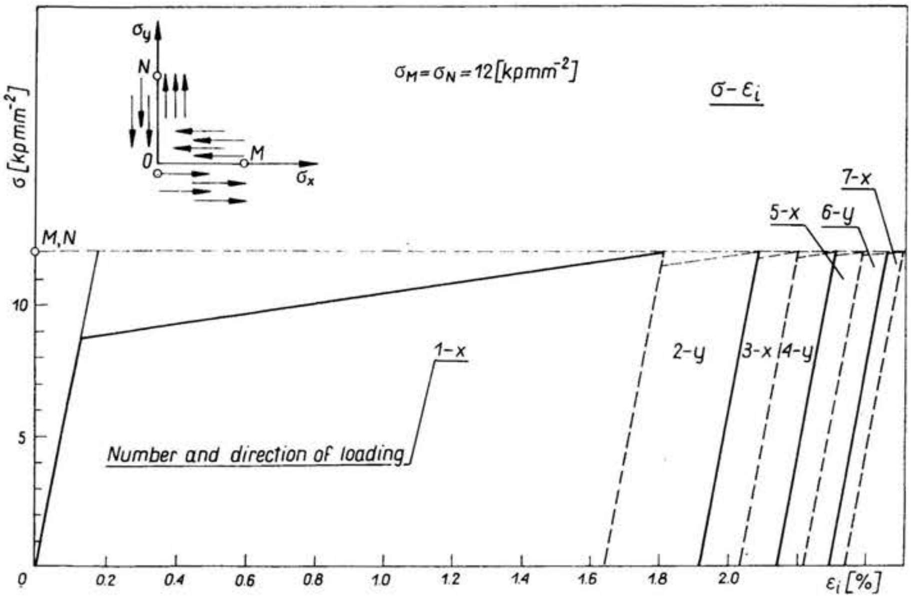


FIG. 10.

5. Theoretical analysis of the problem on the basis of the kinematic work-hardening hypothesis

In the present paper an attempt was made to describe theoretically the behaviour of the yield surface of the material subjected to repetitive cyclic loadings on the basis of the kinematic work-hardening hypothesis. In Fig. 11 the travel of the yield surfaces during the cyclic loadings along the path *OMONO* was shown. In the considerations presented the proportionality limit σ_{prop} was assumed. Points M and N correspond, similarly as in the experiments, to the extensional stress equal to 12 kp/mm². From the theoretical description it results that the stabilization of the yield surface has already occurred practically during the fifth half-cycle. In the case of a loading magnitude 10 kp/mm² the stabilization of the surface occurred after 5 cycles. But for loadings $\sigma_P = \sigma_R = 15$ kp/mm² stabilization took place after three cycles (Fig. 12). It is worth mentioning that the yield surfaces were considered as stabilized when the transition of their centers did not exceed 5% of the initial plasticity limit. Comparing the theoretical results obtained with the experimental data, from which it was obtained that seven, five and seven halt-cycles are necessary for the evanescence of the differences between compared surfaces, one should confirm that although for the loadings $\sigma_K = \sigma_L = 10$ kp/mm² and $\sigma_M = \sigma_N = 12$ kp/mm² the results were in agreement for the larger loadings $\sigma_P = \sigma_R = 15$ kp/mm²; however, the discrepancy between a theory and experimental results is significant (three half-cycles from a theory, seven half-cycles from the experiment) [2].

Much better results were obtained using the analogous theoretical analysis performed on the basis of the conventional plasticity limit $\sigma_{0,1}$ (Fig. 13). For loading *OMONO*, similarly as for two the remaining loadings, the theoretical prediction of the yield surface stabilization after 7 half-cycles is in good agreement with experimental data.

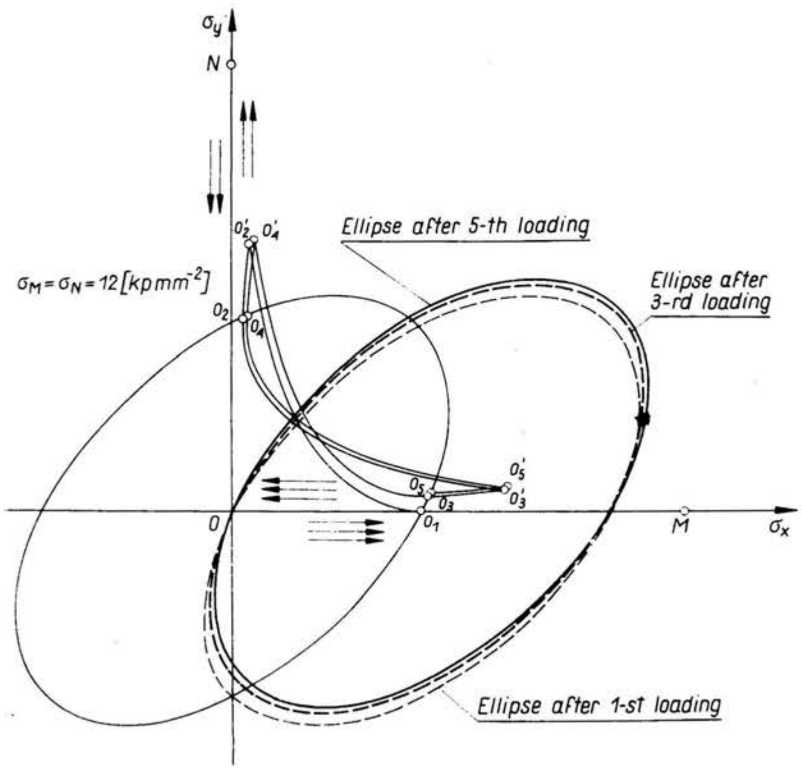

σy
N
O'2
O'4
σM = σN = 12 [kp mm^-2]
O2
O4
Ellipse after 5-th loading
Ellipse after 3-rd loading
O5
O'5
O3
O'3
O
O1
M
σx
Ellipse after 1-st loading

FIG. 11.

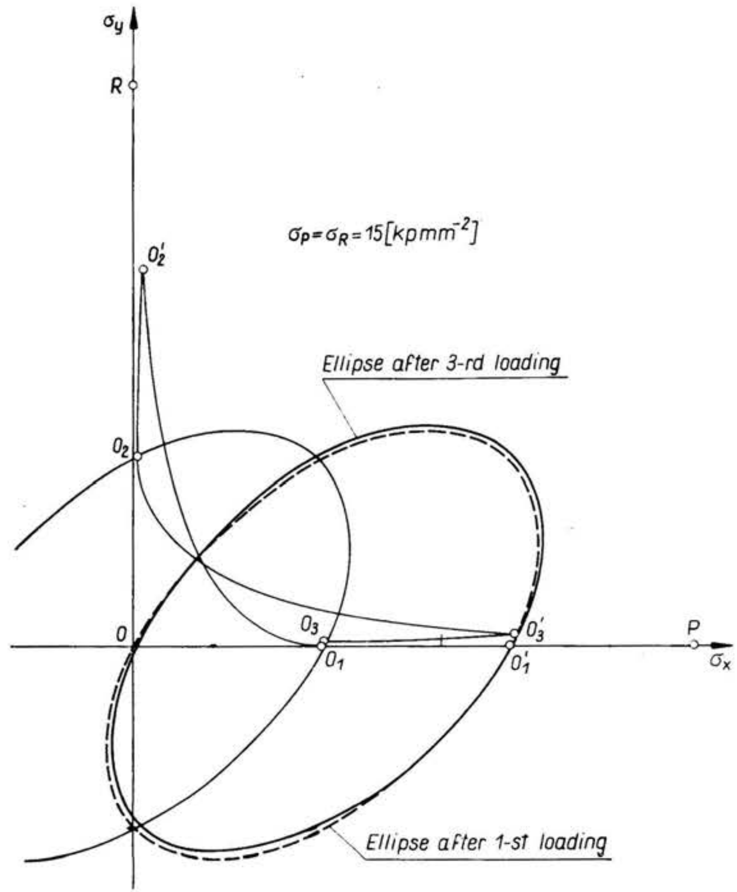
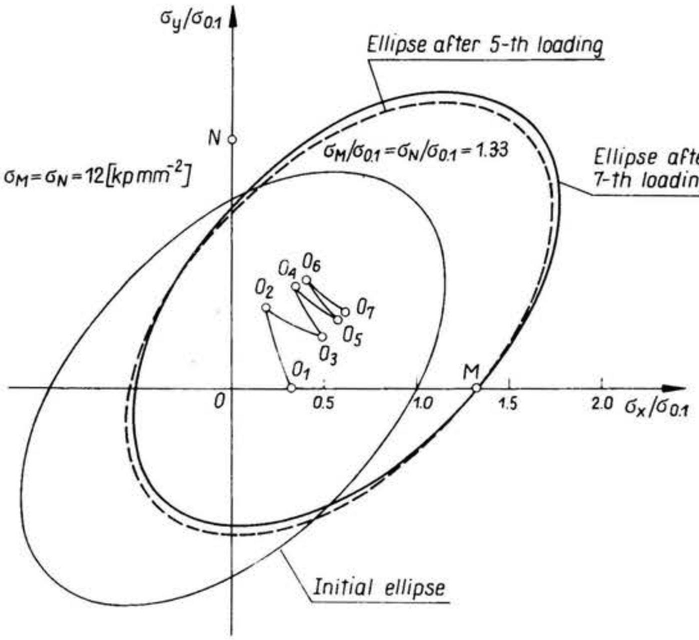
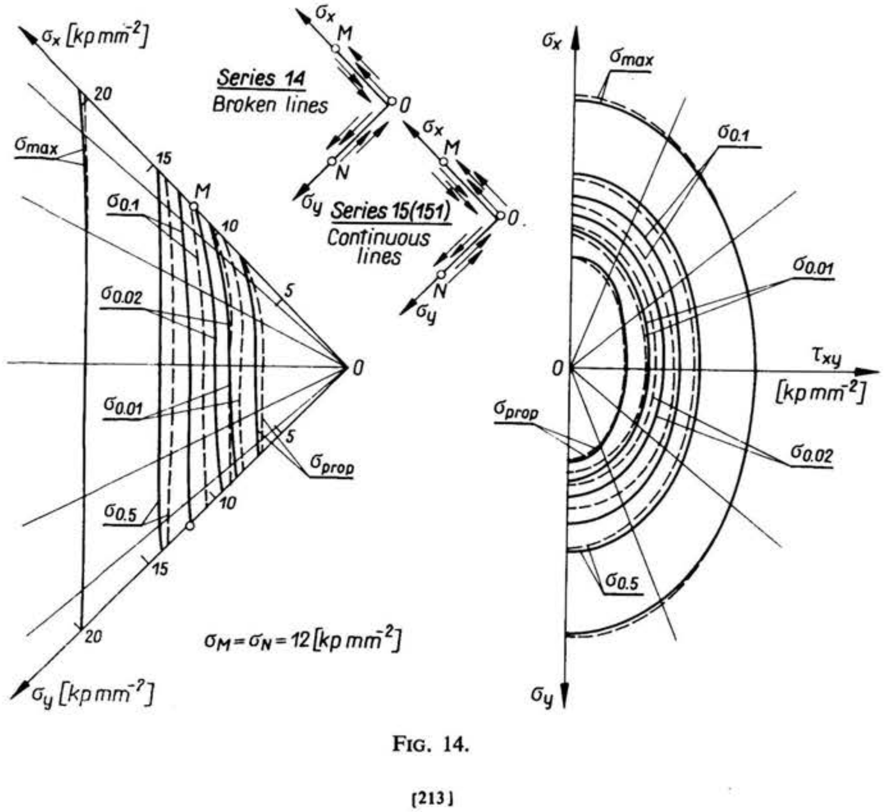

σy
R
σP = σR = 15 [kp mm^-2]
O'2
O2
Ellipse after 3-rd loading
O
O3
O'3
P
O1
O'1
σx
Ellipse after 1-st loading

FIG. 12.



σy/σ0.1
Ellipse after 5-th loading
N
σM/σ0.1=σN/σ0.1=1.33
Ellipse after
7-th loading
σM=σN=12[kp mm-2]
O4
O6
O2
O7
O5
O3
O1
M
0
0.5
1.0
1.5
2.0
σx/σ0.1
Initial ellipse

FIG. 13.



σx [kp mm-2]
Series 14
Broken lines
Series 15(151)
Continuous
lines
σmax
σ0.1
σ0.02
σ0.01
σ0.5
σprop
M
N
0
20
15
10
5
τxy
[kp mm-2]
σM=σN=12[kp mm-2]
σy [kp mm-2]
σx
σy

FIG. 14.

Assuming the kinematic work-hardening hypothesis for the description of the material behaviour at the combined cyclic loadings one should pay attention to the fact that this hypothesis is very sensitive to the direction of loading. This is easily observed in Fig. 11 considering the situation of the centers of the ellipses after successive half-cycles of loading. The ellipses after the third and fifth loadings have a consistent course, while after the fourth and fifth half-cycles their situations are extremely different. This is caused by the fact that in the first case the terminal segments of the loading paths of the compared ellipses have the same direction (along the x-axis) while in the second case they are different (directions of the x and y axes). In the meantime the results of the experimental investigations indicate that the discrepancy between the surfaces obtained after the fourth and fifth half-cycles of loading is small (Fig. 14).

The results presented in the paper indicate that the character of the processes occurring in the material during cyclic loadings is very complex. For their description, even in a frame of the same theoretical concept, the conclusions appear different, sometimes inconsistent. Therefore, consideration of the particular cases of cyclic loading on the basis of the kinematic work-hardening hypothesis requires great care.

6. Conclusions

On the basis of the performed analysis of the results of investigations it was confirmed that after multiple cyclicly repetitive loadings of the material along two different paths the difference between the yield surfaces gradually decreases. The growth of the loading magnitude is associated with a solely insignificant increase of the number of half-cycles necessary for the effacement of the differences between two compared surfaces.

On the basis of the proportionality limit the kinematic work-hardening hypothesis is not able to describe the behaviour of the material for every magnitude of loading. Instead, on the basis of the conventional plasticity limit $\sigma_{0,1}$, the concept of the kinematic work-hardening predicts the changes occurring in the material for all cases considered in the paper.

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