ON THE MOTION OF A BODY UNDER NO FORCES.

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It is well known that in the motion of a body under no forces, and with one point fixed, the extremity of the vector representing the angular velocity traces out a curve in the body, which is the intersection of two quadrics; and that the coordinates of any point on this curve can be expressed as elliptic functions of the time. It is also known by Abel's theorem (Halphen, *Fonctions Elliptiques*, II. p. 450), that if β_1, β_2, β_3, β_4 are the elliptic function parameters of four coplanar points on such a curve

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 \equiv 4mK + 4niK' :$$

and that, if the sum of the parameters of two points on the curve is constant, the line joining the points is a generator of a fixed quadric through the given quartic. It may be easily deduced that if the difference of the parameters of two points on the quartic is constant, the tangent plane, which can be drawn through the line joining these points, to touch the quartic curve, touches a fixed quadric, drawn through the curve. I do not know if the statement of a theorem combining these results may be equally well known. Employing the notation of Routh's *Rigid Dynamics* we have 'the tangent plane, at any point on the polhode, to any given one of the quadrics,

$$(A^2 + \lambda A)\, x^2 + (B^2 + \lambda B)\, y^2 + (C^2 + \lambda C)\, z^2 = G^2 + \lambda T,$$

cuts the curve again in two points, such that the time taken by the instantaneous axis to pass from one to the other is constant.'

Again, if the line joining two points on the polhode is a generator of a fixed quadric, through the polhode, the sum of the times to these points is constant. This might be thus stated—'If $PQRS$ are four points on the polhode, such that the time taken by the axis to pass from P to Q is the same as the time from R to S, then PS and QR will be generators of the same quadric through the polhode.' For if $t_2 - t_1 = t_4 - t_3$, then $t_2 + t_3 = t_1 + t_4$, that is, the sum of the parameters of P and S is equal to the sum of the parameters of Q and R.

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