


 20, $2 x+3$





 $x^{2} x^{2}$






 $\operatorname{coc}^{2}+4.4$


 2 2.20

 Coser act 7.14 .420



 co 3
 the






## THE

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## CONTENTS.

ARITHMETIC AND ALGEBRA.
page
A new proof that a general quadric may be reduced to its canonical form (i.e. a linear function of squares) by means of a real orthogonal substitution. By Prof. J, J. Sylvester ..... 1
Note on the sums of two series. By Prof. Cayley ..... 29
On the reduction of a bilinear quantic of the $n$th order to the form of a sum of $n$ products by a double orthogonal substitution. By Prof. J. J. Sylfester ..... 43
On an arithmetical theorem in periodic continued fractions. By Prof. J. J. Sylvester ..... 63
Notes on modern higher algebra. By T. S. Fiske ..... 89
Gordan’s series. By H. F. Baker ..... 91
Solution of $(a, b, \ldots, c)=\left(a^{p}, b^{p}, \ldots, c^{p}\right)$. By H. W. Lloyd.Tanner ..... 118
On Latin squares. By Prof. Cayley - ..... 135
Note on series whose coefficients involve powers of the Bernoullian numbers By J. W. L. Glaisher ..... 138
A proof of the theorem of reciprocity for quadratic residues. By F . Franklin ..... 176
On the equation $x^{17}-1=0$. By Prof. Cayley ..... 184
Some inequalities. By W. Segar ..... 189
GEOMETRY.
On the converses of some theorems in elementary geometry. By C. Leudesdorf ..... 14
The solution of a special case of the problem of the establishment of a correlation between two figures. By J. Brilit ..... 57
On an extension of a problem of Pappus's. By Istrota Kenji ..... 67
Geometrical interpretation of a condition of integrability. By W. Burwside ..... 96
The lines of zero length on a surface as curvilinear coordinates. By W. Burnside ..... 99
Note on the intersection of lines with curves of an odd degree. ByH. G. Dawson110
On the focals of a quadric surface. By Prof. Cayley ..... 113
The solution of a special case of the problem of the establishment of a correlation between two plane figures (second paper), By J. Brilu - ..... 151
Note on reciprocal lines. By Prof. Cayley - ..... 174
Note de géométrie, à propos d'un thêorème de M. Stewart. By Prof. Mannheim ..... 178
Quaternion proofs of theorems relating to asymptotic lines. By T. Motoda ..... 188
DIFFERENTIAL AND INTEGRAL, CALCULUS, AND FINITE DIFFERENCES.
Note on the differential equation of a conic. By E. B. Eliliotr ..... 5
On differential expressions which persist in form after the transformation $x=\frac{1}{x}, y=\frac{y^{\prime}}{x}, \quad$ By E. B. Elliot ..... 7
On Stirling's formula and other interpolation formulæ. By G. H. Stuart ..... 19
THEORX OF ELLIPTIC FUNCTIONS.
Note on the sums of two series. By Prof. Cayley ..... 29
Expansions of $K, I, G, E$ in powers of $k^{\prime 2}-k^{2}$. BJ J. W. L. Glaisher ..... 146
On a direct relation between the definite elliptic integrals of the first and second order. By G. F. Childe ..... 155
On the expansion of $\frac{G}{K}, \frac{K}{G}, \frac{l}{E}$, dcc., in ascending powers of $k^{2}$. By
J. W. L. Glaisher ..... 164
APPLIED MATHEMATICS.
Note on energy in an elastic solid. By Karl Pearson ..... 31
The parabolic trajectory. By Prof. Greenhill ..... 47
The small deformation of curves and surfaces with application to the vibra- tions of a helix and a circular ring. By J. H. Miohrll ..... 68
On the exhaustion of Neumann's mode of solution for the motion of solids of revolution in liquids, and similar problems. By J. H. Mrchell ..... 83
Vibrations of a string stretched on a surface. By J. H. Michell ..... 87
Propagation of energy in the electro-magnetic field. By W. Burnside ..... 98
On the resultant of two finite displacements of a rigid body. By W. Burnside - - . . . - . - 102
Note on Clapeyron's theorem of the three moments. By Kari Pearson ..... 129
On the stability of a bent and twisted wire. By J. H. Michell ..... 181

