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Maximization of circumferential opening resultants
Another aspect of the stress approach to mixed mode fracture

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A NEW MAGNITUDE, the pair of stress resultants R^D, R^E along two complementary arcs ADB AEB of the singular circle (i.e. a small circle centered at the crack tip, Fig. 4) is introduced and investigated. It is shown that their components R_0^D, R_0^E in direction perpendicular to OB obtain maxima when OB coincides with the direction ϑ_p of crack propagation. On the basis of this result a new criterion for mixed mode crack propagation is established with an excellent physical interpretation, as R_0^D, R_0^E are the maxima of the opening force resultants acting on the singular circle. This is the first "stress criterion" which takes under consideration the complete stress field around the crack tip .

Wprowadzono i zbadano nową wielkość charakterystyczną w teorii pękania, mianowicie parę naprężeń wypadkowych R^D, R^E skierowanych wzdłuż łuków ADB, AEB koła osobliwego (czyli małego koła o środku w wierzchołku szczeliny, rys. 4). Pokazano, że ich składowe R_0^D, R_0^E w kierunku prostopadłym do OB osiągają wartości maksymalne gdy OB pokrywa się z kierunkiem propagacji szczeliny ϑ_p. Na tej podstawie zbudowano nowe kryterium propagacji szczeliny o modach mieszanych, charakteryzujące się doskonałą interpretacją fizyczną, gdyż R_0^D i R_0^E odpowiadają maksymalnym siłom rozwierającym działającym na kole osobliwym. Jest to pierwsze „kryterium naprężeniowe" uwzględniające kompletne pole naprężenia wokół wierzchołka szczeliny.

Введена и исследована новая величина, характерная в теории разрушения, а именно пара результирующих напряжений R^D, R^E направленных вдоль дуг ADB, AEB особого круга (т. е. малого круга с центром в вершине трещины, рис. 4). Показано, что составляющие R_0^D, R_0^E в перпендикулярном направлении к OB достигают максимальных значений, когда OB совпадает с направлением распространения трещины ϑ_p. На этой основе построен новый критерий распространения трещины со смешанными модами, характеризующийся хорошей физической интерпретацией, т. к. R_0^D и R_0^E отвечают максимальным раскрывающим силам, действующим на особом круге. Это первый „критерий в напряжениях", учитывающий полное поле напряжения вокруг вершины трещины.

1. Introduction

The solution of the problem of mixed mode brittle fracture consists in determining the direction ϑ_p of crack propagation and the load at fracture $\sigma^{cr}/\sigma_I^{cr}$, where σ^{cr}, σ_I^{cr} are the values of external load for which propagation starts in the mixed mode and mode-I configurations respectively. The study of the above problem can be restricted to the model of the uniaxially loaded inclined crack (Fig. 1) if the theoretical analysis is based on the singular expressions of stresses, as has been pointed out, among others, by CHRYSAKIS [4].

Two approaches for the solution of the problem have been proposed:

(i) the stress approach by ERDOGAN and SIH [2] in 1963, and

(ii) the energy approach by SIH [3] in 1973.

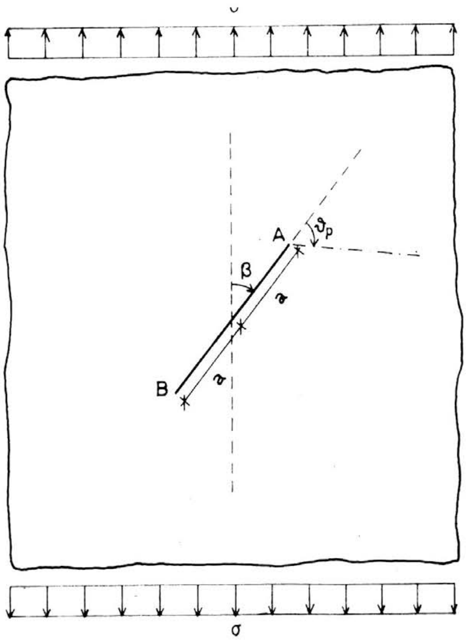

FIG. 1. Uniaxially-loaded inclined crack.

Both approaches are based on the singular expressions of stresses. In other words, the investigations take place along the circumference of the *singular circle*: a circle centered at the crack tip 0, of radius r small enough, so that the expressions of stresses are restricted to their singular terms only, but not so small, so that the elastic solution is valid.

The stress approach has been founded on two hypotheses [2]:

(a) The crack tip extension starts at its tip in radial direction.

(b) The crack extension starts in the plane perpendicular to the direction of greatest tension.

ERDOGAN and SIH [2] implemented hypothesis (b) on the model shown in Fig. 2a: they considered a polar element $(dr, d\vartheta)$ on the radial direction OB of the expected, according to hypothesis (a), crack extension so that the "tension" which by hypothesis (b) would cause separation of the material in this element, is the stress component σ_ϑ. Hence they proposed ϑ_p to be the direction of $\max\sigma_\vartheta$.

The stress approach was identified with the $\max\sigma_\vartheta$ criterion—more precisely, nobody talked about "stress approach" as no alternative to the maximization of σ_ϑ was thought until CHRYSAKIS [4, 5] proposed that hypothesis (b) could also be implemented in the model shown in Fig. 2b: the elementary areas are considered in pairs, one on arc ADB, the other on arc AEB, where OB is again the radial direction of expected propagation. Then the pairs of stresses (σ_D, σ_E) exert an "opening action" (in the terminology of [4, 5]) or "tension" (in the terminology of hypothesis (b) of [2]), leading to separation of the elastic material, enclosed in the singular circle, along the radius OB somewhere halfway between D and E. This model has been applied to the two usual systems of stresses:

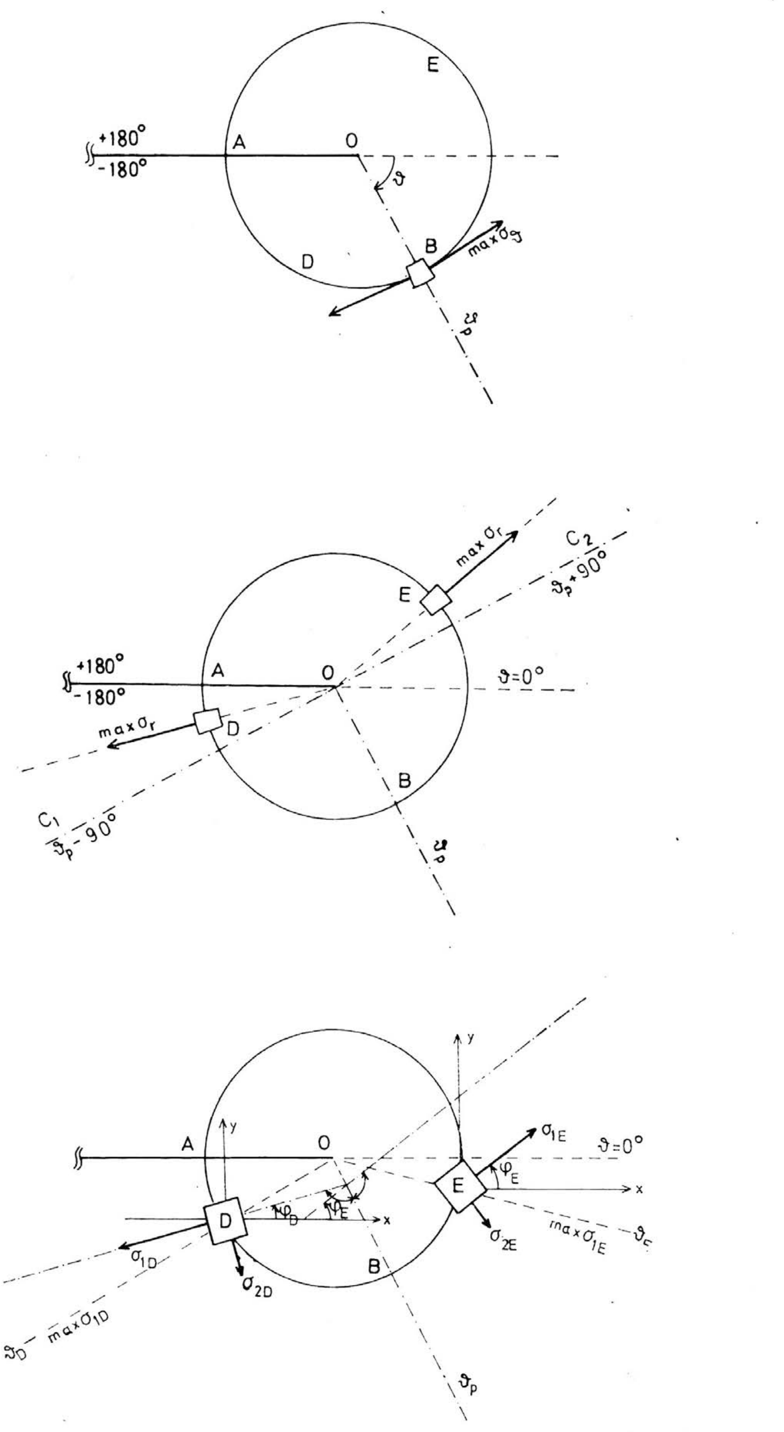

FIG. 2. Model of crack propagation; (a) for the $\max\sigma_\vartheta$ criterion, (b) for the $\max\sigma_r$ criterion and (c) for the $\max\sigma_1$ criterion.

(i) In [4] to the remaining polar components σ_r and $\tau_{r\vartheta}$. The elementary areas were the polar ones $(dr, d\vartheta)$, the search for extrema of σ_r gave for each value of β two directions of $\max\sigma_r$, ϑ_D in arc ADB, ϑ_E in arc AEB and the direction ϑ_B of the bisector of the angle of ϑ_D, ϑ_E coincided with the direction ϑ_p of propagation given by other criteria and experiments [2, 3]. The same technique was applied to $\tau_{r\vartheta}$ and again two absolutely maximum values of $\tau_{r\vartheta}$ were found at ϑ_D, ϑ_E; the coincidence of ϑ_B to ϑ_p was good except for small values of β. For the determination of load at fracture the projections of σ_r, $\tau_{r\vartheta}$ on ϑ_p-90 were integrated along arc ADB and their projections on ϑ_p+90 along arc AEB, giving the corresponding components of stress resultants along the direction $\vartheta_p\pm 90$ of the opening forces.

(ii) In [5] the principal stresses were investigated. Two directions of $\max\sigma_1$ (σ_1 is the larger of σ_1, σ_2) were found for each value of β: ϑ_D in arc ADB and ϑ_E in AEB. At these positions the corresponding angles φ_D, φ_E of σ_1 with the x-axis were evaluated and the

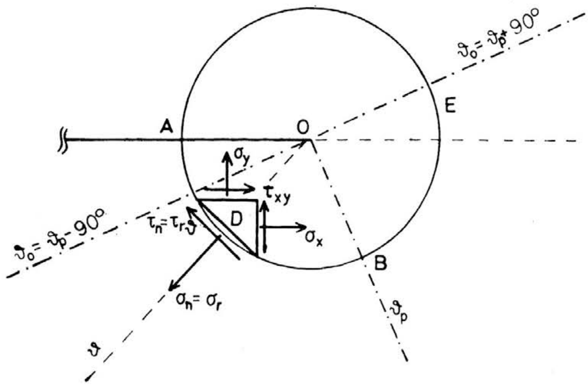

FIG. 3. Boundary stresses for determining load at fracture.

bisector ϑ_B of the angle of directions φ_D, φ_E was found again to coincide with ϑ_p (Fig. 2c). For the determination of load at fracture, the triangular boundary elements were considered with boundary stresses σ_n, τ_n (Fig. 3). The projections of σ_n, τ_n on the directions ϑ_p-90, ϑ_p+90 were integrated along the arcs ADB, AEB, respectively, giving the stress resultants $R_0^D(\vartheta_B)$, $R_0^E(\vartheta_B)$ corresponding to these arcs. In both cases [4, 5] it is assumed that propagation starts when these "circumferential stress resultants" of the mixed mode configuration reach the corresponding value of the mode I configuration — and this equation determines $\sigma^{cr}/\sigma_{\rm I}^{cr}$.

Thus two new criteria have been established in [4]: the $\max\sigma_r$ and $\max|\tau_{r\vartheta}|$ and one new in [5]: the $\max\sigma_1$ criterion, showing that σ_ϑ is not the only component whose maximization can form a basis for the prediction of mixed-mode propagation. The common point of all the above criteria is that they consider the maximum value (s) of a stress component, at one point B in the case of $\max\sigma_\vartheta$, at two points D, E in the cases of $\max\sigma_r$, $\max|\tau_{r\vartheta}|$, $\max\sigma_1$ and they rely on these isolated stress values for their predictions. But crack propagation is not due to the action of a certain stress component at one or two points, but to the action of the complete stress field. If the material is cut along the circumference of a singular circle and the latter is loaded by the polar stresses which existed along this circumference in the infinite plane configuration, then the propagation of the crack along OB will be due to the stress resultants R^D, R^E of stresses along arcs ADB, AEB, respectively (Fig. 4). These remarks bring up the idea of investigating the possibility to predict crack propagation on the basis of R^D, R^E, which is the objective of the present paper.

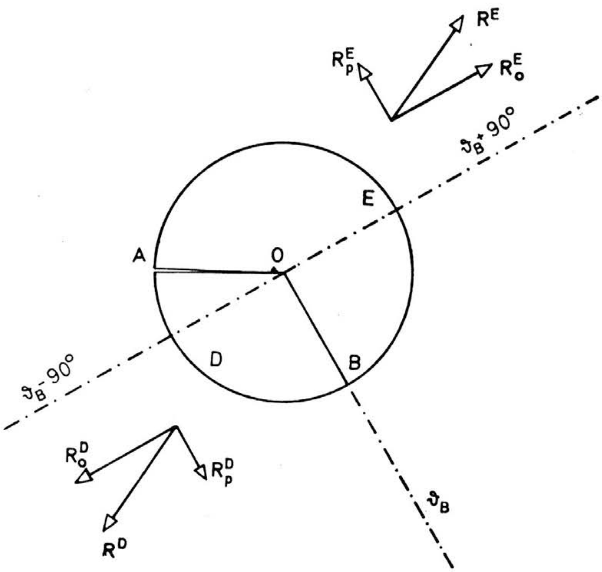

FIG. 4. Equilibrium of the singular circle under the action of the circumferential stress resultants.

For reasons of equilibrium it must be:

$$R^D(\vartheta_B) = R^E(\vartheta_B) \tag{1.1}$$

for any value of ϑ_B. If R^D_p, R^E_p are the components of R^D, R^E along ϑ_B and R^D_0, R^E_0 the components along $\vartheta_B \pm 90$, then Eq. (1.1) is equivalent to:

$$R^D_p(\vartheta_B) = R^E_p(\vartheta_B), \qquad R^D_0(\vartheta_B) = R^E_0(\vartheta_B) \qquad \forall\vartheta_B. \tag{1.1$_1$}$$

In this paper it is shown that in the direction $\vartheta_B = \vartheta_p$ of propagation it is:

$$R^D_0(\vartheta_p) = \max_{\vartheta_B} R^D_0(\vartheta_B) \tag{1.2$_1$}$$

and

$$R^D_p(\vartheta_p) = 0. \tag{1.2$_2$}$$

Hence a new criterion can be established on the basis the present results: that of the *maximum circumferential opening resultant* in which ϑ_p is a root of $dR^D_0(\vartheta_B)/d\vartheta_B = 0$ or, alternatively, of Eq. $(1.2)_2$, while the load at fracture is found by simply evaluating this stress resultant for $\vartheta_B = \vartheta_p$.

2. The circumferential stress resultants

As already explained in the Introduction, the model for the study of mixed mode crack propagation is based on the singular circle. The material enclosed in this circle, already separated along radius OA by the existing crack, is expected to separate along another radius OB by the propagating branch of the crack under the "opening action" exerted on it by the boundary stresses. The obvious choice for expressing boundary stresses along a circumference are their polar components, so that the normal boundary stress is $\sigma_n = \sigma_r$ and the tangential one $\tau_n = \tau_{r\vartheta}$. If one employs Cartesian components, then the expressions of σ_n, τ_n in terms of σ_x, σ_y, σ_{xy} are essentially the transformation formulae of Cartesian to polar components. Engaging in the [4, 5] usual notation:

$$\begin{aligned} s &= \sin\left(\frac{\vartheta}{2}\right), & c &= \cos\left(\frac{\vartheta}{2}\right), & t &= \tan\left(\frac{\vartheta}{2}\right), \\ s_n &= \sin\left(\frac{n\vartheta}{2}\right), & c_n &= \cos\left(\frac{n\vartheta}{2}\right), & t_n &= \tan\left(\frac{n\vartheta}{2}\right), & n &= 2, 3, \ldots \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} s_B &= \sin\vartheta_B, & c_B &= \cos\vartheta_B, \\ s_p &= \sin\vartheta_p, & c_p &= \cos\vartheta_p \end{aligned} \tag{2.2}$$

the singular expressions of stresses are [1]

$$\begin{aligned} \sqrt{2\pi r}\sigma_r &= K_1 c(1+s^2)+K_2 s(1-3s^2), \\ \sqrt{2\pi r}\,\tau_{r\vartheta} &= K_1 sc^2+K_2 c(1-3s^2), \\ K_1 = \left(\sigma\sqrt{\pi a}\right)\sin^2\beta, &\qquad K_2 = \left(\sigma\sqrt{\pi a}\right)\sin\beta\cos\beta. \end{aligned} \tag{2.3}$$

The projection of the force, per unit thickness of the plate, due to σ_r, on $\vartheta_B \pm 90$ is [4]

$$dp_r(\vartheta) = \pm\sigma_r \sin(\vartheta-\vartheta_B) r d\vartheta$$

and, substituting from the expressions (2.1), (2.2) and evaluating the indefinite integral,

$$\sqrt{\frac{\pi}{2r}}\, p_r(\vartheta) = \pm\left[\frac{s^5}{5}(2K_1 s_B-6K_2 c_B)+\frac{s^3}{3}(K_1 s_B+2K_2 c_B)-s(K_1 s_B) + \frac{c^5}{5}(2K_1 c_B+6K_2 s_B)-\frac{c^3}{3}(4K_1 c_B+7K_2 s_B)+c(2K_2 s_B)\right]. \tag{2.4}$$

Similarly, because of $\tau_{r\vartheta}$ one finds

$$dp_{r\vartheta}(\vartheta) = \pm\tau_{r\vartheta}\cos(\vartheta_B-\vartheta) r d\vartheta$$

and

$$\sqrt{\frac{\pi}{2r}}\, p_{r\vartheta}(\vartheta) = \pm\left[\frac{s^5}{5}(-2K_1 s_B+6K_2 c_B)+\frac{s^3}{3}(2K_1 s_B-5K_2 c_B)+s(K_2 c_B) + \frac{c^5}{5}(-2K_1 c_B-6K_2 s_B)+\frac{c^3}{3}(K_1 c_B+4K_2 s_B)\right]. \tag{2.5}$$

Adding Eqs. (2.4) and (2.5) one finds the indefinite integral representing the combined action of σ_r and $\tau_{r\vartheta}$:

$$\sqrt{\frac{\pi}{2r}}\, p_\pm(\vartheta) = \pm\left[s^3(K_1 s_B-K_2 c_B)-s(K_1 s_B-K_2 c_B)-c^3(K_1 c_B+K_2 s_B) + c(2K_2 s_B)\right]. \tag{2.6}$$

The upper sign corresponds to ϑ_B+90 (arc AEB) and the lower to ϑ_B-90 (arc ADB). The resultant of these forces for arc ADB is given by the definite integral of Eq. (2.6):

$$(2.7)_1 \qquad R_0^D(\vartheta_B) = \sqrt{\frac{\pi}{2r}}\,[p_-(\vartheta_B)-p_-(-180)] = \sqrt{\frac{\pi}{2r}}\,p_-(\vartheta_B)$$

and that of arc AEB:

$$(2.7)_2 \qquad R_0^E(\vartheta_B) = \sqrt{\frac{\pi}{2r}}\,[p_+(180)-p_+(\vartheta_B)] = -\sqrt{\frac{\pi}{2r}}\,p_+(\vartheta_B)$$

since it is easily seen that $p_\pm(\pm 180) = 0$.

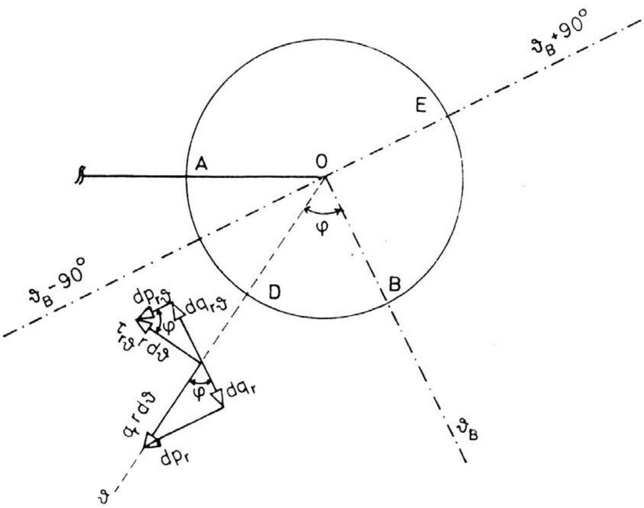

FIG. 5. Components of boundary stresses along the opening direction ϑ_0 and the direction of propagation ϑ_p.

Working similarly with the projections along ϑ_B, one has (Fig. 5):

$$(2.8) \qquad dq_{r\vartheta}(\vartheta) = \tau_{r\vartheta}\sin(\vartheta_B-\vartheta)\,rd\vartheta,$$

$$(2.9) \qquad dq_r(\vartheta) = \sigma_r\cos(\vartheta_B-\vartheta)\,rd\vartheta$$

with the indefinite integral of their sum $(dq_r+dq_{r\vartheta})$:

$$(2.10) \qquad \sqrt{\frac{\pi}{2r}}\,q(\vartheta) = -s^3(K_1c_B+K_2s_B)+s(K_1c_B+K_2s_B)+c^3(K_2c_B-K_1s_B)-c(2K_2c_B)$$

and resultants along the arcs ADB, AEB, respectively:

$$(2.11) \qquad \begin{aligned} R_p^D(\vartheta_B) &= q(\vartheta_B)-q(-180) = q(\vartheta_B),\\ R_p^E(\vartheta_B) &= q(180)-q(\vartheta_B) = -q(\vartheta_B). \end{aligned}$$

At this point one verifies from Eqs. (2.7) and (2.11) that these resultants are of equal magnitude and opposite sense, as expected. Actually, Fig. 6 represents the distribution of σ_r along the circumference of the singular circle (the scale of lengths is exaggerated

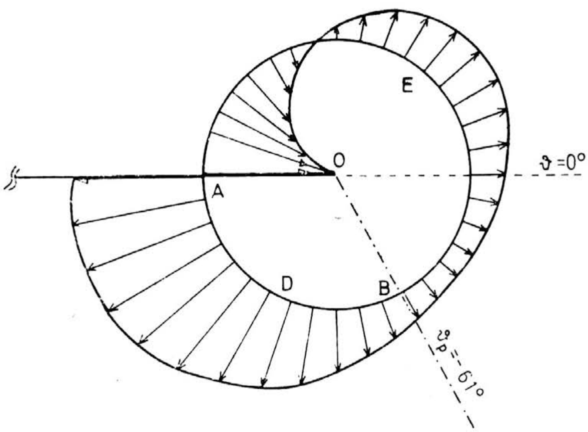

FIG. 6. Distribution of $\sigma\sqrt{r}$ for $\beta = 30$ along the singular circumference. The scale of the singular circle radius is exaggerated.

in comparison with that of stresses for the sake of clarity). Similar diagrams hold for the distribution of $\tau_{r\vartheta}$ and the combined action "$\sigma_r+\tau_{r\vartheta}$". Since there is equilibrium, the resultant of "$\sigma_r+\tau_{r\vartheta}$" along the circumference (C) must be zero, hence their resultant along any arc ADB must be opposite to the resultant along the complementary arc AEB. This, in terms of their components along the directions $\vartheta_p \pm 90$ and ϑ_p, is written as

$$\begin{gathered} R_0^D(\vartheta_B) = R_0^E(\vartheta_B) = R_0(\vartheta_B), \\ R_p^D(\vartheta_p) = -R_p^E(\vartheta_B) = R_p(\vartheta_B) \end{gathered} \tag{2.12}$$

for all values of ϑ_B.

Now it will be shown that for any given crack inclination β, the value of ϑ_B maximizing $R_0(\vartheta_B)$ is the root of $R_p(\vartheta_B)$ and is also equal to the direction of propagation ϑ_p, determined by other criteria as well as experimentally. First of all, from Eqs. (2.5), (2.7) and (2.12) it is

$$\sqrt{\frac{\pi}{2r}}\, R_0(\vartheta_B) = \sin^3\frac{\vartheta_B}{2}(K_1 s_B - K_2 c_B) - \sin\frac{\vartheta_B}{2}(K_1 s_B - K_2 c_B) \\ - \cos^3\frac{\vartheta_B}{2}(K_1 c_B + K_2 s_B) + \cos\frac{\vartheta_B}{2}(2K_2 s_B). \tag{2.13}$$

s_B, c_B are expressed in terms of $(\vartheta_B/2)$ and, introducing the notation

$$S = \sin\frac{\vartheta_B}{2}, \qquad C = \cos\frac{\vartheta_B}{2}, \qquad T = \tan\frac{\vartheta_B}{2}, \tag{2.14}$$

after some algebra Eq. (2.13) becomes:

$$\sqrt{\frac{\pi}{2r}}\, R_0(\vartheta_B) = -3K_2 S^3 + 3K_2 S - K_1 C^3. \tag{2.15}$$

The equation $dR_0/d\vartheta_B = 0$ in terms of T is found from Eq. (2.15) to be

$$2T^2 - \mu T - 1 = 0, \tag{2.16}$$

where

$$\mu = \frac{K_1}{K_2} = \tan\beta. \tag{2.17}$$

Similarly, from Eqs. (2.10), (2.11), (2.12) and, in the notation of Eq. (2.14), it is found that:

$$\sqrt{\frac{\pi}{2r}}\, R_p(\vartheta_B) = -K_1 SC^2 + K_2 C(2-3C^2)$$

or, in terms of T,

$$\sqrt{\frac{\pi}{2r}}\, R_p(\vartheta_B) = \frac{CK_2}{1+T^2}(2T^2-\mu T-1). \tag{2.18}$$

Comparison of Eqs. (2.16) and (2.18) shows that $dR_0/d\vartheta_B$ and $R_p(\vartheta_B)$ have common roots. The physical interpretation of this result is obvious: for the directions ϑ_B, which give extrema of the opening component (R_0) of the circumferential stress resultant, the other component (R_p) vanishes.

Furthermore, for the determination of load at fracture, the mode I case is considered too. The stress resultant is obtained from the general equation (2.15) for $\vartheta_B = 0$:

$$R_0^{\mathrm{I}} = -K_1^{\mathrm{I}} = -\sigma_{\mathrm{I}}^{\mathrm{cr}}\sqrt{\pi a}\,. \tag{2.19}$$

On the other hand for each solution ϑ_p of Eq. (2.16) the corresponding S, C given by the notation (2.14), are substituted in Eq. (2.15) and give

$$R_0(\vartheta_p) = -\sigma^{\mathrm{cr}}\sqrt{\pi a}\,[3K_2(S^3-S)+K_1C^3]. \tag{2.20}$$

Hence, on the basis of the usual assumption $R_0^{\mathrm{I}} = R_0(\vartheta_p)$, one obtains from Eqs. (2.19) and (2.20)

$$\frac{\sigma^{\mathrm{cr}}}{\sigma_{\mathrm{I}}^{\mathrm{cr}}} = [3K_2(S^3-S)+K_1C^3]^{-1}. \tag{2.21}$$

3. Numerical results and conclusions

A new criterion for mixed mode crack propagation can be stated on the basis of the theory developed in Section 2:

Initiation of crack propagation under mixed mode conditions:

(i) *takes place in the direction ϑ_p, in which the opening component R_0 of the circumferential stress resultant becomes maximum* (*i.e. root of* Eq. (2.16)); *then its other component R_p along the direction of propagation vanishes* (*since ϑ_p is a root of* Eq. (2.18), *too*);

(ii) *takes place at the moment when the imposed load obtains a critical value σ^{cr}, given by* Eq. (2.21).

Numerical values of ϑ_p, obtained by the above criterion for various values of β, are given in line (1) of Table 1, while the coresponding values of $\sigma^{\mathrm{cr}}/\sigma_{\mathrm{I}}^{\mathrm{cr}}$ are given in line (1) of Table 2. They are seen to be in good agreement with the corresponding values determined by other criteria or experimentally. The results are plotted in Figs. 7 and 8.

Table 1. Direction of propagation.

β		2	4	6	8	10	20	30	40	50	60	70	80	82	84	86	88	90
Present results	1	−69.9	−69.2	−68.5	−67.9	−67.2	−63.8	−60.0	−55.6	−50.3	−43.2	−33.3	−18.9	−15.4	−11.7	−7.9	−4.0	0.0
max σ_r	2	−70.0	−69.4	−68.8	−68.2	−67.6	−64.5	−61.0	−56.9	−51.8	−44.7	−34.5	−19.3	−15.7	−11.9	−8.0	−4.0	0.0
max $\tau_{r\vartheta}$	3	−61.4	−60.9	−60.4	−60.0	−59.5	−56.9	−54.1	−50.8	−46.5	−40.7	−32.0	−18.7	−15.3	−11.7	−7.9	−4.0	0.0
max σ_1	4	−76.5	−75.6	−74.6	−73.7	−72.8	−68.1	−63.1	−57.7	−51.4	−43.7	−33.4	−18.9	−15.5	−11.8	−7.9	−4.0	0.0
max σ_ϑ	5							−60.2	−55.7	−50.2	−43.2	−33.2	−19.3					
S	6							−63.5	−56.7	−49.5	−41.5	−31.8	−18.5					
Experim.	7							−62.4	−55.6	−51.1	−43.1	−30.7	−17.3					

Table 2. Critical loading $\sigma^{cr}/\sigma_I^{cr}$.

β		2	4	6	8	10	20	30	40	50	60	70	80	82	84	86	88	90
Present results	1	24.4	12.0	7.9	5.9	4.7	2.3	1.5	1.2	1.1	0.98	0.97	0.99	0.99	0.99	0.99	1.0	1.0
Ref. [5]	2	24.7	12.2	8.0	5.9	4.7	2.3	1.5	1.2	1.15	0.98	0.97	0.99	0.99	0.99	1.0	1.0	1.0
max σ_ϑ	3	24.4	12.0	7.9	5.9	4.7	2.3	1.5	1.2	1.1	0.98	0.97	0.99	0.99	0.99	0.99	1.0	1.0
S	4	26.4	13.1	8.7	6.5	5.2	2.6	1.8	1.4	1.2	1.1	1.1	1,0	1.0	1.0	1.0	1.0	1.0

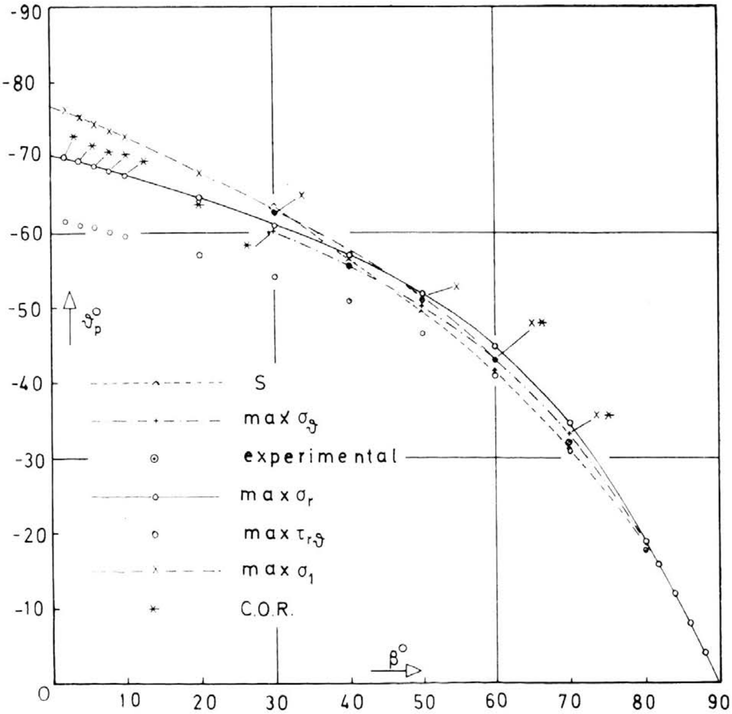

FIG. 7. Angle of propagation ϑ_p versus crack inclination β. C.O.R. stands for Circumferential Opening Resultant (present results).

The small difference in the values of $\sigma^{cr}/\sigma_I^{cr}$ reported in Table 2 of Ref. [4] is due to the fact that the evaluation of R_0^D, R_0^E due to $\tau_{r\vartheta}$ was based on an inappropriately chosen boundary element.

In conclusion, the following remarks should be made. In order that somebody be able to judge the importance of the new criteria proposed by CHRYSAKIS in [4, 5] and in the present paper, one should go back to 1973 (S-criterion [3]) and 1963 (max σ_ϑ criterion (2)). An extensive literature has been created, devoted to modifications, applications to particular cases and comparisons of the S and max σ_ϑ criteria. Even more important, in [2], p. 520, it is stated that "According to hypotheses (a) and (b), only the tangential components of these stresses can initiate crack growth...". This restriction to σ_ϑ raised the question, how the remaining stress components in the vicinity of the crack tip, although singular, wouldn't and/or couldn't participate in the phenomenon of fracture. And it was considered one of the main advantages of the energy approach, compared to the max σ_ϑ criterion, that the expression of the strain energy density factor S was taking under consideration the contribution of all the stress components to the initiation of propagation.

In [4, 5] and the present paper for the first time the above-mentioned restriction to max σ_ϑ has been questioned and it has been shown that if the model of Fig. 2b is adopted instead of that of Fig. 2a, then there are alternatives to the max σ_ϑ criterion: the max σ_r and max $|\tau_{r\vartheta}|$ in [4], the maximum principal stress σ_1 [5] and the maximum circumferential opening resultant in this paper. They cover all the cases of prediction of (ϑ_p, σ^{cr}) on the basis

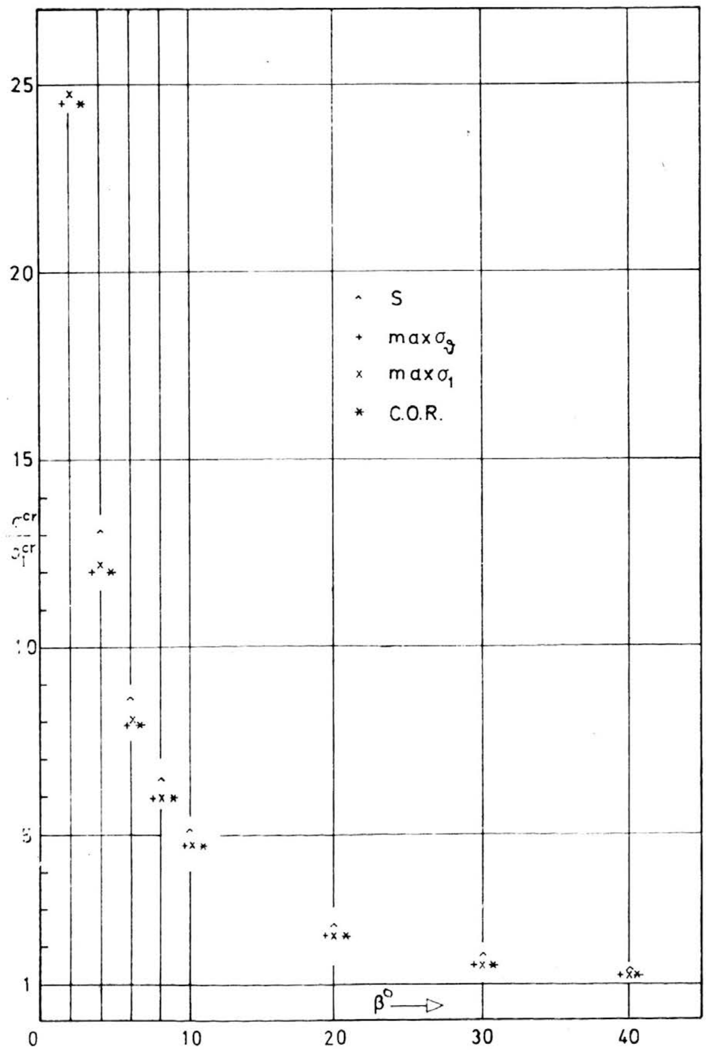

FIG. 8. Normalized load at fracture $\sigma^{cr}/\sigma_{I}^{cr}$ versus crack inclination β. C.O.R. stands for Circumferential Opening Resultant (present results).

of the variation of the singular stresses and, all together, form what is called here "the stress approach to mixed mode fracture".

Finally two basic differences should be pointed out.

(i) In [2] propagation is identified with separation of the material of the polar element (dr, $d\vartheta$) under the action of $\max\sigma_\vartheta$ (Fig. 2a). In [4, 5] and the present paper it is identified with separation of the material enclosed in the singular circle under the action of stresses at its circumference (Fig. 2b).

(ii) In [4, 5] the determination of ϑ_p is based on two isolated stress values: the extrema of a certain stress component at two points D, E of the singular circumference. On the contrary, in the present paper the determination of ϑ_p is based on the maximization of the resultant action of *both* boundary stresses (σ_n, τ_n) along the *whole* circumference. Thus it is observed that the maxima of the circumferential stress resultants R_0^D, R_0^E hav the same axis of symmetry (radius OB in direction ϑ_p) with the extrema of their particular stress components (i.e. $\sigma_r = \sigma_n$, $\tau_{r\vartheta} = \tau_n$) — a result neither self-evident nor derivable from general equilibrium considerations.

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