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Some visco-elastic aspects of liquid foams of high void fraction and possibilities of their application(*)

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LIQUID FOAMS of high dryness fraction belong to two-phase media. A foam composed of very small bubbles is shown experimentally to behave like a non-Newtonian fluid. For this purpose tests were carried out on flowing foams in smooth pipes of various diameters in the laminar range. An expression has been derived for a dynamic similarity number in foams which claims to possess a correct physical meaning and it takes into account the surface tension of the bubbles, their diameter, viscosity of the fluid composing the bubbles, the diameter of the pipe, the foam velocity and density. Using this dynamic similarity number relevant for foams, a very good correlation has been achieved for the friction coefficient of foams in pipes similar to the classical Hagen–Poiseuille one. Applying this newly-derived friction law, a good correlation has been obtained between the measured and estimated data of the impulse loss across a honey-comb immersed in foam and crossed by a moving shock wave. The unusually high impulse loss in the above case reported in [1] finds its justification due to the non-Newtonian character of the foam.

Płynne piany o dużej zawartości gazu należą do ośrodków dwufazowych. Piana złożona z bardzo małych banieczek zachowuje się jak ciecz nienewtonowska, co zostało stwierdzone doświadczalnie w niniejszej pracy. W tym celu przeprowadzono badania płynnych pian w gładkich rurach o różnych średnicach i przy różnych szybkościach przepływu. Wyprowadzono wyrażenie na liczbę podobieństwa dynamicznego R_F dla pian, posiadającą prawdziwe znaczenie fizyczne oraz uwzględniającą naprężenia powierzchniowe banieczek, ich średnicę, lepkość cieczy, średnicę przewodu, szybkość i gęstość piany. Stosując nową liczbę podobieństwa dynamicznego, uzyskano bardzo dobrą korelację między współczynnikiem tarcia pian w rurach a liczbą R_F, podobną do klasycznej korelacji Hagena–Poiseuille'a dla cieczy newtonowskich. Wykorzystując powyższe prawo tarcia uzyskano również dobrą korelację między obliczoną a zmierzoną stratą impulsu fali uderzeniowej przechodzącej przez cienkościenny ulowy zespół rurek zanurzonych w pianie. Niezwykle silny spadek impulsu notowany w tych warunkach [1] daje się wyjaśnić nienewtonowskim charakterem tarcia w pianach.

Жидкие пены с большим содержанием газа принадлежат к двухфазным средам. Пена, состоящая из очень малых пузырьков, ведется как неньютоновая жидкость, что констатировано экспериментально в настоящей работе. С этой целью проведены исследования жидких пен в гладких трубах с разными диаметрами и при разных скоростях течения. Выведено выражение для числа динамического подобия R_F для пен, обладающего действительным физическим значением и учитывающего поверхностное напряжение пузырьков, их диаметр, вязкость жидкости, диаметр провода, скорость и плотность пены. Применяя новое число динамического подобия, получена очень хорошая корреляция между коэффициентом трения пен в трубах и числом R_F, аналогичная классической корреляции Хагена–Пуазейля для ньютоновой жидкости. Используя вышеприведенный закон трения, получена тоже хорошая корреляция между рассчитанной и измеренной потерями импульса ударной волны проходящей через тонкостенную улейбразную систему трубок, погруженных в пене. Необыкновенно сильное падение импульса, отмеченное в этих условиях [1], объясняется неньютоновым характером трения в пенах.

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1. Introduction

A CONSIDERABLE amount of research has been made on liquid-gas mixtures of low void fraction called frothy mixtures, but comparatively little attention has been drawn to high void fraction liquid foams. The most remarkable feature of liquid-gas mixtures is that the velocity of sound in such a medium can attain extremely low values, one or two orders of magnitude lower than the components. A finely structured foam of high void fraction can be considered as a continuum, it has a very long life duration and does not collapse under the stress of a shock wave. Pressure pulses propagate in it at extremely low velocities, a most unusual situation seldom encountered in nature. This leads to a paradoxical situation like the achievement of high Mach Numbers with small relative velocities [2]. Many more interesting features are worth noting like powerful sound attenuation when compared to the well-known polyurethane dry foams, good shock wave attenuation in longer passages filled with foams, isothermal characteristics across the normal shock front, possibility of extracting high rates of frictional work at the boundary of a shock wave crossing a foam-filled thin walled honey-comb, etc. [1, 2, 3].

The void fraction is often quoted as the principal parameter defining foam characteristics. It appears that this is not sufficient, the bubble diameter is also of fundamental importance because it is related to the foam stability and the stress produced by the surface tension. Small bubbles, high liquid viscosities and void fraction result in foams resistant to very high pressure gradients and having long life spans.

Little is known about the non-Newtonian characteristics of liquid foams of high void fraction. A more detailed study of it shown below may result in a better understanding of other non-Newtonian liquids because of a newly-formulated approach. This study has also been applied with success to estimate the loss of a shock wave crossing a thin walled honey-comb filled with foam. The high impulse loss in this case is partly due to the non-Newtonian character of the foam.

2. Results

2.1. Friction in pipes

Experiments have been conducted in a special rig to verify the friction characteristics of liquid foams and covered a range of bubble diameters, void fractions, liquid viscosities and pipe diameters. Table 1 indicates the pertinent data. Tests carried at low speeds gave nonlinear relations between shear stresses and shear rates pointing at a non-Newtonian behaviour. To correlate data, Metzner's classical method was adopted [4, 5] which assumes

$$\tau = K'\left(\frac{8V}{D}\right)^{n'}, \tag{1}$$

where V — mean velocity, D — diameter of the pipe, K' and n' found experimentally for the foams are approximately 18.5 and 0.48, respectively. Typical results of such tests are shown in Fig. 1. Figure 2 presents more data plotted against the shear rate for various

Table 1. Foam characteristics.

Type of foam	Bubble diameter d (mm)	Void fraction α	Liquid viscosity μ kg/m s	Exponent n in Eq. (6)
Water + detergent or shampoo	0.52 to 0.4	0.9 to 0.4	0.001 to 0.00015	0.542
Commercial (Gillette)	0.02 to 0.035	0.93	0.148	0.507
Water + detergent + glycerol	0.52 to 0.4	0.9 to 0.94	0.0064 to 0.01	0.542

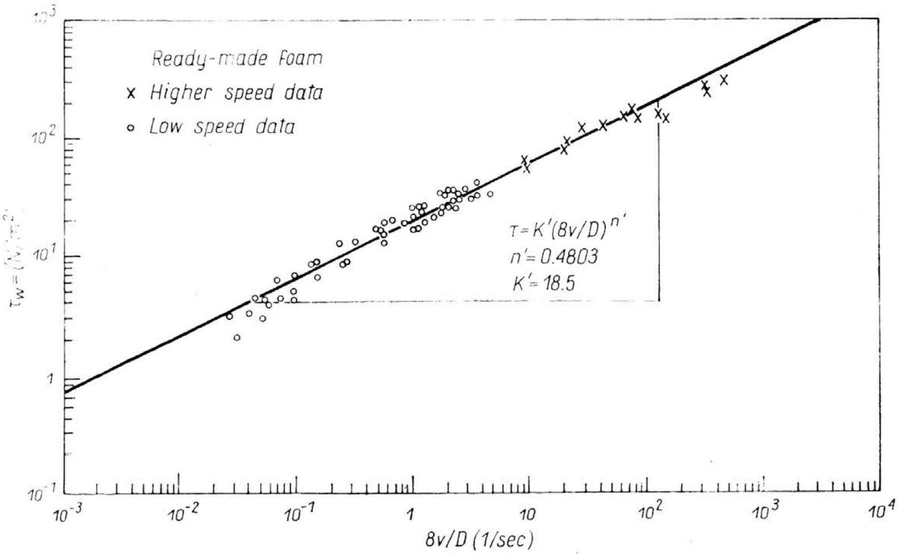

FIG. 1. Wall stress in pipes with moving ready-made foam as $f(U/D)$ using Metzner's power law for non-Newtonian fluids.

foams and pipe diameters. Comparison is made of the shear stress which would be obtained if the liquids alone were used. One observes that the foams for the same shear rate may have about two orders of magnitude more shear stress than the liquid component.

Figure 3 shows the results in terms of the friction coefficient C_f against the "Reynolds Number" R' adopted by Metzner. Again the correlation is very good. In this case

$$R' = \frac{D^{n'} V^{2-n'}}{K' 8^{n'-1}}, \tag{2}$$

$$C_f = \frac{16}{R'}. \tag{3}$$

The last expression is identical with the Hagen–Poiseuille for Newtonian fluids.

Notwithstanding the good correlation shown above, one may object that the classical expression [1] commonly used for non-Newtonian fluids as well as the "Reynolds Number"

R' defined above are objectionable from the fundamental point of view involving principles of dynamic similarity of all systems which obey Newton's laws of motion. It can be shown that for such systems the generalized dynamic similarity number [6] is

$$\frac{\text{Inertia Stresses}}{\text{Stress of the "Typical Force"}}. \tag{4}$$

If the typical force relates to the viscous stress, the above ratio is the Reynolds Number. If the compressibility comes in the foreground because of increased velocities, this ratio

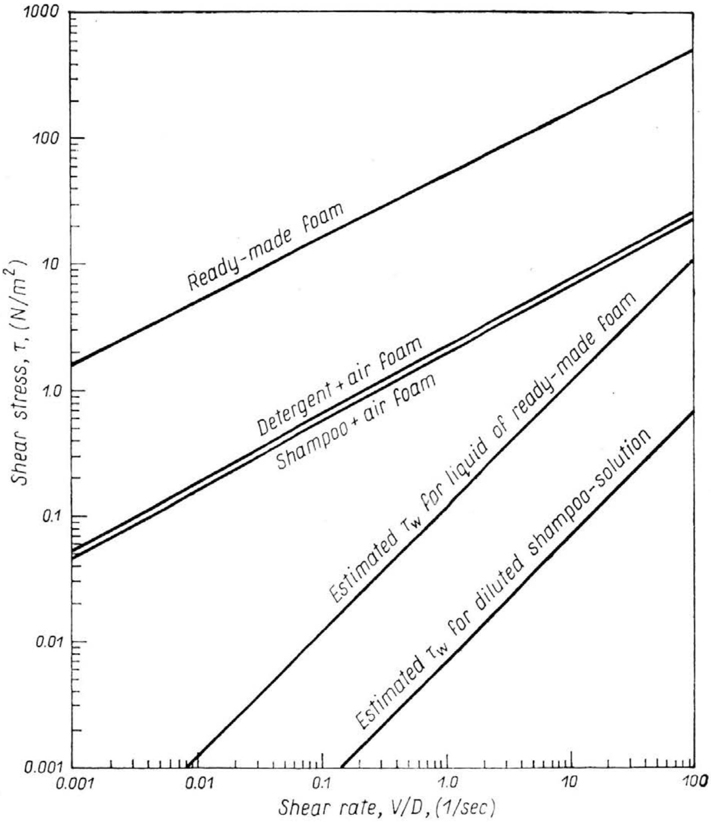

FIG. 2. Measured shear stress of foams in pipes compared with the stress of liquids forming these foams as $f(U/D)$.

becomes the Mach Number, if gravity, it is the Froude Number and so on. In the expression for R' used commonly for the non-Newtonian fluids one does not recognize in it this fundamental feature. It rather becomes a correlation parameter with an arbitrarily chosen number 8 to fit the classical Hagen–Poiseuille formula for the friction coefficient. Also the numbers K' and n' must be found experimentally in a tedious way for each fluid with a dimensionally obscure meaning of the K' standing for viscosity. In other words it is a method forcing non-Newtonian fluids into a Newtonian frame!

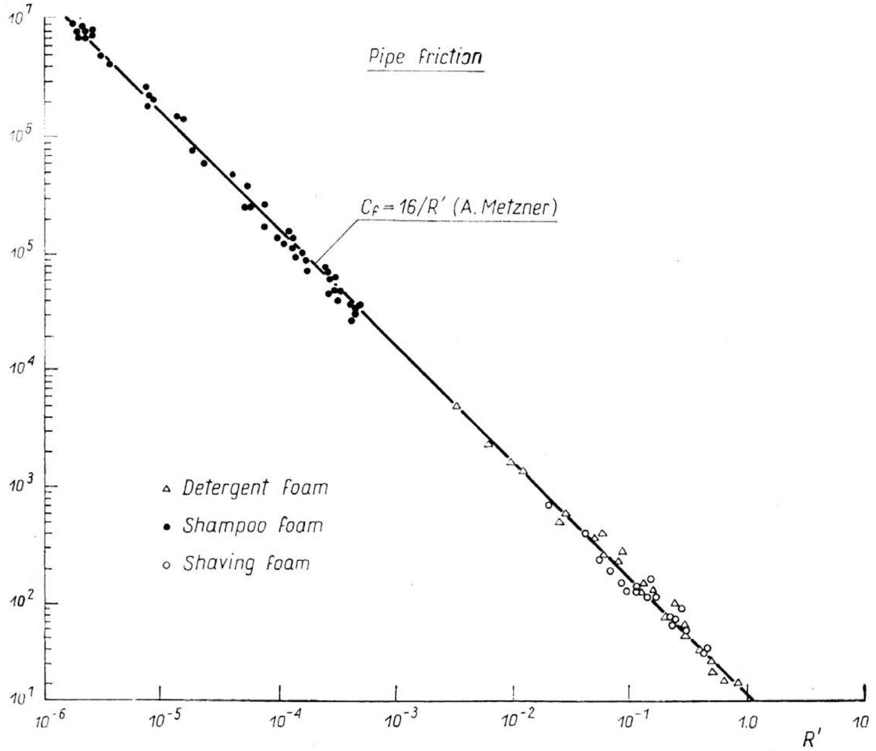

FIG. 3. The friction coefficient c_f obtained from Metzner's formula and his definition of the Reynolds Number.

2.2. Derivation of the similarity number for foams — R_F

A different approach was sought. In a two-component medium consisting of a finely dispersed fluid in a gas, yet forming a continuum through the interaction of bubbles, the "typical stresses" are represented through the surface tension acting on the normal to the bubble surface and tangential stresses due to friction between the bubbles and the walls, both are of importance even at very slow motion. The relation between σ_{ii} and τ_{ij} of the stress tensor is theoretically unknown. One may venture, however, that the generalized dynamical similarity number, expressed in Eq. (4) must hold. Thus for the foam mixture it should take the form

$$\text{(5)} \qquad \frac{\text{Inertia stresses}}{(\text{viscous stresses})^n,\ (\text{surface tension stresses})^{1-n}} = R_F!$$

and would represent truly the equivalent of the Reynolds Number. Here the typical stress must be composed of two parts, as it is a two-component mixture. For the foams this new similarity number is

$$\text{(6)} \qquad R_F = \frac{\varrho_F V^2}{\left(\frac{\mu V}{D}\right)^n \left(\frac{\sigma}{d}\right)^{1-n}},$$

where D is the pipe diameter, μ viscosity of the liquid, σ surface tension of the bubble, d bubble diameter, ϱ_F foam density. It should be noted that R_F properly understood cannot contain any arbitrary numbers.

If the reasoning is correct, one should also obtain a good correlation for various foams and tube diameters using this new concept.

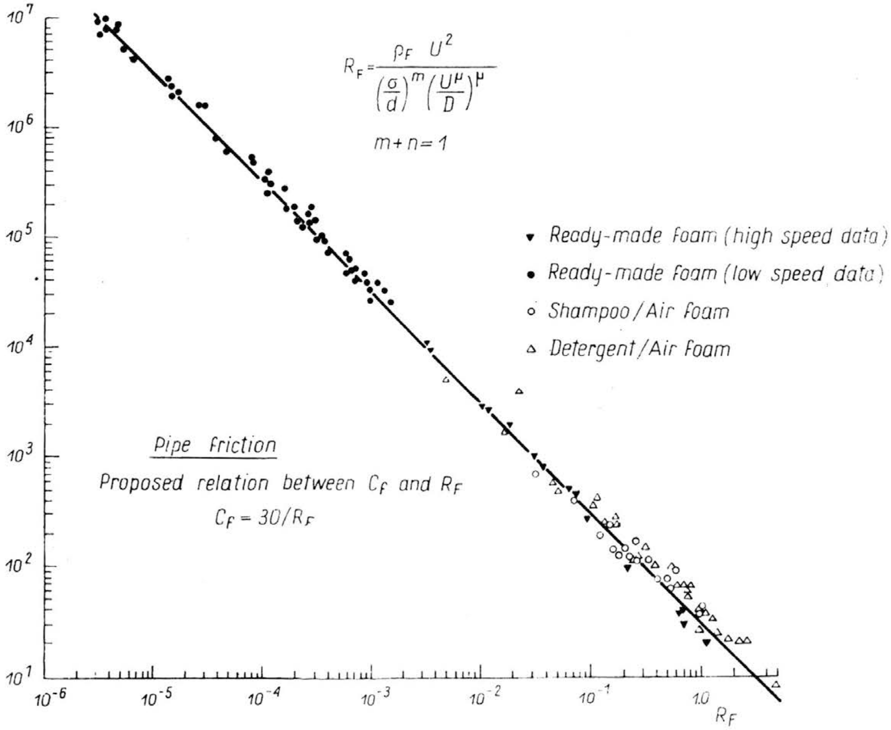

FIG. 4. Relation between C_f and R_F using the correct definition of the Reynolds Number, proposed in the paper.

Figure 4 shows such a correlation in terms of the friction coefficient. It is very satisfactory and leads to an expression similar to the Hagen-Poiseuille formula

$$C_f = \frac{30}{R_F} \tag{7}$$

which justifies this new approach.

The exponent n seems to vary very little within the range of the tested foams and is likely to be a weak function of the void fraction and bubble size. Experimental values of n are listed in Table 1. The advantage of this method is that it deals with only one empirical parameter n, provides an insight into the contribution of the surface tension, bubble diameter, viscosity of the fluid (which is Newtonian) and above all does not violate the principle of dynamical similarity. This method should be tested, if possible, for other non-Newtonian fluids. The expression (6) appears to be more satisfactory than the usually adopted definition of "Reynolds Number" in Eq. (2) for non-Newtonian fluids.

2.3. Shock waves in foams

Besides various aspects of shock waves in foams developed previously [1, 2], the above findings might be used to explain the unusually strong shock wave attenuation observed when a normal shock moving in a tube with foam traverses a thin-walled honey-comb immersed in foam (see Fig. 5). Before the arrival of the shock, the foam inside the honey-comb is stationary. It may be assumed that for a moderately short honey-comb, say (length of the cell) ÷ (cell diameter) = 10, the particle velocity will be imparted to the foam in

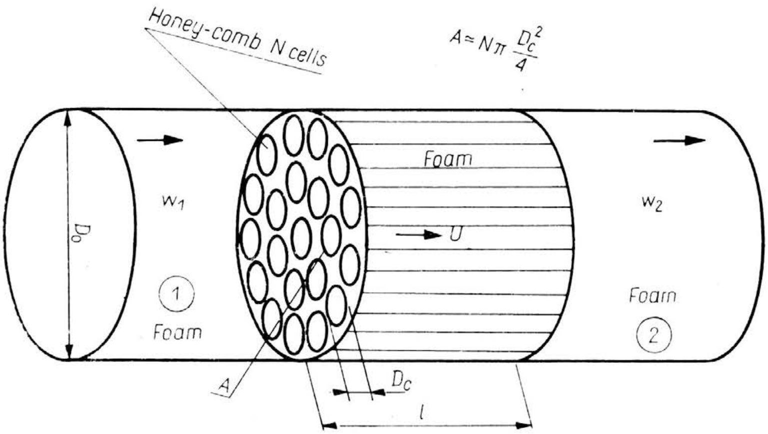

FIG. 5. A schematic outlay of shock tube experiments using a honey-comb immersed in foam and traversed by a shock wave.

the cells and the flow will still be laminar. In the tube of diameter D_0 there are N cells having each the diameter D_c and length l. Thus $ND_c^2 \cong D_0^2$. The wetted area of the cells is $ND_c\pi l$. Initially the foam in the honey-comb has a velocity $u_1 = 0$ after the shock foam reaches (for moderately long cells) the particle velocity u_2, which is calculable from normal shock relations [3]. For the change of u a linear relation may be assumed $u = bt$, with b some constant, thus if the time required to cross the honey-comb is T, $u_2 = bT = \frac{bl}{\overline{W}}$, with $\overline{W}$ = mean shock wave velocity across the honey-comb, say $(W_1+W_2)/2$. For a foam with surface tension σ, liquid viscosity μ, bubble diameter d, one obtains from the previous consideration using Eqs. (6) and (7) to calculate the wall stress $T_w = C_f \varrho_F \frac{u_F^2}{2}$ the loss of impulse

$$\Delta I = 15\,(\text{Wet Area}) \left(\frac{\sigma}{d}\right)^{1-n} \left(\frac{\mu}{D_c}\right)^n l \int_0^T b^n t^b dt = \frac{15\pi N D_c l^2 \left(\frac{\sigma}{d}\right)^{1-n} \left(\frac{\mu V}{D_c}\right)^n}{\overline{W}(n+1)}. \tag{8}$$

If the cross section of the shock tube is A_0, the loss of impulse per unit area with $ND_c^2 \simeq D_0^2$

$$(9) \qquad \frac{\Delta I}{A_0} = \frac{60\, ND_c l^2 \left(\frac{\sigma}{d}\right)^{1-n} \left(\frac{\mu V}{D_c}\right)^n}{D_0^2 \overline{W}(n+1)} = \frac{60\, l^2 \left(\frac{\sigma}{d}\right)^{1-n} \left(\frac{\mu V}{D_c}\right)^n}{D_c \overline{W}(n+1)}.$$

One observes the importance of the ratio l/D_0 and of the length l of the honey-comb cells as $\frac{\Delta I}{A_0} \sim \left(\frac{l}{D_0}\right)^2 \sim l^2$ for $D_0 = \text{const}$. Experiments were carried out in a shock tube to verify these assumptions using data shown in Table 2.

Table 2. Details of shock impulse loss.

Honey-comb	Foam (ready made)
$N = 51$	$n = 0.493$ (Eq. 5)
$t = 0.2$ mm (wall thickness of the honey-comb)	$d = 0.02$ mm
$D_c = 6.5$ mm	$\sigma = 0.025$ N/m
$l = 80$ mm	
Wet. Area: 51 $D_c \pi l$	$\mu = 0.014$ Pa s

Figure 6 gives the estimated and measured values as per Eq. (8) of the impulse loss against the Mach Number of the shock wave propagating in the foam. The agreement is very good. For longer cells the particle velocity u_2 estimated from normal shock formulae was too high and the estimated losses were also too high. It appears that for long cells $\frac{\Delta I}{A_0} \sim l^2$ doesn't hold with the assumption that u_2 = particle velocity.

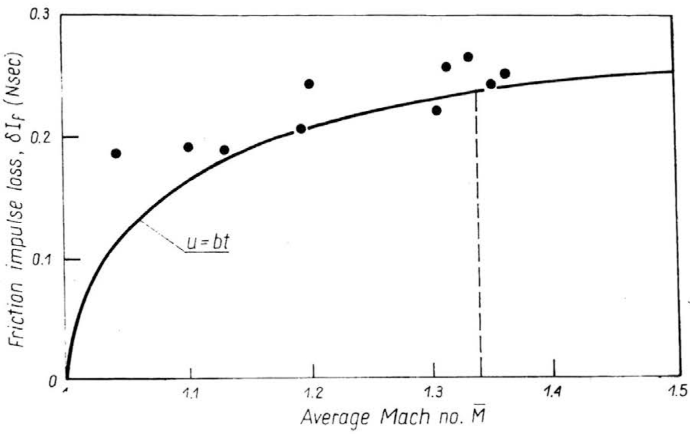

FIG. 6. Frictioni mpulse loss through a honey-comb immersed in foam and traversed by a shock wave. Comparison between formula and measurements in a shock tube.

3. Conclusions

Liquid foams of high void fraction have a comparatively simpler internal structure than many other non-Newtonian fluids. This provides the possibility for a more rational approach to estimate the friction coefficient C_f in pipes in the laminar range because the Reynolds Number proposed in this paper for foams R_F is properly formulated. Excellent correlation has been obtained using the newly developed expression between C_f and R_F for a wide range of pipe diameters, fluid viscosities and bubble diameters. Using these findings a formula has been developed to estimate the impulse drop due to frictional dissipation for a shock wave crossing to a thin-walled honey-comb in liquid foam. It provides a very high resistance to flow much more so than the liquid component itself. It is hoped that this new approach may be adopted to other non-Newtonian fluids and explain better their behaviour.

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