Archives of Mechanics • Archiwum Mechaniki Stosowanej • 29, 3, pp. 497–501, Warszawa 1977

Asymptotic shock wave propagation in electromagnetic field

W. A. LEVIN (MOSCOW)

THE PROPAGATION of magnetohydrodynamic shock waves in a transverse electric and magnetic field is considered for small magnetic Reynolds numbers. The conditions for the decay of the shock wave and its degeneration into the acoustic wave are established. It has been found that such a degeneration takes place in a finite distance and after a finite time-lapse.

CONSIDER the propagation of shock waves in a tube of constant cross section in the presence of external electric and magnetic fields. We assume that the conductivity of gas behind the shock wave is other than zero and the magnetic Reynolds numbers Re_m are small. Under these restrictions the induced electric and magnetic fields may be neglected and regarded as prescribed [1, 2]. The flow scheme is shown in Fig. 1. Let us determine the conditions under which the shock wave is weakened during propagation along a tube and degenerates into an acoustic wave. We shall also establish the asymptotic laws for such degeneration. The decaying process of shock waves is sufficiently well studied in [3–7].

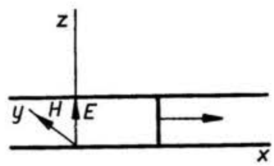

FIG. 1.

The shock wave, when it is weakened, may degenerate into an acoustic wave only asymptotically. Besides, the trajectory of the shock wave has no asymptote in the plane r, t (r stands for distance and t time). However, the presence of the transverse magnetic field of the constant intensity at $\mathrm{Re}_m \ll 1$ leads to the exponential decay of the shock wave [8]. In the presence of only the external magnetic field, the asymptotic propagation of the shock wave was considered in the papers [9, 10] for the numbers $\mathrm{Re}_m \sim 1$. The asymptotic propagation of the detonation wave in usual conditions and in the presence of the electromagnetic field was investigated in [11–15].

The system of equations describing the motion of the conductive gas behind the wave may be written in the form

$$\frac{\partial \varrho}{\partial t}+\frac{\partial \varrho v}{\partial r}=0, \quad \varrho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right)+\frac{\partial p}{\partial r}=-\frac{\sigma H}{c}\left(E+\frac{vH}{c}\right),$$
$$\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial r}+\gamma p\frac{\partial v}{\partial r}=(\gamma-1)\sigma\left(E+\frac{vH}{c}\right)^2. \tag{1}$$

http://rcin.org.pl

Here, ϱ, v and p denote the density, velocity and pressure, respectively; E and H are the strengths of the electric and magnetic fields, c is the velocity of light, and $\sigma = \sigma(p, \varrho)$ — the conductivity of the medium — is a known function of pressure and density.

Let us introduce the new dimensionless variable by means of the formulae

$$v = a_0 V, \quad \varrho = \varrho_0 R, \quad p = \varrho_0 a_0^2 P, \quad \sigma = \sigma_* \varphi, \quad H = H_* \mathscr{H},$$

$$E = E_* \mathscr{E}, \quad t = t_0 \tau, \quad r = a_0 t_0 x, \quad t_0 = \frac{\varrho_0 c^2}{\sigma_* H_*^2},$$

where u_0 and ϱ_0 are the sound velocity and the density in the undisturbed medium, respectively; and the quantities with stars denote certain characteristic values of the corresponding parameters.

The system of Eqs. (1) in terms of new variables has the form

$$\frac{\partial R}{\partial \tau} + \frac{\partial RV}{\partial x} = 0, \quad R\left(\frac{\partial V}{\partial \tau} + V\frac{\partial V}{\partial x}\right) + \frac{\partial P}{\partial x} = -\varphi(\omega\mathscr{E} + V\mathscr{H})\mathscr{H},$$

(2)
$$\frac{\partial P}{\partial \tau} + V\frac{\partial P}{\partial x} + \gamma P\frac{\partial V}{\partial x} = (\gamma-1)\varphi(\omega\mathscr{E} + V\mathscr{H})^2,$$

$$\omega = \frac{cE_*}{a_0 H_*}.$$

On the shock wave the usual conservation laws should be satisfied which, may be written as

(3)
$$v = \frac{2a_0}{\gamma+1}\frac{1-q}{\sqrt{q}}, \quad p = p_0\frac{2\gamma-(\gamma-1)q}{(\gamma+1)q}, \quad \varrho = \varrho_0\frac{\gamma+1}{\gamma-1+2q}.$$

Here, $q = \dfrac{a_0^2}{D^2}$ and D is the shock wave velocity. In terms of new variables the wave velocity is $\dot{x}_s = \dfrac{1}{\sqrt{q}}$.

Using the system of Eqs. (2), the derivative of the gas velocity behind the jump with respect to the space coordinate may be expressed in terms of the values of the parameters determined on the wave and the wave velocity. This expression will be used further in the analysis of shock wave motion.

(4)
$$\left.\frac{\partial V}{\partial x}\right|_s = \frac{-(V-\dot{x}_s)[R\dot{V} + \varphi\mathscr{H}(\omega\mathscr{E} + V\mathscr{H})] + \dot{P} - (\gamma-1)\varphi(\omega\mathscr{E} + V\mathscr{H})^2}{R(V-\dot{x}_s)^2 - \gamma P}.$$

As a measure of the deviation of the shock wave from the acoustic wave we introduce the parameter $\varepsilon = 1-q$. We call the wave weak if $\varepsilon \ll 1$. From the relations derived on the front of the wave it results, that with the second order of accuracy ε^2 (inclusively), the parameters of gas evaluated on the wave satisfy the relations of the simple Riemann and their values are

(5)
$$V = \frac{2\varepsilon}{\gamma+1} + \ldots, \quad P = \frac{1}{\gamma} + \frac{2\varepsilon}{\gamma+1} + \ldots, \quad R = 1 + \frac{2\varepsilon}{\gamma+1} + \ldots.$$

http://rcin.org.pl

By the annex of the Riemann flow to the surface of the wave in a known manner [4] and taking into account only terms of principal order, the following equation describing the variation of the shock wave intensity is obtained from [4]:

$$\text{(6)}\qquad \frac{4}{\gamma+1}\dot{\varepsilon}+\frac{2\varepsilon}{\gamma+1}\frac{1}{\tau}+\varphi\mathscr{H}\left(\omega\mathscr{E}+\frac{2\varepsilon}{\gamma+1}\mathscr{H}\right)-(\gamma-1)\varphi\left(\omega^2\mathscr{E}^2+\frac{4\varepsilon}{\gamma+1}\omega\mathscr{E}\mathscr{H}\right)=0$$

or, in equivalent form,

$$\text{(6')}\qquad \dot{\varepsilon}+\frac{\varepsilon}{2\tau}+\frac{\varepsilon}{2}\varphi\mathscr{H}[\mathscr{H}-2(\gamma-1)\omega\mathscr{E}]+\frac{\gamma+1}{4}\varphi\omega\mathscr{E}[\mathscr{H}-(\gamma-1)\omega\mathscr{E}]=0.$$

Let us consider the various particular cases. In a case of the usual shock wave $\varphi = 0$, the solution of Eq. (6') is $\varepsilon = \varepsilon_0\sqrt{\frac{\tau_0}{\tau}}$, what is in agreement with the known results [3-7]. In the presence of only the constant magnetic field $H = 1$, $\mathscr{E} = 0$ and the constant conductivity $\varphi = 1$, the solution becomes $\varepsilon = \frac{A}{\sqrt{\tau}}e^{-\frac{\tau}{2}}$. This exponential form of decaying of the shock wave also coincides with the results obtained earlier in the paper [8].

However, if the external electric and magnetic fields of the constant intensity $E = H = 1$ are applied, the behaviour of the shock waves depending on the parameter ω at the constant conductivity $\varphi = 1$ may be different. The general solution of Eq. (6') in this case may be written in the form:

$$\text{(7)}\qquad \varepsilon = \varepsilon_0\sqrt{\frac{\tau_0}{\tau}}\,e^{-\frac{1-2(\gamma-1)\omega}{2}(\tau-\tau_0)} + \frac{\gamma+1}{4}\omega[(\gamma-1)\omega-1]\frac{1}{\sqrt{\tau}}e^{-\frac{1-2(\gamma-1)\omega}{2}\tau}\int_{\tau_0}^{\tau}\sqrt{\xi}\,e^{\frac{1-2(\gamma-1)\omega}{2}\xi}\,d\xi,$$

where ε_0 is the value of parameter ε in the moment $\tau = \tau_0$. From this solution it results that for the values of the parameter ω from the interval $0 < \omega \leqslant \frac{1}{2(\gamma-1)}$ the shock wave is weakened and on the finite distance it degenerates into an acoustic wave. For the values of the parameter ω from the interval $\frac{1}{2(\gamma-1)} < \omega < \frac{1}{\gamma-1}$ the shock wave degenerates into an acoustic wave, if the initial shock wave intensity satisfies the inequality

$$\text{(8)}\qquad \varepsilon_0\sqrt{\tau_0}+\frac{\gamma+1}{4}\omega[(\gamma-1)\omega-1]\int_{\tau_0}^{\infty}\sqrt{\xi}\,e^{\frac{1-2(\gamma-1)\omega}{2}\xi}\,d\xi<0.$$

For the inverse inequality, i.e. for the values $\omega < 0$ or $\omega > \frac{1}{\gamma-1}$, the degeneration of the shock wave into an acoustic wave does not occur. Therefore the electric field, depending on its intensity, may accelerate the transition of the wave or strengthen it. Here, the situation is absolutely analogous to that observed in the propagation of the detonation wave in the electromagnetic field [13]. Depending on the value of the parameter ω, the

http://rcin.org.pl

precompressed detonation wave transfers into Chepman-Juge region or remains precompressed.

In a general case the conductivity of the medium depends on its thermodynamic properties and is the function of pressure and density $\varphi = \varphi(P, R)$. It may happen that the conductivity of the medium before the shock wave equals zero, i.e. $\varphi\left(\frac{1}{\gamma}, 1\right) = 0$. In this case the asymptotic character of the behaviour of the shock wave will depend on the shape of the function $\varphi(P, R)$. If the function φ is analytic in the vicinity of the point $\left(\frac{1}{\gamma}, 1\right)$ then, in this case, neglecting the small terms of the high order of smallness, the following relation for the shock wave intensity is obtained from Eq. (6′):

$$\dot{\varepsilon}+\frac{\varepsilon}{2\tau}+\frac{\varepsilon}{2}(m+n)\omega\mathscr{E}[\mathscr{H}-(\gamma-1)\omega\mathscr{E}] = 0, \tag{9}$$

where $m = \left(\frac{\partial\varphi}{\partial P}\right)_0$, $n = \left(\frac{\partial\varphi}{\partial R}\right)_0$.

For $H = E = 1$, this solution becomes

$$\varepsilon = \frac{A}{\sqrt{\tau}}e^{-\frac{m+n}{2}\omega[1-(\gamma-1)\omega]\tau} \tag{10}$$

Since $m+n > 0$ then, for the values of the parameter ω from the range $0 < \omega < \frac{1}{\gamma-1}$, the shock way decays exponentially and degenerates into an acoustic wave. For the remaining parameters ω, i.e. for $\omega < 0$ or $\omega > \frac{1}{\gamma-1}$, damping of the wave does not occur.

In several cases the dependence of the conductivity of the medium on its thermodynamic properties may be approximated by the power function, i.e. in the neighbourhood of the initial parameters of the state we have $\varphi = \varphi_*\varepsilon^\alpha$, $\alpha > 0$. The variation of the shock wave intensity will then be described by the equation

$$\dot{\varepsilon}+\frac{\varepsilon}{2\tau}+\frac{\gamma+1}{4}\varepsilon^\alpha\varphi_*\omega\mathscr{E}[\mathscr{H}-(\gamma-1)\omega\mathscr{E}] = 0. \tag{11}$$

Here the parameter α may assume the values from the range $0 < \alpha < 2$. If $\alpha \geqslant 2$, the damping of the weak shock wave, in the first approximation, will occur in the same manner as in usual gasdynamics. The solution of Eq. (11) may be written in the form

$$\varepsilon = \left\{A-\frac{(\gamma+1)(1-\alpha)}{2(3-\alpha)}\varphi_*\omega[1-(\gamma-1)\omega]\tau^{\frac{3-\alpha}{2}}\right\}^{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\tau}}, \tag{12}$$
$$\mathscr{E} = \mathscr{H} = 1.$$

From this expression it follows that for $0 < \omega < \frac{1}{\gamma-1}$, the wave is always weakened. However, the laws of damping depend essentially on the magnitude of the parameter α. For $0 < \alpha < 1$, the wave degenerates into an acoustic wave at a finite moment of time,

http://rcin.org.pl

and for $\alpha > 1$ only asymptotically. If $\omega < 0$ or $\omega > \frac{1}{\gamma-1}$, the intensity of the wave does not change.

Thus the shock wave in all cases is weakned and degenerates into an acoustic wave if the parameter ω satisfies the inequality $0 < \omega < \frac{1}{\gamma-1}$. However, the laws of such weakening depend essentially on the conductivity properties of the medium. In certain cases the wave can degenerate into an acoustic wave in a finite period of time. For other values of the parameter ω the weakening of the wave does not occur. Upon analyzing the stationary solutions of Eqs. (1), as performed in [14, 16], one may demonstrate that in this case the finite stationary solutions do not extist in the whole interval of variation of the space coordinate ranging from 0 to ∞.

Acknowledgment

I wish to express my gratitude to Professor W. FISZDON for fruitful discussion of the results presented.

References

1. А. Г. Куликовский, Г. А. Любимов, *Магнитная гидродинамика*, М., Физматгиз, 1962.
2. А. Б. Ватажин, Г. А. Любимов, С. А. Регирер, *Магнитогидродинамические течения в каналах*, М. Наука, 1970.
3. Л. Д. Ландау, *Об ударных волнах на далеких расстояниях от места их возникновения*, ПММ, **9**, 4, 1945.
4. Л. И. Седов, *Методы подобия и размерности в механике*, М., Наука, 1967.
5. Ю. Л. Якимов, *Об асимптотических решениях уравнений одномерного неустановившегося движения идеального газа и об асимптотических законах затухания ударных волн*, ПММ, 19, 6, 1955.
6. К. Е. Губкин, *Распространение разрывов в звуковых волнах*, ПММ, **22**, 4, 1958.
7. О. С. Рыжов, *Затухание ударных волн в неоднородных средах*, ПМТФ, 2, 1961.
8. А. А. Луговцов, *Распространение слабых ударных волн в магнитном поле*, ПМТФ, 4, 1966.
9. В. П. Коробейников, *Затухание слабых магнитогидродинамических ударных волн*, Магнитная гидродинамика, 2, 1967.
10. В. П. Коробейников, *Задачи теории точечного взрыва в газах*, Труды Мат. ин-та им. В. А. Стеклова, 69, 1973.
11. Г. Г. Черный, *Асимптотическое поведение плоской детонационной волны*, ДАН СССР, **172**, 1967.
12. В. А. Левин, Г. Г. Черный, *Асимптотические законы поведения детонационных волн*, ПММ, **31**, 3, 1967.
13. В. А. Левин, *Распространение детонационных волн в электрическом и магнитном полях*, Отчет, 972, ин-т механики МГУ, 1969.
14. В. А. Левин, *Распространение детонационных волн в электрическом и магнитном полях*, Отчет 1103, ин-т механики МГУ, 1970.
15. А. А. Бармин, В. А. Левин, *Асимптотическое поведение плоской магнитогидродинамической детонационной волны*, Научные труды, N1, ин-т механики МГУ, 1970.
16. E. Resler, W. Sears, *The prospects for magnetoaerodynamics*, JAS, **25**, 4, 1958.

INSTITUTE OF MECHANICS
UNIVERSITY, MOSCOW, USSR.

Received November 12, 1976.

http://rcin.org.pl