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NOTE ON THE (2, 2) CORRESPONDENCE OF TWO VARIABLES.

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In connection with my paper "On the porism of the in-and-circumscribed polygon and the (2, 2) correspondence of points on a conic," *Quar. Math. Jour.*, t. XI. (1871), pp. 83—91, [489], I remark that if (θ, ϕ) have a symmetrical (2, 2) correspondence, and also (ϕ, χ) the same symmetrical (2, 2) correspondence, then (θ, χ) will have a (not in general the same) symmetrical (2, 2) correspondence. In fact, to a given value θ there correspond, say the values ϕ_1, ϕ_2 of ϕ; then to ϕ_1 correspond the values θ, χ_1 of χ (viz. one of the two values is $=\theta$), and to ϕ_2 the values θ, χ_2 of χ (viz. one of the values is here again $=\theta$); that is, to the given value θ there correspond the two values χ_1, χ_2 of χ; and similarly to any value of χ there correspond two values of θ; viz. to χ_1 the value θ and say θ_1; to χ_2 the value θ and say θ_2; that is, the correspondence of θ, χ is a (2, 2) correspondence and is symmetrical.

Analytically, if we have

$$(a,\ b,\ c,\ f,\ g,\ h \between \theta\phi,\ \theta+\phi,\ 1)^2=0,$$

and

$$(a,\ b,\ c,\ f,\ g,\ h \between \phi\chi,\ \phi+\chi,\ 1)^2=0,$$

then writing

$$(a, \ldots \between \phi u,\ \phi+u,\ 1)^2=0,$$

the roots hereof are $u=\theta$, $u=\chi$; i.e. we have

$$(a, \ldots \between \phi u,\ \phi+u,\ 1)^2=(a, \ldots \between \phi,\ 1,\ 0)^2 (u-\theta)(u-\chi);$$

or, what is the same thing, we have

$$1 : -(\theta+\chi) : \theta\chi = (a, \ldots)(\phi, 1, 0)^2 : 2(a, \ldots)(\phi, 1, 0)(0, \phi, 1) : (a, \ldots)(0, \phi, 1)^2$$

$$= a\phi^2 + 2h\phi + b \quad : \quad 2(h\phi^2 + \overline{b+g}\,\phi + f) \quad : \quad b\phi^2 + 2f\phi + c,$$

giving $\phi^2 : \phi : 1$ proportional to linear functions of 1, $\theta+\chi$, $\theta\chi$, and therefore a quadric relation $(*)(\theta\chi, \theta+\chi, 1)^2 = 0$, with coefficients which are not in general (a, b, c, f, g, h).

Suppose, however, that the coefficients have these values, or that the correspondence is

$$(a, b, c, f, g, h)(\theta\chi, \theta+\chi, 1)^2 = 0,$$

we must have

$$(a, b, c, f, g, h)(a\phi^2 + 2h\phi + b, \ -2(h\phi^2 + \overline{b+g}\,\phi + f), \ b\phi^2 + 2f\phi + c)^2 = 0,$$

that is,

$$(ac + b^2 + 2bg - 4fh)(a, b, c, f, g, h)(\phi^2, -2\phi, 1)^2 = 0,$$

or, we have

$$ac + b^2 + 2bg - 4fh = 0,$$

as the condition in order that the symmetrical (2, 2) correspondence between θ and χ may be the same correspondence as that between θ and ϕ, or between ϕ and χ.