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On bubble motion through liquid under reduced gravity

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THE present note deals with the analytical and experimental investigation of the motion of a gas bubble through liquid enclosed between parallel plane walls very close together. An improved reduced-gravity simulator is described. Comparison between theoretical and experimental findings shows excelient agreement. An analogy between the bubble motion in the plate model and bubbles rising freely through liquid is discussed.

Niniejszy artykuł dotyczy badań doświadczalnych i teoretycznych ruchu pęcherzyka gazu w cieczy zawartej między dwiema równoległymi, płaskimi ściankami znajdującymi się w bardzo bliskiej odległości. Opisano udoskonalony modulator zredukowanej siły ciężkości. Porównanie wyników doświadczalnych i teoretycznych daje bardzo dobrą zgodność. Przedyskutowano analogię między ruchem pęcherzyka w modelu płytowym, a pęcherzykami tworzącymi się swobodnie w płynie.

Настоящая заметка касается экспериментальных и теоретических исследований движения пузырька газа в жидкости, содержащейся между двумя параллельными плоскими стенками, находящимися на очень близком расстоянии друг от друга. Описан усовершенствованный модулятор приведенной силы тяжести. Сравнение экспериментальных и теоретических результатов дает хорошее совпадение. Обсуждена аналогия между движением пузырька в пластинчатой модели и пузырьками образующимися в свободной жидкости.

1. Introduction

SPACE processes utilize the low-gravity environment with its unequalled features (weightlessness, vacuum, temperature, pressure and radiation) for the manufacturing of products the characteristics of which are superior to terrestrial ones, or are not attainable on earth [1]. One major problem of materials processing in zero gravity is the degassing of the molten matter. In order to investigate proposed space processes in an earthbound workshop under reduced gravity conditions, a simple test apparatus (the so-called reduced gravity- or zero-g-simulator, respectively) has been designed to study the motion of bubbles through liquid. The experimental arrangement consists essentially of two parallel plane glass plates of variable distance. The channel built by the plates is filled with liquid. By virtue of a syringe bubbles are injected into the fluid. If the plates are positioned horizontally, the liquid-gas system is in a state of effective weightlessness. Slight inclination of the plates with respect to the horizontal line generates a weak force field which causes the bubbles to move in a translatory fashion. The rising of the bubble due to buoyancy can easily be observed. In the following the flow within the plate model is analysed theoretically and experimentally, yielding the migration velocity as a function of the angle of inclination of the plates and other relevant parameters. Of special interest is, of course, the question whether the results obtained from analysis and experimentation permit the prediction of the behaviour of bubbles under real weightlessness. It will be shown that there exists an analogy between the bubble motion in the simulator and spherical bubbles rising freely through liquid.

2. Theoretical considerations

In order to determine the migration velocity U_0 of the bubble through the surrounding liquid, the following assumptions are made (Fig. 1):

(i) the bubble touches both plates ($z = \pm h$), except a very thin liquid film covering the walls,

(ii) the bubble contour in a plane parallel to the plates remains nearly circular,

(iii) the liquid motion is slow (creeping flow); moreover, the distance ($2\,h$) between the walls is small in comparison with the bubble diameter ($2\,R$).

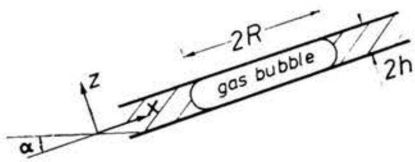

FIG. 1. Sketch of flow situation and notations.

With the assumption of closely spaced plates and slow viscous flow, the equations of fluid motion reduce to those governing the flow in a Hele-Shaw cell. Consequently, there exists a parabolic velocity distribution normal to the plates, and the piezometric pressure

$$p^* = p + \varrho g z^* \tag{2.1}$$

is essentially a function of the coordinates x, y (where the (x, y) plane is parallel to the plates and located in the midst between the plates), and z, with $z^* = x \sin\alpha + z\cos\alpha$. We further denote the pressure by p, the density by ϱ and the acceleration due to gravity by g. The positive z^*-axis is directed opposite to the gravitational force and intersects the z-axis under the angle of inclination α. Let us denote the liquid velocity relative to the plates and averaged with respect to the stratum by $\mathbf{V}$. The components of this flow field are functions of x and y only, furthermore, they are derivable from a potential Φ^*, given by

$$\Phi^* = -\frac{h^2}{3\eta} p^*(x, y),$$

with η as the viscosity of the liquid. Thus the mean flow defines a two-dimensional velocity field which is irrotational. Perturbations of this flow occur in the immediate neighbourhood of solid walls as well as at free surfaces (liquid-gas interface). The domain of perturbation is of the order of magnitude of the distance between the plates. Referring to a coordinate system fixed within the centre of the bubble, we obtain for the liquid velocity [2]

$$\mathbf{v} = \frac{3}{2}(\mathbf{v}^* - \mathbf{U}_0)\left(1 - \frac{z^2}{h^2}\right) + \mathbf{U}_0, \quad \mathbf{U}_0 = U_0\,\mathbf{i}, \tag{2.2}$$

where $\mathbf{i}$ is the unit vector in the x-direction and $\mathbf{v}^*$ is the velocity of liquid, averaged with respect to z, as seen by an observer moving with the bubble. Hence

$$\mathbf{v}^* = \mathbf{U}_0+\mathbf{V}. \tag{2.3}$$

The flow field $\mathbf{v}^*$ is subject to the same boundary conditions as the motion $\mathbf{u}^*$ of an inviscid incompressible fluid around a cylinder. Since both flows are potential flows, they are identical. Restricting ourselves to cylindrical bubbles of circular shape, migrating with velocity U_0, we find in terms of bubble-centred polar coordinates (Fig. 2)

$$u_r^* = U_0\left(1-\frac{R^2}{r^2}\right)\cos\varphi, \tag{2.4}$$

$$u_\varphi^* = -U_0\left(1+\frac{R^2}{r^2}\right)\sin\varphi, \tag{2.4'}$$

whence

$$v_r = \left[1-\frac{3}{2}\frac{R^2}{r^2}\left(1-\frac{z^2}{h^2}\right)\right]U_0\cos\varphi, \tag{2.5}$$

$$v_\varphi = \left[-1-\frac{3}{2}\frac{R^2}{r^2}\left(1-\frac{z^2}{h^2}\right)\right]U_0\sin\varphi \tag{2.5'}$$

follows, since $v_r^* = u_r^*$ and $v_\varphi^* = u_\varphi^*$.

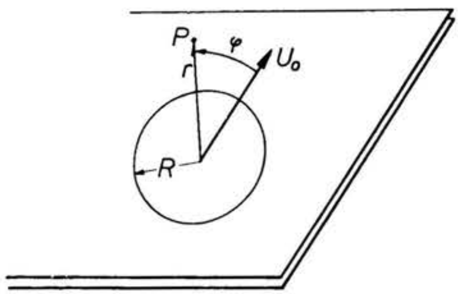

FIG. 2. Bubble-fixed coordinate system.

If the bubble rises steadily between the plates, the potential energy of the liquid lost per unit of time will be dissipated completely. Thus

$$\varrho g\sin\alpha\cdot\pi R^2 2hU_0 = \Psi, \tag{2.6}$$

where Ψ denotes the total dissipation. The latter results by integrating the dissipation function

$$\psi = \eta\left[\left(\frac{\partial v_r}{\partial z}\right)^2+\left(\frac{\partial v_\varphi}{\partial z}\right)^2\right] \tag{2.7}$$

over the entire liquid volume Ω:

$$\Psi = \int\limits_\Omega \psi d\Omega = 9\eta U_0^2\left(\frac{R}{h}\right)^4 \int\limits_{\varphi=0}^{2\pi}\int\limits_{r=R}^{\infty}\int\limits_{z=-h}^{h}\frac{z^2}{r^3}\,dzdrd\varphi = 6\pi\eta U_0^2\frac{R^2}{h}. \tag{2.8}$$

Substitution into Eq. (2.6) yields for the migration velocity of the bubble

$$U_0 = \frac{1}{3}\frac{\varrho}{\eta}h^2 g \sin\alpha. \tag{2.9}$$

For the terminal velocity $\bar{U}$ of a spherical gas bubble of radius $\bar{R}$, rising freely through unbounded liquid, we have in case of small Reynolds numbers [3]

$$\bar{U} = \frac{1}{3}\frac{\varrho}{\eta}\bar{R}^2 g, \tag{2.10}$$

with $\mathrm{Re} = \bar{R}\bar{U}\varrho/\eta$. Comparison between Eqs. (2.9) and (2.10) suggests an analogy between both types of flow, if we put $2h = 2\bar{R}$ and interpret $g \sin\alpha$ as the effective gravity. However, this analogy is disturbed by the peculiarities of the simulator. As already pointed out, the equations of the Hele-Shaw flow do not render the real flow pattern in the immediate neighbourhood of the bubble correctly. Besides viscosity the interfacial (surface) tension plays an important role. In the problem under study we are concerned with menisci moving relative to fixed solid walls. If the liquid meets the wall under a contact angle of zero degree, i.e., in case of a completely wetting liquid, we are able to evaluate the effect caused by this flow. There results an additional dissipative drag force, namely

$$D = 4\pi R\sigma\left(\frac{U_0\eta}{\sigma}\right)^{2/3}, \tag{2.11}$$

where σ denotes the surface tension. This formula is based on a paper by FRIZ [4], up to the factor 4, which was determined according to the experiments described below.

Taking into consideration perturbation effects and experimental data, the migration (rise) velocity can be written in dimensionless form as follows

$$\frac{h^2\varrho g\sin\alpha}{U_0\eta} = 3+2\left(\frac{\sigma}{\eta U_0}\right)^{1/3}\frac{h}{R}. \tag{2.12}$$

This relation indicates that the influence due to the above mentioned disturbances, represented by the second term on the right-hand side, decreases with increasing bubble radius and increasing migration velocity.

3. Experimental procedure

An improved test set-up, the original version of which was described by STONG [5] and subsequently, including some modifications, by SIEKMANN, ECK and JOHANN [6], is shown in Fig. 3. The apparatus consists essentially of two parallel flat glass plates whose thickness amounts to 10 mm. By means of rails the distance between the plates may be varied from 1 to 3 mm. The space between the plates is occupied by the test fluid. Boreholes in the upper plate allow the injection of air bubbles into the liquid-filled channel. The entire experimental arrangement is mounted on a solid rectangular iron frame (40 cm × 70 cm) which rests on a swiveling device. The bearings were designed in such a way that effects due to clearance were eliminated. If the plates are tilted towards the horizontal line, the bubbles begin to rise. The migration velocity can be determined by

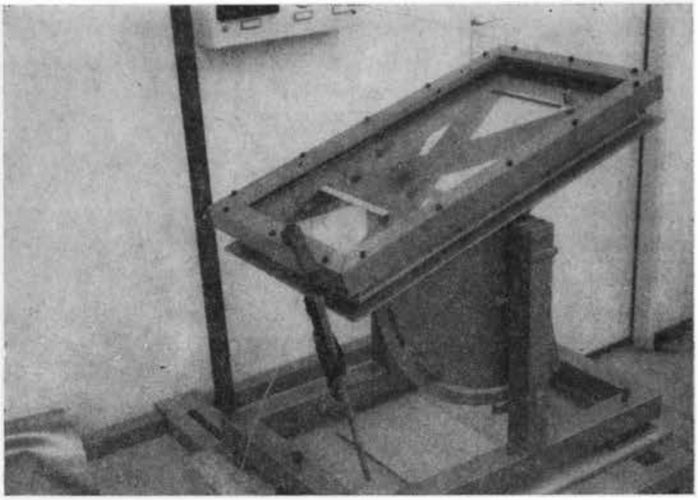

FIG. 3. Reduced gravity simulator.

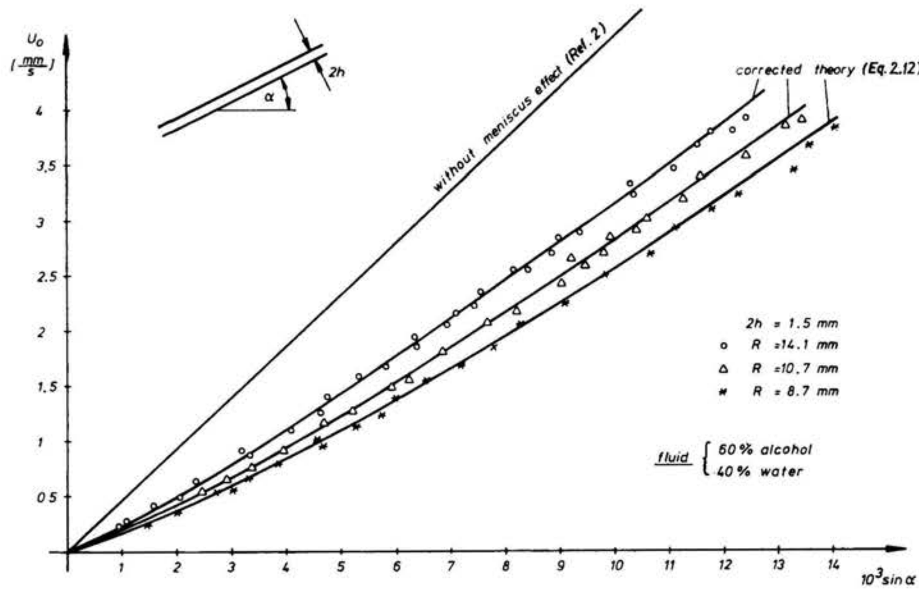

FIG. 4. Velocity of circular bubbles as function of the angle of inclination.

time measurements. In Fig. 4 the migration velocity of the bubbles is plotted vs. the inclination angle of the plates. The distance between the plates was 1.5 mm. The corrected theory (Eq. (2.12)) is based on a graphical fitting of the test data. The figure exhibits clearly the influence of the "meniscus effect" on the flow in the immediate neighbourhood of the bubble. Neglect of this effect yields a linear relationship between the migration velocity and the inclination of the plates. Figure 5 shows in dimensionless form experimental data for two different fluids as function of the relevant parameters (bubble diameter,

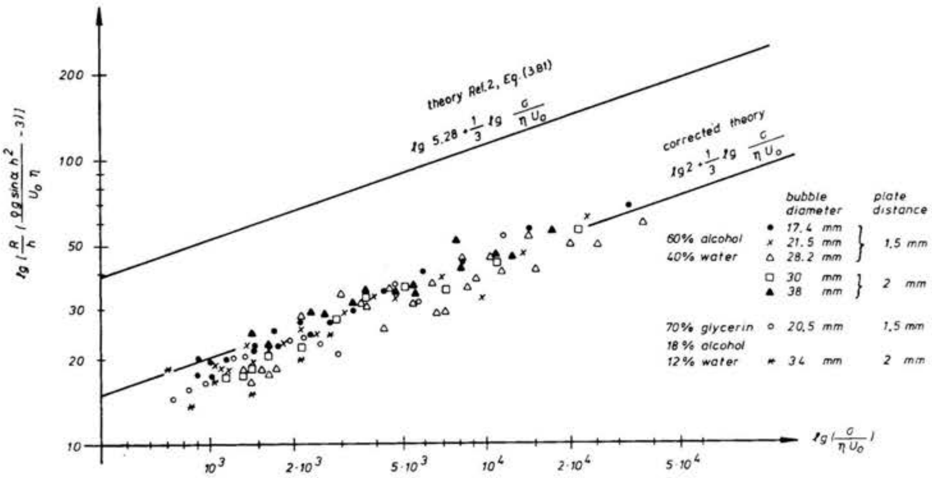

FIG. 5. Comparison of theory and experiment in dimensionless form.

plate distance). The figure demonstrates also the experimental modification (lower solid line) of the theory (upper solid line). This correction is restricted to a numerical factor only. It should be pointed out that the experiments confirm the validity of this theory, which allows the calculation of the contour of dynamic menisci in case of wetting liquids.

4. Conclusions

In the preceding sections we have seen that the migration velocity of gas bubbles in a liquid, enclosed between close parallel plates and placed into a weak gravitational field, can be determined theoretically with great accuracy. By means of an analogy between this bubble motion and the motion of freely rising spherical bubbles it is possible to predict the behaviour of bubbles under the condition of reduced gravity, i. e., in space, provided that the results of the plate model are corrected accordingly. This is of some importance with respect to materials processing in the low gravity gradient field of an orbital laboratory, i.e., degassing of molten materials. With the help of the simulator we can already on earth study the intensity of the force fields which are necessary to collect and to drive the bubbles distributed in the liquid.

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References

1. *Proceedings of the Third Space Processing Symposium, Skylab Results*, Vol. I, II, NASA M-74-5, June 1974.

2. W. ECK, *Blasenbewegung zwischen parallelen, ebenen Wänden*, Dissertation, Technische Hochschule Darmstadt, 1975.
3. V. G. LEVICH, *Physicochemical hydrodynamics*, Prentice Hall, Inc., 1962.
4. G. FRIZ, *Über den dynamischen Randwinkel im Fall der vollständingen Benetzung*, Zeitschrift für Angewandte Physik, **19**, 4, 374–378, 1965.
5. C. L. STONG, *What hapens to the fluid in the tanks of spacecraft falling freely in space*, Scientific American, 106–11, April 1972.
6. J. SIEKMAN, W. ECK, W. JOHANN, *Experimentelle Untersuchungen über das Verhalten von Gasblasen in einem Null-g-Simulator*, Z. Flugwiss., **22**, 3, 83–92, 1974.

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