Arch. Mech., 35, 3, pp. 423-436, Warszawa 1983

Modified theory of viscoplasticity Physical foundations and identification of material functions for advanced strains

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THE MAIN objective of the present paper is to study the physical foundations of the modified theory of viscoplasticity and identification of the material functions and constants in the case of advanced strains. The modified theory of viscoplasticity satisfies the requirement that during the deformation process the response of a material modelled becomes elastic-plastic for the assumed, non-zero, quasi-static value of the effective strain rate. Particular attention is devoted to the physical interpretation of the transition from the strain rate dependent plastic flow to the elastic-plastic response and a scalar control function which plays essential role in the description of such a transition. The least square technique was used in order to determine material constants of one-dimensional constitutive equation of viscoplasticity for AISI 316L stainless steel on the basis of available experimental results of tension test at room temperature and at different strain rates.

Głównym celem pracy jest zbadanie fizycznych podstaw zmodyfikowanej teorii lepkoplastyczności oraz identyfikacja funkcji i stałych materiałowych w przypadku zaawansowanych odkształceń plastycznych. Zmodyfikowana teoria lepkoplastyczności spełnia warunek, że podczas procesu deformacji reakcja modelowego materiału staje się sprężysto-plastyczna dla przyjętej, niezerowej, quasi-statycznej wartości efektywnej prędkości odkształcenia. Szczególną uwagę zwrócono na fizyczną interpretację przejścia, zależnego od prędkości odkształcenia płynięcia plastycznego do reakcji sprężysto-plastycznej oraz na skalarną funkcję kontrolną, która odgrywa istotną rolę w opisie tego przejścia. Zastosowano metodę najmniejszych kwadratów aby określić stałe materiałowe jednowymiarowego równania lepkoplastyczności dla stali nierdzewnej AISI 316L na podstawie dostępnych danych doświadczalnych próby rozciągania w temperaturze pokojowej przy różnych prędkościach odkształcenia.

Основной целью работы является исследование физических основ модифицированной теории вязкопластичности и идентификация материальных функций и констант для случая развитых пластических деформаций. Модифицированная теория пластичности удолветворает следующему условию: в течение процесса деформации ответ моделированного материала становится упруго-пластическим для некоторого принятого, отличного от нуля квазистатическо значения эфективной скорости деформации. Особое внимание уделяется физическому истолкованию перехода пластитического течения, (зависимого от скорости деформации) к упруго-пластическому режиму, а также скалярно-контрольной функции, играющей существенную роль при описании этого перехода. С использованием метода наименьших квадратов определялись материальные константы одномерного уравнения вязкопластичности для нержавеющей стали AISI 316L на основе доступных экспериментальных результатов испытаний на растяжение при ей комнатной температуре и при различных скоростях деформации.

1. Introduction

THE NEW THEORY of viscoplasticity which can be applicable in a study of the influence of strain rate effects on the instability of plastic flow was formulated recently by PERZYNA [12]. The theory contains two essential modifications in comparison with the former one (cf. PERZYNA [11, 13]). Firstly, the effect of defects, inclusions and imperfections is taken into account. Secondly, the modified theory satisfies also the requirement that during the deformation process the response of a material thereby modelled becomes elastic-plastic for the assumed, non-zero, quasi-static value of the effective strain rate. The material functions and constants were determined on the basis of available experimental data obtained under the condition of dynamic loading for rate sensitive, plastic materials. These results were pertinent to the yield point and small plastic strains, only.

The aim of the present paper is to study the physical foundations of the mentioned modified theory of viscoplasticity and identification of material functions and constants in the case of advanced strains. Particular attention is devoted to the physical interpretation of the transition from the strain rate dependent plastic flow to the elastic-plastic response and a scalar control function which plays essential role in the description of such a transition.

2. Material structure with internal state variables

In what follows we shall consider only isothermal processes. Let us assume that the intrinsic state σ of a particle X consists of its local configuration $(\mathbf{E}(t), \vartheta(t))$ and its method of preparation $\boldsymbol{\omega}(t)$, i.e. (cf. PERZYNA [13])

$$\sigma = \big(\mathbf{E}(t), \vartheta(t), \boldsymbol{\omega}(t)\big), \tag{2.1}$$

where $\mathbf{E}(t)$ denotes the strain tensor, $\vartheta(t)$ temperature and $\boldsymbol{\omega}(t)$ is the internal state vector. It is postulated that the internal state vector $\boldsymbol{\omega}(t)$ can be assumed in the form (1)

$$\boldsymbol{\omega}(t) = \big(\mathbf{E}_p(t), \varkappa(t)\big), \tag{2.2}$$

where $\mathbf{E}_p(t)$ denotes the inelastic strain tensor and $\varkappa(t)$ is an isotropic work-hardening parameter.

The constitutive equation for the Piola–Kirchhoff stress tensor $\mathbf{T}(t)$ is assumed in the form as follows:

$$\mathbf{T}(t) = \hat{\mathbf{T}}(\sigma). \tag{2.3}$$

The tensorial material function $\hat{\mathbf{T}}$ is assumed to be differentiable with respect to all components of the intrinsic state σ.

The evolution equation for the internal state vector $\boldsymbol{\omega}(t)$ is postulated in the form

$$\dot{\boldsymbol{\omega}}(t) = \hat{\boldsymbol{\Omega}}\big(\mathbf{E}(t), \vartheta(t), \dot{\mathbf{E}}(t), \boldsymbol{\omega}(t), \varphi\big), \qquad t \in [0, d_{\mathrm{P}}] \tag{2.4}$$

with the initial value as follows

$$\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0 = (\mathbf{E}_p^0, \varkappa^0), \tag{2.5}$$

where d_{P} denotes the duration of the process considered.

(1) In the paper [12] an additional internal state variable $\xi(t)$ was considered. The parameter $\xi(t$ was interpreted as a scalar measure of the concentration of defects, inclusions and imperfections. The extension of present theory for this parameter and pertinent evolution equation of diffusion type is straightforward.

The vectorial function $\hat{\mathbf{\Omega}}$ depends on the strain tensor $\mathbf{E}(t)$, the strain rate tensor $\dot{\mathbf{E}}(t)$, the internal state vector $\boldsymbol{\omega}(t)$ and the scalar control function $\varphi \in U$, where

$$(2.6)\qquad U = \{\varphi\colon \max|\varphi| < M,\quad \lim_{\mathrm{I}_2 \to \mathrm{I}_2^s} \varphi = 0,\quad \varphi(0) = 0,\quad \varphi(\cdot) = 0,\quad \text{for}\quad \mathrm{I}_2 < \mathrm{I}_2^s\},$$

$$(2.7)\qquad \mathrm{I}_2 = (\mathrm{II}_{\dot{\mathbf{E}}})^{\frac{1}{2}},\ \mathrm{I}_2^s = (\mathrm{II}_{\dot{\mathbf{E}}_s})^{\frac{1}{2}}.$$

The second invariant I_2^s is called the static value of strain rate measure. We define I_2^s as such value for which, in a test under combined stress conditions with $\mathrm{I}_2 \leqslant \mathrm{I}_2^s$, there is no rate sensitivity effect observed.

The reason why we introduce the scalar control function φ in the evolution equation (2.4) is that it helps to describe the properties of a material in a range of strain rates near the static value (say $\mathrm{I}_2 = \mathrm{I}_2^s$).

3. Physical foundations(2)

It is assumed that viscoplastic strain is produced by the expansion of dislocation loops over the glide planes of active slip systems. This means that our considerations are limited to the region of moderate temperature in which the mechanisms of twinning, dislocation climb or diffusion are negligible. The mean value of the viscoplastic shear strain produced by the expansion of dislocation loops in a single slip system can be expressed as follows (cf. GILMAN [6], KRÖNER and TEODOSIU [8] and KOCKS *et al.* [7]):

$$(3.1)\qquad \gamma(t) = \frac{\mathrm{b}S(t)}{\Delta V},$$

where b denotes the length of Burgers vector and $S(t)$ is the area swept out by all dislocation loops of the considered slip system, in the time interval $[t_0, t]$, which can be determined in the following way

$$(3.2)\qquad S(t) = \int_{l_0}^{l(t)} u(\lambda, t)\,d\lambda,$$

where $l_0 = l(t_0)$, $l(t)$ is the total length of the mobile dislocation lines at time t and $u(\lambda, t)$ is the displacement of the dislocation line at the point parametrized by λ, $\lambda \in [l_0, l(t)]$.

The product $\mathrm{b}S(t)$ is averaged over certain volume of crystalline material ΔV. The dimension of this volume should be larger in comparison with the mean separation distance between crystal defects.

Differentiating (3.1) with respect to t, for the fixed crystalline volume ΔV we obtain the relation for the shear strain rate

$$(3.3)\qquad \dot{\gamma}(t) = \frac{\mathrm{b}\dot{S}(t)}{\Delta V}.$$

(2) This point is discussed more thoroughly elswhere (cf. PĘCHERSKI [17]).

Application to the integral with parameter (3.2) of the Leibniz rule of differentiation yields

$$\dot{S}(t) = \int_{l_0}^{l(t)} \dot{u}(\lambda, t)d\lambda + u[l(t), t]\dot{l}(t). \tag{3.4}$$

The first part of (3.4) corresponds to the rate of change of the glide area swept out by all dislocation loops moving at the instant t. The symbol $\dot{u}(\lambda, t)$ denotes the expansion velocity of the dislocation line at the point parametrized by λ. The mean expansion velocity can be expressed as follows

$$v(t) = \frac{1}{l(t)} \int_{l_0}^{l(t)} \dot{u}(\lambda, t)d\lambda. \tag{3.5}$$

The second part of (3.4) pertains to the rate of change of the glide area produced by the change of the total length of the mobile dislocation lines at time t. In the case of the net increase of the total length $l(t)$ this means that in the time increment Δt the newly-generated dislocation segment of the length $\dot{l}(t)\Delta t$ moves at the distance $u[l(t), t]$ and sweeps out the new glide area

$$\Delta S(t) = u[l(t), t]\dot{l}(t)\Delta t. \tag{3.6}$$

Taking into account the fact that the new dislocation loops generated by the dislocation source expand very fast comparing with the waiting time t_w at short range obstacles and the transition time along the mean free path L we can assume that the newly-generated dislocations appear immediately at the nearest short range obstacle. Then, the displacement $u[l(t), t]$ corresponds in such a case to the mean separation distance of short range obstacles d.

From (3.4) and (3.5) we have

$$\dot{S}(t) = l(t)v(t) + d\dot{l}(t) \tag{3.7}$$

and due to (3.3)

$$\dot{\gamma}(t) = \mathrm{b}\varrho_M(t)v(t) + \mathrm{b}d\dot{\varrho}_M(t), \tag{3.8}$$

where

$$\varrho_M(t) = \frac{\mathrm{b}l(t)}{\Delta V} \quad \text{and} \quad \dot{\varrho}_M(t) = \frac{\mathrm{b}\dot{l}(t)}{\Delta V} \tag{3.9}$$

correspond to the mobile dislocation density and the rate of change of mobile dislocation density, respectively.

For $\varrho_M(t) = \text{const}$ (3.8) transforms into the well known Orowan's relation

$$\dot{\gamma}(t) = \mathrm{b}\varrho_M v(t). \tag{3.10}$$

The rate of change of the mobile dislocation density $\dot{\varrho}_M(t)$ is the result of the production rate of dislocation sources $\dot{\varrho}_M^+(t)$ and the rate of dislocation immobilization after the flight with the mean expansion velocity $v(t)$ over the mean free path L (cf. e.g. KOCKS *et al.* [7] and KRÖNER and TEODOSIU [8]):

$$\dot{\varrho}_M(t) = \dot{\varrho}_+^M(t) - \frac{\varrho_M(t)v(t)}{L}. \tag{3.11}$$

If $\varrho_M(t) = \text{const}$

$$\dot{\varrho}_M^+(t)L = \varrho_M v(t) \tag{3.12}$$

and (3.10) yields the following equivalent Orowan's relation expressed in terms of the production rate of dislocation sources

$$\dot{\gamma}(t) = \mathrm{b}\dot{\varrho}_M^+(t)L. \tag{3.13}$$

From (3.8) and (3.11) we have

$$\dot{\gamma}(t) = \mathrm{b}d\dot{\varrho}_M^+(t) + \mathrm{b}\varrho_m(t)v(t), \tag{3.14}$$

where

$$\varrho_m(t) = \left(1 - \frac{d}{L}\right)\varrho_M(t). \tag{3.15}$$

Usually $d \ll L$ but in the case when $d \approx L$ the relation (3.14) transforms into the Orowan's relation (3.13).

Due to (3.14) the viscoplastic shear strain rate $\dot{\gamma}(t)$ is determined by the production rate of new mobile dislocations and the motion of dislocation lines opposed by short range obstacles over the mean free path L.

The analysis of the known mechanisms of dislocation generation leads to the conclusion that they are governed by the shear stress equal to the critical athermal strength. On the other hand, the dislocation mobility is controlled by the mechanisms of thermally activated surmounting of short range obstacles. In this mechanism the effective stress, $\tau^* = \tau - \tau_\mu$ plays the decisive role.

Taking into account (3.14) and specifying the relation for the velocity $v(t)$ on the basis of the theory of thermally activated mobility of diclocations with the reverse jumps at very low effective stress (cf. KOCKS *et al.* [7]) we have the following relation

$$\dot{\gamma}(t) = \mathrm{b}d\dot{\varrho}_M^+(t) + \mathrm{b}\varrho_m(t)v_0 \exp\left\{ -\frac{\Delta G\left[A^*\left(\frac{\tau(t)}{\tau_\mu} - 1\right)\right]}{k\vartheta} \right\} \times \left[1 - \exp\frac{-\left(\frac{\tau(t)}{\tau_\mu} - 1\right)\tau_\mu A_r b}{k\vartheta}\right], \tag{3.16}$$

where the function $\Delta G(\cdot)$ denotes the free activation enthalpy, $\tau(t)$ is the applied shear stress and τ_μ is the athermal shear strength. The symbol A_r corresponds to the activation area swept out after a succesful reverse jump of a dislocation segment, k is the Boltzmann constant, ϑ denotes temperature whereas v_0 and A^* are the material parameters which can be specified for the particular mechanism of thermally activated dislocation mobility (cf. KOCKS *et al.* [7]).

4. Phenomenological model for a single slip system

The discussed physical relations make a basis for the formulation of the phenomenological model suitable for the application in the constitutive modelling of crystalline material behaviour. Thus, equation (3.16) leads to the following phenomenological generalization (cf. PĘCHERSKI [16]):

$$\bar{\gamma}(t) = \dot{\gamma}_{qs}+\eta\left\langle\Phi^*\left[A^*(\vartheta, \tau_\mu)\left(\frac{\tau}{\tau_\mu}-1\right)\right]\right\rangle\frac{\tau}{|\tau|}, \tag{4.1}$$

where

$$\dot{\gamma}_{qs} \equiv \mathrm{b}L\dot{\varrho}_m^+ \tag{4.2}$$

is the quasi-static strain rate pertinent to the athermal strength τ_μ and

$$\eta \equiv \mathrm{b}\varrho_m v_0 \tag{4.3}$$

is the viscosity parameter whereas the excess stress function $\Phi^*(\cdot)$ and the material function $A^*(\cdot)$ correspond to the phenomenological generalization of the second term in microscopic relation (3.16)

$$\Phi^*\left[A^*(\vartheta, \tau_\mu)\left(\frac{\tau}{\tau_\mu}-1\right)\right] \equiv \exp\frac{-\Delta G\left[A^*\left(\frac{\tau}{\tau_\mu}-1\right)\right]}{k\vartheta}\left[1-\exp\frac{-\left(\frac{\tau}{\tau_\mu}-1\right)\tau_\mu A_r b}{k\vartheta}\right]. \tag{4.4}$$

The idea of the phenomenological excess stress function $\Phi^*(\cdot)$ is known from the theory of viscoplasticity (PERZYNA [11]).

The symbol $\Phi^*(\cdot)$ is defined as follows

$$\langle\Phi^*(\cdot)\rangle = \begin{cases}\Phi^*(\cdot), & \tau > \tau_\mu,\\ 0, & \tau \leqslant \tau_\mu,\end{cases} \tag{4.5}$$

where the property $\Phi^*(0) = 0$ results directly from (4.4).

The evolution equation (4.1) for the strain rate $\dot{\gamma}(t)$ can be rewritten into the following equivalent form (PĘCHERSKI [16])

$$\dot{\gamma}(t) = \frac{\eta}{1-\dfrac{\dot{\gamma}_{qs}}{\dot{\gamma}(t)}}\left\langle\Phi^*\left[A^*(\vartheta, \tau_\mu)\left(\frac{\tau_\mu}{\tau}-1\right)\right]\right\rangle\frac{\tau}{|\tau|}. \tag{4.6}$$

To describe phenomenologically internal structural changes on the level of a single slip system additional structural variable should be specified. This is the athermal strength τ_μ which is responsible for strain-hardening. The analysis of the physical theories of strain-hardening (cf. e.g. KUHLMANN–WILSDORF [9] and MEKING and KOCKS [10]) leads to the phenomenological evolution equation

$$\dot{\tau}_\mu(t) = H(\vartheta, \gamma, \tau_\mu)\dot{\gamma}(t), \qquad \tau_\mu(t_0) = \tau_{\mu 0}, \tag{4.7}$$

where the symbol $H(\cdot)$ denotes the material function which should be determined experimentally.

Let us observe that according to the general material structure considered on the level of a single slip system the internal state vector $\boldsymbol{\omega}(t)$ can be identified as follows

$$\boldsymbol{\omega}(t) = \big(\gamma(t),\, \tau_\mu(t)\big) \tag{4.8}$$

and the evolution equation for the internal state vector $\boldsymbol{\omega}(t)$ is represented by the equations (4.6) and (4.7). The scalar control function φ considered in (2.4) and (2.6) is represented by

$$\varphi(\cdot) = 1 - \frac{\dot{\gamma}_{qs}}{\dot{\gamma}(t)}. \tag{4.9}$$

Let us consider the limit case when $\tau \to \tau_\mu$. In such a situation (cf. PĘCHERSKI [16])

$$\frac{\eta}{1-\dfrac{\dot{\gamma}_{qs}}{\dot{\gamma}(t)}} \to \infty \quad \text{and} \quad \Phi^*\left[A^*(\vartheta, \tau_\mu)\left(\frac{\tau}{\tau_\mu}-1\right)\right] \to 0 \tag{4.10}$$

and according to (4.5) the strain rate $\dot{\gamma}(t)$ becomes undetermined. This corresponds to the rate independent plastic glide which occurs if the applied shear stress satisfies the condition

$$\tau(t) = \tau_\mu(t) \quad \text{and} \quad \dot{\tau}(t) = \dot{\tau}_\mu(t). \tag{4.11}$$

In such a case the strain rate $\dot{\gamma}(t)$ is determined from (4.7) and (4.11)

$$\dot{\gamma}(t) = \frac{1}{H(\vartheta, \gamma, \tau_\mu)}\{\dot{\tau}(t)\}, \qquad \gamma(t_0) = \gamma_0, \tag{4.12}$$

where

$$\{\dot{\tau}(t)\} = \begin{cases} \dot{\tau}(t), & \tau(t) = \tau_\mu(t) \quad \text{and} \quad \dot{\tau}(t) \geqslant 0, \\ 0, & \tau(t) = \tau_\mu(t) \quad \text{and} \quad \dot{\tau}(t) < 0 \quad \text{or} \quad \tau(t) < \tau_\mu(t). \end{cases} \tag{4.13}$$

The phenomenological model of plastic glide in a single slip system derived from the physical considerations gives the uniform description of plastic deformation in an extensive range of strain rates and encompasses rate-sensitive as well as rate-independent behaviour of material. As it was underlined previously (PERZYNA [13]), such a uniform description appears very useful in the analysis of deformation instability phenomena in rate sensitive materials.

5. Formulation of constitutive equations of modified theory of viscoplasticity

The equations of phenomenological model for a single slip system motivate the formulation of the following constitutive equations of modified theory of viscoplasticity (cf. PERZYNA and WOJNO [14] and PERZYNA [12])

$$\dot{\mathbf{E}}_p(t) = \frac{\eta\big(\vartheta(t), \mathbf{E}_p(t)\big)}{\varphi(\cdot)}\left\langle \Phi\left[A\big(\vartheta(t), \varkappa(t)\big)\left(\frac{f(\cdot)}{\varkappa(t)}-1\right)\right]\right\rangle \frac{\partial f(\cdot)}{\partial \mathbf{T}(t)},$$
$$\mathbf{E}_p(t_0) = \mathbf{E}_{p0}, \tag{5.1}$$

$$\dot{\varkappa}(t) = \operatorname{tr}[\hat{\mathbf{K}}_1(\sigma)\dot{\mathbf{E}}_p(t)], \qquad \varkappa(t_0) = \varkappa_0, \tag{5.2}$$

where $\eta(\vartheta(t), \mathbf{E}(t))$ and $A(\vartheta(t), \varkappa(t))$ are material functions,

$$f(\cdot) = f(\mathbf{T}(t), \mathbf{E}_p(t), \vartheta(t)) \tag{5.3}$$

denotes the quasi-static yield function (loading function) and the symbol $\langle\Phi(\cdot)\rangle$ is understood according to the definition

$$\langle\Phi(\cdot)\rangle = \begin{cases} 0 & \text{if} \quad f(\cdot) \leqslant \varkappa(t), \\ \Phi(\cdot) & \text{if} \quad f(\cdot) > \varkappa(t). \end{cases} \tag{5.4}$$

The scalar control function $\varphi(\cdot)$ can be assumed, due to (4.6), as follows

$$\varphi(\cdot) = 1 - \frac{\mathrm{I}_2^s}{\mathrm{I}_2(t)} \tag{5.5}$$

or as it was postulated previously (PERZYNA [12])

$$\varphi(\cdot) = \varphi\left(\frac{\mathrm{I}_2(t)}{\mathrm{I}_2^s} - 1\right). \tag{5.6}$$

It is noteworthy that for (3)

$$\|\dot{\mathbf{E}}_p(t) - \dot{\mathbf{E}}_s\| = 0 \Rightarrow \mathrm{I}_2(t) = \mathrm{I}_2^s \tag{5.7}$$

the evolution equations (5.1) and (5.2) as well as the constitutive equation (2.3) lead to the following results describing the elastic-plastic work-hardening materials

$$\dot{\mathbf{E}}_p(t) = \Lambda \partial_{\mathbf{T}(t)} f(\cdot), \quad f(\cdot) = \varkappa(t), \quad \mathrm{tr}(\partial_{\mathbf{T}} f \dot{\mathbf{T}}) + \partial_\vartheta f \dot{\vartheta} > 0, \tag{5.8}$$

$$\dot{\varkappa}(t) = \mathrm{tr}[\hat{\mathbf{K}}(\sigma) \dot{\mathbf{E}}_p(t)], \tag{5.9}$$

where

$$\Lambda = \{\mathrm{tr}[(\hat{\mathbf{K}} - \partial_{\mathbf{E}_p} f) \partial_{\mathbf{T}} f]\}^{-1} [\mathrm{tr}(\partial_{\mathbf{T}} f \dot{\mathbf{T}}) + \partial_\vartheta f \dot{\vartheta}]. \tag{5.10}$$

Since the tensorial material functions $\hat{\mathbf{T}}$ and $\hat{\mathbf{K}}$ are the same for the elastic-viscoplastic response of a material as well as for the elastic-plastic range, we can assume these functions based on the results for the theory of plasticity.

6. Discussion of experimental results

The discussion of experimental results obtained for different materials in the extensive range of strain rates and determination of the material functions of modified viscoplasticity theory are given in [12]. These results are pertinent to the yield point and small plastic strains, only.

Recently, the new vast experimental data have been published by ALBERTINI *et al.* [4], ALBERTINI and MONTAGNANI [3] and ALBERTINI *et al.* [5] for advanced strains up to fracture in the form of the uniaxial flow curves of the steels AISI 304 L, AISI 316 L and AISI 321 virgin, welded and irradiated at strain rates ranging between 10^{-2} and 10^3 s^{-1} and at the test temperature 20°C, 400°C, 550°C, 750°C and 950°C. In order to relate the flow stress to the applied strain and strain rate, the following two devices to perform uniaxial tension tests at constant strain rate were used (cf. ALBERTINI *et al.* [4]):

(3) The norm $\|\cdot\|$ is understood as the natural norm in the space of strain rate tensor $\dot{\mathbf{E}}$.

1. A hydropneumatic machine where the displacement velocity of a gas-driven light weight piston is controlled by the flow of water through a calibrated orifice. This machine gives strain rates ranging between 10^{-1} and 10^{2} s^{-1}.

2. A modified Hopkinson bar with a prestressed bar loading device. A tension wave with a rise time shorter than $25 \cdot 10^{-6}$ s is transmitted along the bar by the rupture of a brittle bridge and acts on the specimen, bringing it to rupture. This machine allows testing at strain rates ranging between 10^{2} and 10^{3} s^{-1}.

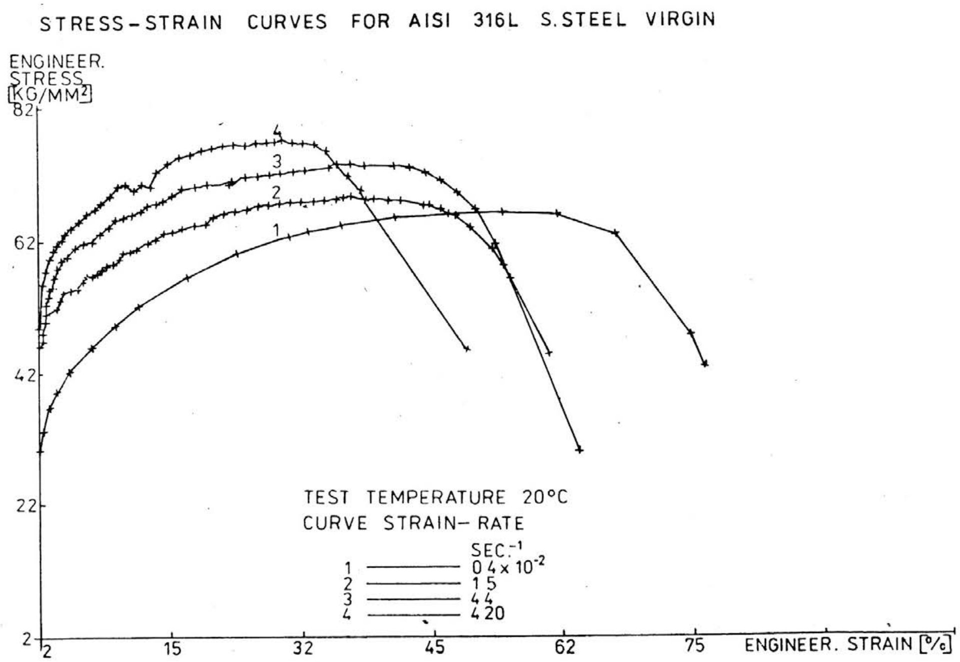

FIG 1. Stress-strain curves for AISI 316L stainless steel (ALBERTINI and MONTAGNANI [3]).

These techniques are discussed more thoroughly by ALBERTINI and MONTAGNANI [1, 2, 3].

As an example of the mentioned results Fig. 1 shows the stress-strain curves of virgin AISI 316L at various strain rates and at the test temperature 20°C.

7. Identification procedure of the material functions

The results shown in Fig. 1 are applied for the demonstration of an example of identification procedure of the material functions occurring in the one-dimensional form of the constitutive equation of modified theory of viscoplasticity (5.1) at room temperature

$$\dot{E}_p = \frac{\eta(E_p)}{\varphi(\cdot)} \left\langle \Phi \left[A(Y_s) \left(\frac{T}{Y_s} - 1 \right) \right] \right\rangle, \tag{7.1}$$

where T and E_p correspond to tensile engineering stress and engineering strain, respectively, whereas Y denotes the quasi-static tensile flow strength.

To fit the experimental results under consideration the evolution equation for work-hardening (5.2) is substituted for simplicity by the quasistatic flow strength-plastic strain function

$$Y_s = \hat{Y}(E_p). \tag{7.2}$$

Two cases of the identification procedure of the material functions of the evolution equation (7.1) are considered. In the first one the power-like excess stress function $\Phi(\cdot)$ and linear form of the scalar control function $\varphi(\cdot)$ as in (5.5) are assumed

$$\dot{E}_p = \frac{\eta(E_p)}{1-\dfrac{\dot{E}_p^s}{\dot{E}_p}}\left\langle\left(\frac{T}{\hat{Y}_s(E_p)}-1\right)^n\right\rangle, \tag{7.3}$$

where $n = 5$, $A(Y_s) = 1$ and the material functions have the form

$$\eta(E_p) = \eta_0 + aE_2^p \quad \text{and} \quad \hat{Y}_s(E_p) = Y_{s0} + KE_p^b. \tag{7.4}$$

The equation (7.3) gives the relation for the dynamic yield strength

$$Y = \hat{Y}_s(E_p)\left\{1+\left[\frac{1}{\eta(E_p)}(\dot{E}_p-\dot{E}_p^s)\right]^{\frac{1}{n}}\right\} \tag{7.5}$$

which is compared in Fig. 2 with the experimental stress-strain curves obtained at four

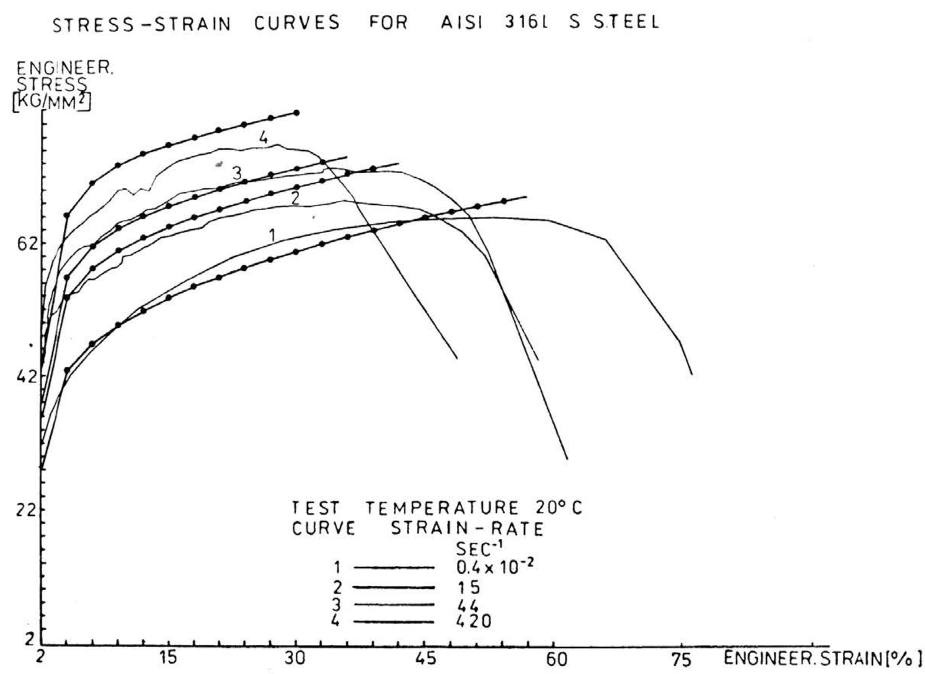

FIG. 2. Comparison of the theoretical prediction due to Eq. (7.5) —·— with experimental curves.

different strain rates. The material constants Y_{s0}, K and b were determined from the quasistatic curve by the power curve fitting method with use of the least square method:

$$Y_{s0} = 25\left[\frac{\text{kG}}{\text{mm}^2}\right], \quad K = 50\left[\frac{\text{kG}}{\text{mm}^2}\right], \quad b = 0.38.$$

Similarily, the linear regression was used in fitting the function (7.5), for the strain rates: $\dot{E} = 4 \cdot 10^{-3}$, 15, 44 and 420 s^{-1} under constant plastic strain and for the given power $n = 5$, to the given experimental results shown in Fig. 1:

$$Y = A + BX, \tag{7.6}$$

where

$$X = (\dot{E}_p - \dot{E}_p^s)^{0,2} \tag{7.7}$$

is a variable and

$$A = Y_s(E_p), \quad B = \frac{Y_s(E_p)}{\eta^{0.2}(E_p)} \tag{7.8}$$

are the linear regression coefficients. Repeating this procedure for the sequence of the values of E_p, the function $\eta(E_p)$ is obtained and constants η_0 and a are determined:

$$\eta_0 = 2556.7355\ (\text{s}^{-1}), \quad a = 380\,000\ (\text{s}^{-1}).$$

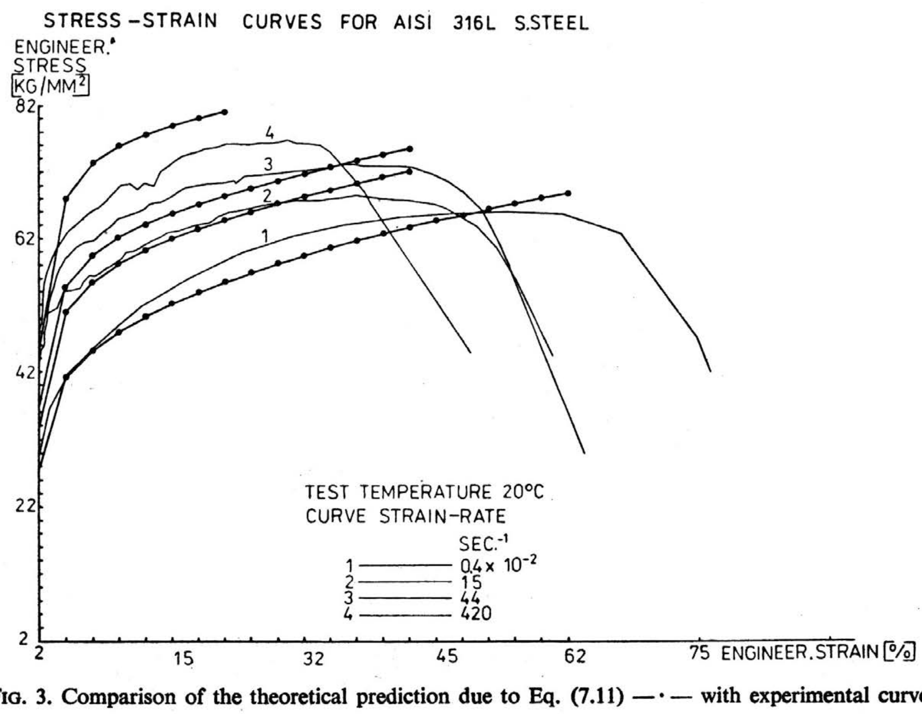

FIG. 3. Comparison of the theoretical prediction due to Eq. (7.11) —·— with experimental curves.

In the second case of identification procedure the powerlike excess-stress function $\Phi(\cdot)$ and linear form of the scalar control function $\varphi(\cdot)$ as in (5.6) are assumed

$$\dot{E}_p = \frac{\eta(E_p)}{\dfrac{\dot{E}_p}{\dot{E}_p^s}-1}\left\langle\left(\frac{T}{\hat{Y}_s(E_p)}-1\right)^n\right\rangle, \tag{7.9}$$

where $n = 7$, $A(Y_s) = 1$ and the quasi-static stress-strain curve is given as in $(7.4)_2$. Following the procedure mentioned the material function $\eta(E_p)$ is determined in the form

$$\eta(E_p) = 1.5\cdot 10^9 + 9.8\cdot 10^{11} E_p^{2.5}. \tag{7.10}$$

The comparison of the relation for the dynamic yield strength obtained from (7.9

$$Y = \hat{Y}_s(E_p)\left\{1+\left[\frac{1}{\eta(E_p)}\left(\frac{\dot{E}^2}{\dot{E}_p^s}-1\right)\right]^{\frac{1}{n}}\right\} \tag{7.11}$$

with the experimental stress-strain curves is shown in Fig. 3.

The plots in Figs. 4 and 5 show the dependence of the yield strength on strain rate calculated from (7.5) and (7.11) for different plastic strains. The pertinent experimental points taken from Fig. 1 are given for comparison.

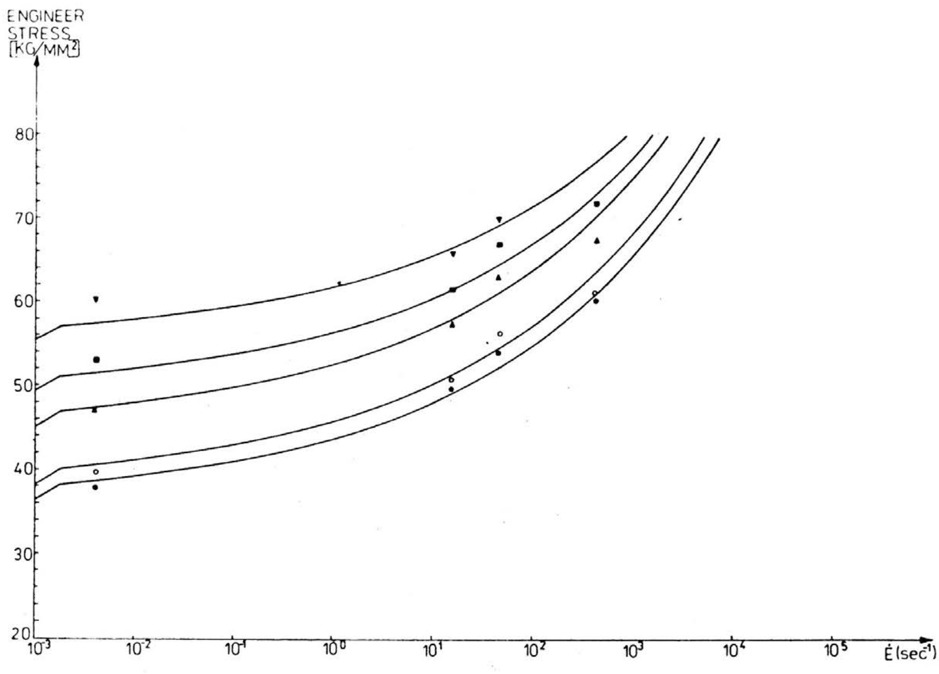

FIG. 4. Comparison of the dependence of the flow strength on strain rate corresponding to Eq. (7.5) with experimental data.

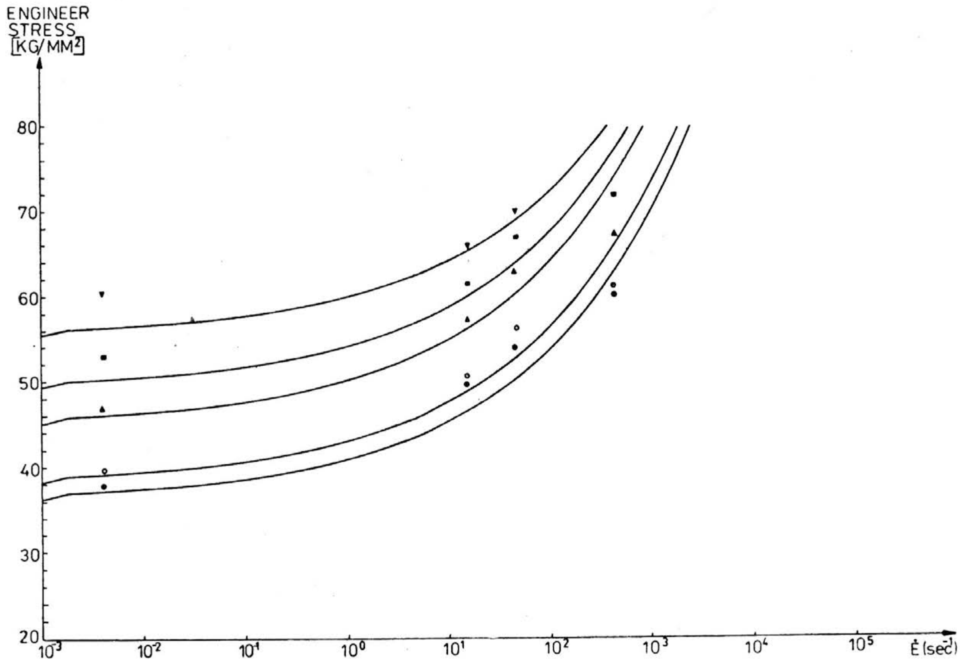

FIG. 5. Comparison of the dependence of the flow strength on strain rate corresponding to Eq. (7.11) with experimental data.

8. Concluding remarks

The theoretical predictions shown in Figs. 2–5 confirm rather good with experiment. However, higher number of tests as far as strain rate is considered would improve the theoretical description of experimental curves. Furthermore, it can be concluded that the first case of identification procedure, given by (7.3), gives more reasonable, from physical point of view, value of viscosity parameter η.

The proposed identification procedure can be applied for the theoretical approximation of the experimental results with an account of temperature and irradiation effects. In such a case the material function $A(\vartheta, Y_s)$ will play an important role (cf. PĘCHERSKI [15]).

The determination of material constants and functions discussed in the paper is based on simple regression methods applicable for calculations with hand-held programmable calculator and should be considered as an example of identification procedure which shows that the constitutive equations of modified theory of viscoplasticity give reasonable prediction of experiment. In the case when higher number of experimental data is considered and the additional effects of temperature, irradiation and imperfections introduced e.g. by welding are included, the more developed computer routines of curve fitting should be implemented.

Aknowledgements

The authors would like to express thanks to Professors Carlo ALBERTINI, Sergio CURIONI and Ricardo CENERINI as well as Mario MONTAGNANI for many fruitful discussions on experimental techniques and results obtained. The financial support of C.N.R. which enabled the cooperation whithin the scientific exchange programme between Polish Academy of Sciences and Consiglio Nazionale delle Ricerche is kindly aknowledged, too.

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POLISH ACADEMY OF SCIENCES
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Received January 11, 1983.