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NOTE ON THE RECTANGULAR HYPERBOLA.

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EVERY conic which passes through the points of intersection of two rectangular hyperbolas is a rectangular hyperbola. In fact if a conic be referred to rectangular axes, the condition that it may be a rectangular hyperbola is *Coeff. of* $x^2 = -$ *Coeff. of* y^2. Hence if U, V be any two quadratic functions of x, y, and if λ be a constant, the condition in question being satisfied for each of the functions U, V, is satisfied for the function $U+\lambda V$: and the equation of any conic through the points of intersection of the conics $U=0$, $V=0$ is $U+\lambda V=0$: which proves the theorem in question.

In particular if from two of the angles of a triangle perpendiculars are let fal on the opposite sides, and if the point of intersection of the perpendiculars and the third angle be joined: then since the first side and the perpendicular upon it are a rectangular hyperbola, and the second side and the perpendicular upon it are a rectangular hyperbola; the third side and the joining line must be a rectangular hyperbola: that is, these two lines must be at right angles to each other. We have thus the well-known theorem that the perpendiculars let fall from the angles of a triangle on the opposite sides meet in a point.

The theorem as to the hyperbolas is a particular case of the theorem that three conics which pass through the same four points are met by any line whatever in six points forming a system in involution. In fact a rectangular hyperbola is a conic meeting the line at infinity in two points harmonically related to the circular points at infinity: hence two of the conics being rectangular hyperbolas, the foci of the involution are the circular points at infinity: hence these points and the points in which the line at infinity meets the third conic are harmonically related to each other; that is, the third conic is a rectangular hyperbola.