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Flow through a porous medium in the presence of mass transfer and free convection flow

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TAKING the general form of the Darcy law, a flow through a porous medium bounded by a plate is analysed in the presence of free convection and mass transfer flow and free stream velocity.

Opierając się na ogólnej postaci prawa Darcy przeanalizowano problem przepływu przez ośrodek porowaty ograniczony płytą, uwzględniając konwekcję swobodną i przepływ masy.

Опираясь на общий вид закона Дарси, проанализирована проблема течения через пористую среду ограниченную плитой, учитывая свободную конвекцию и перенос массы.

Nomenclature

C^+ species concentration,
C nondimensional species concentration,
C_w^+ species concentration at the plate,
C_∞^+ species concentration at the free stream,
c_p specific heat of the fluid at constant pressure,
D chemical molecular diffusivity,
E Eckert number,
Gr Grashof number,
Gc modified Grashof number,
g_x^+ gravitational acceleration,
K^+ permeability of the porous medium,
K nondimensional permeability,
k thermal conductivity,
P Prandtl number,
p^+ pressure,
Sc Schmidt number,
T^+ dimensional temperature of the fluid,
T nondimensional temperature,
T_w^+ temperature of the plate,
T_∞^+ temperature of the fluid in the free stream,
u^+ velocity component in the x^+-direction,
U_∞^+ free stream velocity,
v^+ velocity component in the y^+-direction,
β volumetric coefficient of thermal expansion,
β^* volumetric coefficient of expansion with concentration,
μ viscosity,
ν kinematic viscosity,
ϱ density.

1. Introduction

RECENTLY published papers by RAPTIS *et al.* [1, 2, 3] and RAPTIS [4] presented an analytical study of the free convection and mass transfer flow through a very porous medium bounded by an infinite plate, when there is no free-stream velocity. The object of the present paper is to study the effects of free convection and mass transfer flow through a very porous medium bounded by an infinite vertical porous plate when there is free-stream velocity. The porous plate is subjected to a constant suction, the temperature and the species concentration at the plate are constant and the flow is steady. The results of the work are important in geophysics [5].

2. Mathematical analysis

In order to formulate the problem mathematically, we write down the equations of fluid motion, for two-dimensional steady free convection and mass transfer flow of an incompresible and viscous fluid through a very porous medium, occupying a semi-infinite region of the space and bounded by an infinite vertical porous plate. We assume also that the fluid properties are not affected by temperature and concentration differences except the denisty in the body force term. With the x^+-axis along the vertical plate in the upward direction and the y^+-axis normal to it and the above assumption, the physical variables are functions of y^+ only, except the pressure p^+, and therefore the equations which govern the problem are [1, 6]:

Continuity equation:

$$\frac{\partial u^+}{\partial y^+} = 0. \tag{2.1}$$

Momentum equations:

$$\varrho u^+ \frac{\partial u^+}{\partial y^+} = -\frac{\partial p^+}{\partial x^+} - \varrho g_{x^+} + \mu \frac{\partial^2 u^+}{\partial y^{+2}} - \frac{\mu}{K^+} u^+, \tag{2.2}$$

$$0 = -\frac{\partial p^+}{\partial y^+} - \frac{\mu}{K^+} v^+. \tag{2.3}$$

Energy equation:

$$v^+ \frac{\partial T^+}{\partial y^+} = \frac{k}{\varrho c_p} \frac{\partial^2 T^+}{\partial y^{+2}} + \frac{\nu}{c_p} \left(\frac{\partial u^+}{\partial y^+}\right)^2. \tag{2.4}$$

Diffusion equation:

$$v^+ \frac{\partial C^+}{\partial y^+} = D \frac{\partial^2 C^+}{\partial y^{+2}}. \tag{2.5}$$

The last terms on the r.h.s of Eqs. (2.2) and (2.3) signify the additional resistance due to the porous medium. The boundary conditions are

$$\begin{gathered} u^+ = 0, \quad T^+ = T_w^+, \quad C^+ = C_w^+, \quad \text{at} \quad y^+ = 0 \\ u^+ \to U_\infty^+, \quad T^+ \to T_\infty^+, \quad C^+ \to C_\infty^+ \quad \text{as} \quad y^+ \to \infty, \\ (T_w^+ > T_\infty^+, \quad C_w^+ > C_\infty^+). \end{gathered} \tag{2.6}$$

From Eq. (2.2) we have for the free-stream

$$0 = -\frac{\partial p^+}{\partial x^+} - \varrho_\infty g_{x^+} - \frac{\mu}{K^+} U_\infty^+. \tag{2.7}$$

Eliminating $-\dfrac{\partial p^+}{\partial x^+}$ between Eqs. (2.2) and (2.7) we obtain

$$\varrho v^+ \frac{\partial u^+}{\partial y^+} = g_{x^+}(\varrho_\infty - \varrho) + \mu \frac{\partial^2 u^+}{\partial y^{+2}} + \frac{\mu}{K^+}(U_\infty^+ - u^+), \tag{2.8}$$

where ϱ_∞ is the density of the flow in the free stream. The equation of state is [6]

$$g_{x^+}(\varrho_\infty - \varrho) = g_{x^+}\beta\varrho(T^+ - T_\infty^+) + g_{x^+}\beta^*\varrho(C^+ - C_\infty^+). \tag{2.9}$$

Substituting Eq. (2.9) in Eq. (2.8), we obtain

$$v^+ \frac{\partial u^+}{\partial y^+} = g_{x^+}\beta(T^+ - T_\infty^+) + g_{x^+}\beta^*(C^+ - C_\infty^+) + \nu \frac{\partial^2 u^+}{\partial y^{+2}} + \frac{\nu}{K^+}(U_\infty^+ - u^+). \tag{2.10}$$

Since the suction is assumed constant, integrating Eq. (2.1) we get

$$v^+ = -v_0, \tag{2.11}$$

where v_0 is the constant suction velocity at the plate. The negative sign in Eq. (2.11) indicates the suction velocity is directed towards the plate. Substituting Eq. (2.11) in Eqs. (2.10), (2.4) and (2.5), we get

$$-v_0 \frac{\partial u^+}{\partial y^+} = g_{x^+}\beta(T^+ - T_\infty^+) + g_{x^+}\beta^*(C^+ - C_\infty^+) + \nu \frac{\partial^2 u^+}{\partial y^{+2}} + \frac{\nu}{K^+}(U_\infty^+ - u^+), \tag{2.12}$$

$$-v_0 \frac{\partial T^+}{\partial y^+} = \frac{k}{\varrho c_p} \frac{\partial^2 T^+}{\partial y^{+2}} + \frac{\nu}{c_p}\left(\frac{\partial u^+}{\partial y^+}\right)^2, \tag{2.13}$$

$$-v_0 \frac{\partial C^+}{\partial y^+} = D \frac{\partial^2 C^+}{\partial y^{+2}}. \tag{2.14}$$

On introducing the following dimensionless parameters

$$u = \frac{u^+}{U_\infty^+}, \quad y = \frac{y^+ v_0}{\nu}, \quad T = \frac{T^+ - T_\infty^+}{T_w^+ - T_\infty^+},$$

$$C = \frac{C^+ - C_\infty^+}{C_w^+ - C_\infty^+}, \quad (C_w^+ > C_\infty^+) \quad \mathrm{Gr} = \frac{\nu g_{x^+}\beta(T_w^+ - T_\infty^+)}{U_\infty^+ v_0^2} \quad (T_\infty^+ > T_\infty^+),$$

$$\mathrm{Gc} = \frac{v g_{x^+}\beta^*(C_w^+ - C_\infty^+)}{U_\infty^+ v_0^2}, \qquad E = \frac{U_\infty^{+2}}{c_p(T_w^+ - T_\infty^+)}, \quad P = \frac{\varrho \nu c_p}{k},$$

$$\mathrm{Sc} = \frac{\nu}{D}, \quad K = \frac{v_0^2}{\nu^2} K^+,$$

equations (2.12), (2.13) and (2.14) become

$$u'' + u' - \frac{1}{K} u = -\mathrm{Gr}\, T - \mathrm{Gc}\, C - \frac{1}{K}, \tag{2.15}$$

$$T'' + \mathrm{P} T' = -\mathrm{P} E u'^2, \tag{2.16}$$

$$C'' + \mathrm{Sc}\, C' = 0, \tag{2.17}$$

where the primes denote differentation with respect to y. The boundary conditions (2.6) in the nondimensional form become

$$\begin{aligned} &u = 0, \quad T = 1, \quad C = 1 \quad \text{at} \quad y = 0, \\ &u \to 1, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.18}$$

In order to solve the system of Eqs. (2.15)–(2.17) under the boundary conditions (2.18), we expand the velocity and the temperature fields in powers of the Eckert number E which, for incompressible viscous fluids is very small. Hence we can write

$$u(y) = u_0(y) + Eu_1(y) + \ldots, \tag{2.19}$$

$$T(y) = T_0(y) + ET_1(y) + \ldots. \tag{2.20}$$

Substituting the above expressions in Eqs. (2.15) and (2.16), we get the following system:

$$u_0'' + u_0' - \frac{1}{K} u_0 = -\mathrm{Gr}\, T_0 - \mathrm{Gc}\, C - \frac{1}{K}, \tag{2.21}$$

$$u_1'' + u_1' - \frac{1}{K} u_1 = -\mathrm{Gr}\, T_1, \tag{2.22}$$

$$T_0'' + \mathrm{P} T_0' = 0, \tag{2.23}$$

$$T_1'' + \mathrm{P} T_1' = -\mathrm{P} u_0'^2, \tag{2.24}$$

$$C'' + \mathrm{Sc}\, C' = 0, \tag{2.25}$$

while the boundary conditions (2.18) become

$$\begin{aligned} &u_0 = 0, \quad u_1 = 0, \quad T_0 = 1, \quad T_1 = 0, \quad C_0 = 1, \quad C_1 = 0 \quad \text{at} \quad y = 0, \\ &u_0 \to 1, \quad u_1 \to 0, \quad T_0 \to 0, \quad T_1 \to 0, \quad C_0 \to 0, \quad C_1 \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{2.26}$$

Thus the solutions of Eqs. (2.21)–(2.25), under the boundary conditions (2.26) and in view of Eqs. (2.19) and (2.20). are given by

$$\begin{aligned} u = {} & C_1 e^{-\mathrm{P}y} + C_2 e^{-\mathrm{Sc}y} + C_3 e^{R_2 y} + 1 + E(C_{11} e^{2R_2 y} + C_{12} e^{-2\mathrm{P}y} + C_{13} e^{-2\mathrm{Sc}y} \\ & + C_{14} e^{(R_2-\mathrm{P})y} + C_{15} e^{(R_2-\mathrm{Sc})y} + C_{16} e^{-(\mathrm{P}+\mathrm{Sc})y} + C_{17} e^{-\mathrm{P}y} + C_{18} e^{R_2 y}), \end{aligned} \tag{2.27}$$

$$\begin{aligned} T = {} & e^{-\mathrm{P}y} + E(C_4 e^{2R_2 y} + C_5 e^{-2\mathrm{P}y} + C_6 e^{-2\mathrm{Sc}y} + C_7 e^{(R_2-\mathrm{P})y} + C_8 e^{(R_2-\mathrm{Sc})y} \\ & + C_9 e^{-(\mathrm{P}+\mathrm{Sc})y} + C_{10} e^{-\mathrm{P}y}), \end{aligned} \tag{2.28}$$

$$C = e^{-\mathrm{Sc}y}, \tag{2.29}$$

where

$$R_1 = \frac{-1 + \left(1 + \frac{4}{K}\right)^{1/2}}{2}, \qquad R_2 = \frac{-1 - \left(1 + \frac{4}{K}\right)^{1/2}}{2},$$

$$C_1 = -\frac{\mathrm{Gr}}{(\mathrm{P}+R_1)(\mathrm{P}+R_2)}, \qquad C_2 = -\frac{\mathrm{Gc}}{(\mathrm{Sc}+R_1)(\mathrm{Sc}+R_2)},$$

$$C_3 = -(C_1 + C_2 + 1), \qquad C_4 = -\frac{\mathrm{P} R_2 C_3^2}{2(2R_2+\mathrm{P})},$$

$$C_5 = -\frac{\mathrm{P} C_1^2}{2}, \qquad C_6 = \frac{P \mathrm{Sc}\, C_2^2}{2(\mathrm{P}-2\mathrm{Sc})},$$

$$C_7 = \frac{2P^2 C_1 C_3}{(R_2 - P)}, \qquad C_8 = \frac{2R_2 C_2 C_3 \mathrm{Sc}\, P}{(R_2 - \mathrm{Sc})(R_2 - \mathrm{Sc} + P)},$$

$$C_9 = -\frac{2P^2 C_1 C_2}{(P + \mathrm{Sc})}, \qquad C_{10} = \sum_{i=4}^{9} C_i,$$

$$C_{11} = -\frac{\mathrm{Gr}\, C_4}{(2R_2 - R_1) R_2}, \qquad C_{12} = -\frac{\mathrm{Gr}\, C_5}{(2P + R_1)(2P + R_2)},$$

$$C_{13} = \frac{\mathrm{Gr}\, C_6}{(2\mathrm{Sc} + R_1)(2\mathrm{Sc} + R_2)}, \qquad C_{14} = \frac{\mathrm{Gr}\, C_7}{(R_2 - P - R_1) P},$$

$$C_{15} = \frac{\mathrm{Gr}\, C_8}{(R_2 - \mathrm{Sc} - R_1)\mathrm{Sc}}, \qquad C_{16} = -\frac{\mathrm{Gr}\, C_9}{(P + \mathrm{Sc} + R_1)(P + \mathrm{Sc} + R_2)},$$

$$C_{17} = -\frac{\mathrm{Gr}\, C_{10}}{(P + R_1)(P + R_2)}, \qquad C_{18} = -\sum_{i=11}^{12} C_i.$$

Expression (2.27) will be used for numerical calculations for the velocity field while the corresponding expression for the rate of heat transfer through the Nusselt number Nu is given, in view of Eq. (2.28), by

$$\text{(2.30)} \qquad \mathrm{Nu} = -\left(\frac{dT}{dy}\right)_{y=0} = P - E[2R_2 C_4 - 2PC_5 - 2\mathrm{Sc}\, C_6 + (R_2 - P) C_7 + (R_2 - \mathrm{Sc}) C_8 - (P + \mathrm{Sc}) C_9 - PC_{10}].$$

3. Results

The velocity profiles are shown in Fig. 1 and the rate of heat transfer through the Nusselt number Nu in Fig. 2 for the case of cooling (Gr > 0) of the plate by free convection cur-

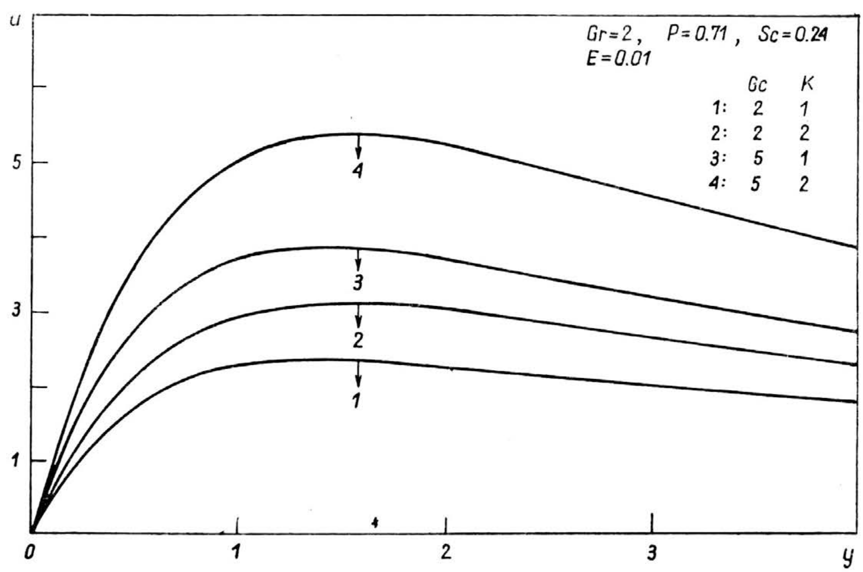

FIG. 1. Velocity profiles.

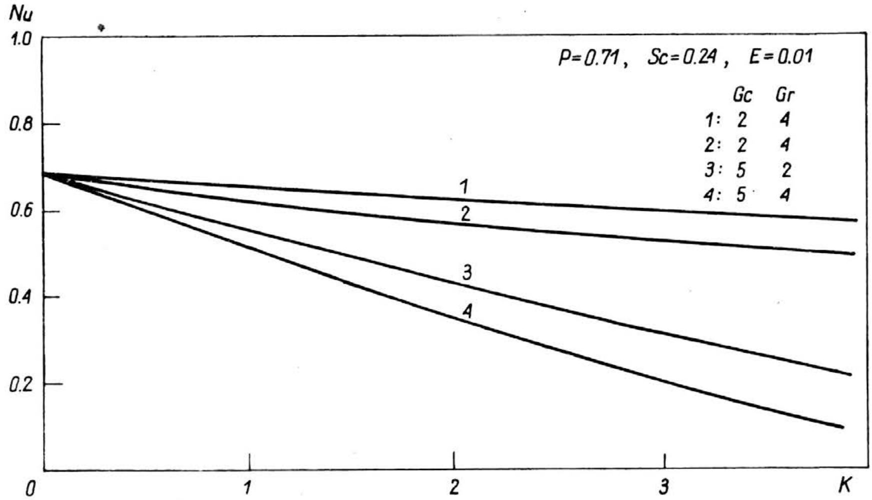

FIG. 2. Nusselt number.

rents. The Prandtl number P is taken equal to 0.71, which corresponds to the air. The Schmidt number Sc(Sc = 0.24) is chosen in such a way as to represent H_2 at low concentration in air at approximately 25°C and 1 atmoshpere. For Fig. 1 we observe that when the permeability parameter K or modified Grashof number Gc increases, the velocity increases, while from Fig. 2 we observe that when the permeability parameter K or modified Grashof number Gc increases, the Nusselt number decreases.

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