550.

PROBLEM AND HYPOTHETICAL THEOREMS IN REGARD TO TWO QUADRIC SURFACES.

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Two conics may be circum-and-inscribable to an n-gon; viz. the conics may be such that there exists a singly infinite series of n-gons each inscribed in the first and circumscribed about the second of the conics. In particular they may be circum-and-inscribable to a triangle.

The following problem arises:

Consider two given quadric surfaces and a given line S; to find the planes through S, which cut the surfaces in two conics circum-and-inscribable to a triangle (it is presumed there are two or more such planes).

Let the surfaces be Θ, Θ', and let the line S a tangent to Θ' meet Θ in the points A, B; if through S we draw two planes as above, then in the first plane the tangents from A, B to the section of Θ' will meet in a point C of Θ; and in the second plane the tangents from A, B to the section of Θ' will meet in a point D of Θ. The points C, D being thus determined the lines AB, AC, BC, AD, BD all touch the surface Θ', and it is presumed that the surfaces Θ, Θ' may be such that CD also touches the surface Θ'; viz. in this case we have a tetrahedron $ABCD$, the summits of which lie in the surface Θ, and the edges touch the surface Θ'; and not only so, but it is further presumed that the surfaces may be such that starting from *any* point A of Θ and using either *any* tangent or a properly selected tangent AB of Θ', it shall be possible to complete the figure as above; or, what is the same thing, the surfaces may be such that there exists a *doubly* or a *triply* infinite series of tetrahedra, the summits of each lying in Θ and its edges touching Θ'. It is also presumed that the faces of the tetrahedra all touch one and the same quadric surface Θ''.