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ACTIVE VIBROACOUSTIC CONTROL OF PLATE STRUCTURES WITH ARBITRARY BOUNDARY CONDITIONS

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Abstract: The paper describes briefly some main aspects of the active feedback control system that has been developed and constructed for reduction of vibroacoustic emission of vibrating plate structures with arbitrary boundary conditions. Relations between the forms and frequency of the vibrations induced by an external harmonic excitation and the distribution of the generated acoustic pressure field are investigated using the developed numerical model based on indirect variational boundary element method. The aim of the control system is to minimize the sound pressure level in a given point of the ambient space. The system uses small, rectangle-shaped piezoelectric transducers as both sensors and actuators. The transducers are connected in a number of independent feedback loops, and the feedback gains are the control parameters which are optimized using the developed optimal control algorithm. The constructed active system has been tested for the stability and control performance during experimental research performed in an anechoic chamber. Results of experiments are presented in the paper, proving a high level of noise reduction and a good agreement with numerical predictions.

Key words: vibroacoustics, active control, plate vibrations.

1. Introduction

The paper describes briefly some main aspects of the active feedback control system that has been developed and constructed for reduction of vibroacoustic emission of vibrating plate structures with arbitrary boundary conditions. The presented investigations fall within the scope of active noise and vibration control methods, which have been intensively developed over the past several decades. A review of various theoretical and practical issues concerning this topic is presented in [1, 2].

In contrast to the most of similar research, this study does not take into account any specific boundary conditions for which analytical solutions of the vibrating plate problem can be given. The vibrational mode shapes and corresponding eigenfrequency values are determined using numerical analyses with the finite element method.

The control system uses piezoelectric sensors and actuators – their position on the surface of the plate determines their ability to sense or induce specific vibrational modes [3, 4]. For the sake of stability it would be most convenient to use a single transducer as both sensor and actuator simultaneously; however, practical implementation of such solutions [5, 6] encounters numerous difficulties resulting from complex properties of the real electronic components. For that reason the described system uses two, asymmetrically placed elements – such a configuration theoretically ensures similar properties without the technological complications mentioned above.

The influence of the inertial loading introduced by the acoustic medium in case of air can be neglected; however, it is important when considering plates submerged in heavy fluids, such as water [7]. For that reason unidirectional structure-medium coupling can be considered. The frequency and form of induced vibrations determine the parameters of the generated acoustic pressure field. Free-field conditions are considered in the present study. The problem of determining the radiation characteristics of the plate is solved numerically, using the developed algorithm implementing indirect variational boundary element method (IVBEM).

The aim of the control system is to modify the form of vibrations in such a way, that the sound pressure level in a selected point of the ambient space would be as low as possible. The only way the system can modify the vibrations is by changing the gains of the feedback loops. The developed algorithm of determining optimal values of those gains will be briefly described below.

2. Free-field acoustic radiation of the vibrating plate structures

2.1. The IVBEM model. The acoustic pressure p in the space surrounding the considered plate structure satisfies the Helmholtz equation:

$$\Delta p + k^2 p = 0, \tag{1}$$

where k is the acoustic wavenumber. The values of the normal velocities for a given form of vibrations determined by the finite element method analysis are imposed as the boundary conditions on the whole surface of the plate, namely:

$$\left.\frac{\partial p}{\partial \mathbf{n}}\right|_{(x,y)} = \omega \rho_a V_n(x, y), \tag{2}$$

where ρ_a is the density of air, $\mathbf{n}$ is the unit vector normal to the surface of the plate and $V_n(x, y)$ is the normal velocity of the point with coordinates (x, y) on the surface of the plate.

The boundary element method is widely used in acoustics (see, for example [10–12]). The only applicable variant of this method in the considered case of an external acoustic problem with open boundary surface is indirect variational boundary element method (IVBEM). The description of IVBEM can be found in [13]. Some examples of practical implementations of this method can be found in [14] and [15].

It can be shown [13] that the sought solution will minimize the following functional:

$$J = 2 \iint_{\Omega} j\omega\rho_a \mu(\mathbf{R}) V_n(\mathbf{R})\, d\Omega(\mathbf{R}) + \iint_{\Omega} \iint_{\Omega} \mu(\mathbf{R})\, \mu(\mathbf{R}_a) \frac{\partial^2 G(\mathbf{R}, \mathbf{R}_a)}{\partial \mathbf{n}(\mathbf{R})\, \partial \mathbf{n}(\mathbf{R}_a)} d\Omega(\mathbf{R})\, d\Omega(\mathbf{R}_a), \tag{3}$$

where $V_n(\mathbf{R})$ denotes the amplitude of the normal velocity at the point on the surface of the plate indicated by the vector $\mathbf{R}$, $\mu(\mathbf{R})$ is the double-layer potential and Ω denotes the surface of the plate.

2.2. Numerical simulations and results. The algorithm solving the problem described with Eqs. (1)–(3) has been implemented in the Matlab environment and used for numerical simulations of the distribution of the acoustic pressure field generated by the vibrating plate. The developed software includes a mesh generator, complete solver as well as post-processing and data visualization modules. The results of the simulations have been compared to the results of the experimental investigations performed in an anechoic chamber. Some exemplary graphs illustrating the predicted and measured distributions of the acoustic pressure in the axis perpendicular to the plate's surface are presented in Fig. 1.

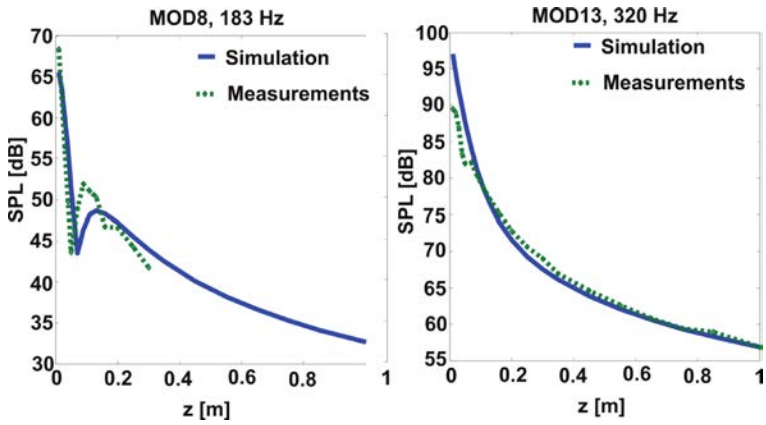

Fig. 1. Results of the measurements and numerical simulations obtained for two (exemplary) vibrational modes.

The comparison of the obtained results reveals good agreement between the numerical predictions and experimental results.

3. The control algorithm

The objective function depends on the modal amplitudes W_N and can be described as follows:

$$f_c(W_1,\ldots,W_N) = p_{re}^2(W_1,\ldots,W_N) + p_{im}^2(W_1,\ldots,W_N) = \left[\sum_{n=1}^{N} P_n^{re} W_n\right]^2 + \left[\sum_{n=1}^{N} P_n^{im} W_n\right]^2, \tag{4}$$

where p_{re} and p_{im} denote the real and imaginary parts of the acoustic pressure which can be decomposed into an infinite series of modal amplitudes with modal radiation coefficients P_n and approximated by the first N components of this series.

The developed optimal control algorithm is complex and includes several subsequent steps. The real and imaginary parts of the acoustic pressure are first computed separately. For each part the gain vector for which a global minimum occurs is determined by iteratively solving Sherman-Morrison equation for the corresponding modal amplitude values. Also a finite number of points with local minimais also determined and saved. Then, the comparison between the results obtained for the real and imaginary parts reveals if there is a common global minimum or – if not – how close is the lowest obtained value to the potentially best global minimum. If the computed value is neither the global minimum nor even close enough to it, then the standard stochastic or heuristic optimization procedures may be used.

4. Results and conclusions

The active control system has been developed and constructed according to the original concept. The system design is completely original and consists of two independent parts. The first, analogue part includes piezoelectric sensor, signal conditioning circuit, variable gain amplifier, power amplifier and piezoactuator. Due to the fact that whole feedback loop does not contain any analog to digital converters no time delays or phase shifts related to conversion and no discretization noise are present in this part of the system. The digital part in which the computations of the optimal gain values occur is just used to drive the variable gain amplifiers.

The system has been tested during the experimental investigations performed in an anechoic chamber. The plate was clamped in the central part of one of its shorter edges while all other edges were free. The excitation force was introduced by one pair of piezoelectric transducers attached to the surface of the plate. The measurements included different points of space inside the chamber and different configurations of involved feedback loops. Some exemplary results obtained in that way are presented in Fig. 2.

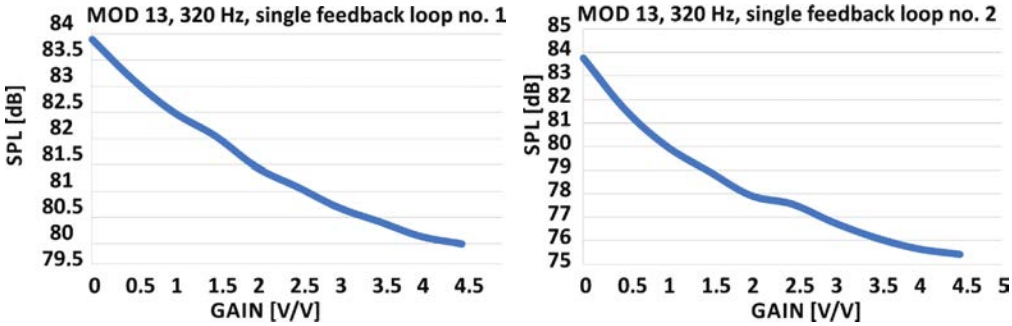

Fig. 2. Measured SPL as a function of feedback gain value, 5 cm from the plate center point in the z axis for two different feedback loops operating alone.

The results clearly state that a high level of noise reduction (up to about 10 dB) may be obtained with the described active control system. However, the attainable control performance depends on many factors, such as the reciprocal relation between the shape functions of the involved vibrational modes and the parameters of the utilized sensor-actuator pairs.

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