892.

NOTE ON THE ORTHOMORPHIC TRANSFORMATION OF A CIRCLE INTO ITSELF.

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THE following is, of course, substantially well known, but it strikes me as rather pretty:—to find the orthomorphic transformation of the circle

$$x^2 + y^2 - 1 = 0$$

into itself. Assume this to be

$$x_1 + iy_1 = \frac{A(x+iy) + B}{1 + C(x+iy)}.$$

Then, writing A', B', C' for the conjugates of A, B, C, we have

$$x_1 - iy_1 = \frac{A'(x-iy) + B'}{1 + C'(x-iy)};$$

and then

$$x_1^2 + y_1^2 = \frac{AA'(x^2+y^2) + AB'(x+iy) + A'B(x-iy) + BB'}{1 + C(x+iy) + C'(x-iy) + CC'(x^2+y^2)},$$

which should be an identity for $x^2 + y^2 = 1$, $x_1^2 + y_1^2 = 1$.

Evidently $C = AB'$, whence $C' = A'B$; and the equation then is

$$1 + AA'BB' = AA' + BB',$$

that is,

$$(1 - AA')(1 - BB') = 0.$$

But $BB' = 1$ gives the illusory result

$$x_1 + iy_1 = B,$$

therefore

$$1 - AA' = 0;$$

and the required solution thus is

$$x_1 + iy_1 = \frac{A(x+iy) + B}{1 + AB'(x+iy)};$$

where A is a unit-vector (say $A = \cos\lambda + i\sin\lambda$) and B, B' are conjugate vectors. Or, writing $B = b + i\beta$, $B' = b - i\beta$, the constants are λ, b, β; three constants as it should be.