893.

THE BITANGENTS OF THE QUINTIC.

(LETTER FROM PROFESSOR CAYLEY TO MR HEAL.)

[From the *Annals of Mathematics*, vol. v. (1890), pp. 109, 110.]

DEAR SIR,

I have to thank you very much for your letter concerning the bitangents of the quintic, and am glad you have obtained more simple numerical coefficients than you had at first. I do not see my way to verifying your result, but assuming it to be correct, it is a very interesting and valuable one. The relation to Salmon's *Higher Plane Curves* (p. 351) is rather curious, viz. the result there given is of the deg-order 18-54, with an extraneous factor $(\alpha x + \beta y + \gamma z)^6$, the rejection of which would reduce it to the proper form, deg-order 18-48; whereas yours is of deg-order 24-66, with an extraneous factor H^2, the rejection of which would reduce it to the same proper form, deg-order 18-48.

But there is another known solution, not by any means a handy one, but which has no extraneous factor*, viz. if

$$\Omega = D^3H - 4D^3H_1 + 6D^3H_2,$$

then the required equation is

$$F\Omega = 0,$$

where the facients of the reciprocant are the first derived functions,

$$\partial_x U, \quad \partial_y U, \quad \partial_z U.$$

Say this is

$$(B^2C^2 - \ldots)(\partial_x U)^6 = 0;$$

the coefficients A, B, C, &c., are those of Ω, viz. these are of deg-order 3-6, and $\partial_x U$, &c., are of deg-order 1-4; so that the deg-order is

$$4\,(3\text{-}6) + 6\,(1\text{-}4) = (12\text{-}24) + (6\text{-}24) = (18\text{-}48),$$

as it should be. It might be worth while to further consider this form, but I doubt whether, practically, the whole question is not too difficult to be worth working at.

I remain, dear sir, yours very sincerely,

A. CAYLEY.

Cambridge, January 17, 1890.

* See *Phil. Trans.*, vol. CXLIX. pp. 193—212, [260].