BOUNDARY LAYER EFFECT AT THE FREE EDGE OF COMPOSITE MATERIAL USING HOMOGENIZATION THEORY

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1. Introduction

Many studies in the theory of composite materials are based on the homogenization approach, which consists of the substitution of the heterogeneous medium by a homogeneous one with certain effective properties [1, 2]. These effective properties are obtained from internal asymptotics which excludes boundary layer effects. On considering the individual ply, at the boundary of the domain one can notice a disruption of periodic arrangement. This disruption leads to possible loss of periodicity, hence the asymptotic approach needs to be corrected [3]. Here, boundary layer study is a correction in the micro solution which accounts for the error in the boundary condition. This error gives rise to a boundary layer which decays very quickly when it travels to the interior of the domain.

2. Asymptotic homogenization for multiple scale analysis

Homogenization methods have proven to be powerful techniques for the study of heterogeneous media [4]. The existence of two length scales such as length scale of the microstructure ε and the length scale of the structure, *L*. The two scales x_i and y_i are spatial variables, where x_i is a macroscopic quantity and $y_i = x_i/\varepsilon$ is a microscopic one as y_i is associated with the length scale of the inclusions or heterogeneities. The solution u_i^ε are approximated with an asymptotic series representation in ε to admit the following ansatz

$$u_i^\varepsilon(\vec{x};\vec{y}) = u_i^{(o)}(\vec{x}) + \varepsilon\left(u_i^{(1)}(\vec{x};\vec{y}) + u_i^{(bl)}(\vec{x};\vec{y})\right) + \text{ h.o.t} \tag{1}$$

where each function $u_i(x, y)$ is Y-periodic function with respect to fast variable *y*, u_i^{bl} is a decaying function in direction normal to free edge and periodic in all other y_j directions.

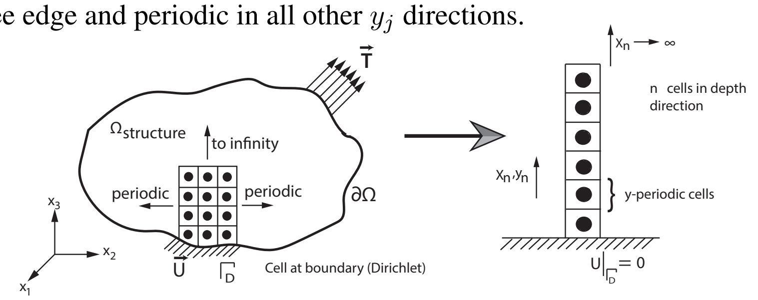


Figure 1: Elastic body with *Y* - periodic cells tends to infinity

The periodic micro correction to $u_1(y)$ can be obtained from each of the six fundamental macro-strains e_x^{ij}, by solving the periodic cell problem. From the equilibrium equation and the consistency condition we get,

$$\left[\frac{1}{V_{RVE}}\left(\int_{V_{RVE}}\left(C_{ijkl}\left(\delta_{km}\delta_{ln} + \chi_{klmn}\right)dv\right)\right)\right](\bar{\varepsilon}_{mn})_{,x_j} + f_i = 0 \tag{2}$$

The expression inside the square bracket is equal to $\bar{C}_{ijkl}$, which is the RVE volume averaged/effective global stiffness and it is a macro problem, where χ_{klmn} is the fully periodic solution.

3. Problems at the vicinity of the boundary

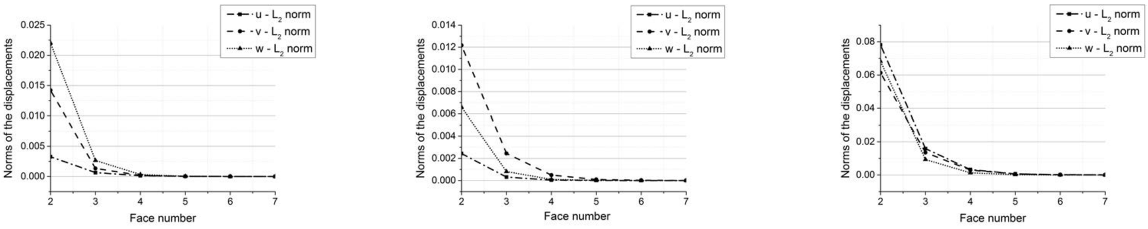
Due to the boundary layer phenomenon, this homogenized system depends in a non trivial way on the boundary [5]. Considering the cell at the boundary of the composite material, as shown in Figure 1 with homogeneous Dirichlet boundary condition, we get $u_i^\varepsilon|_{\Gamma_D} = u_i^0 + \varepsilon\left(u_i^1 + u_i^b l\right) = 0$. But $u_i^0|_{\Gamma_D} = 0$ which gives $u_i^{bl}|_{\Gamma_D} = -u_i^1|_{\Gamma_D}$ as displacement boundary condition for the boundary layer correction with the enforcement of $u_i^{(o)}(\vec{x}) = 0$ but $u_i^{(1)}(\vec{x};\vec{y}) \neq 0$. This is coupled with the consistency enforced equilibrium equation as

(3) $$\frac{\partial}{\partial y_i} C_{ijkl} \bar{\varepsilon}^{bl}_{kl} = 0$$

The above condition means that, u_i^{bl} is not required to be fully Y - periodic and therefore, one needs to solve u_i^{bl} for only one column of cells starting from Γ_D as shown in Figure 2.

4. Results

Selected results for the decay of the displacement fields for three different fibre orientations are shown in Figure 3. Variation of stress fields for both fully periodic and boundary layer problem are shown in Figure 4 for unit macro strain of ϵ_{zz}. Microstructural effects for different fibre orientation have been investigated according to the procedure described in [5].



(a) (b) (c)

Figure 2: Decay of the displacement norms for unit macro strain (ϵ_{zz}) over the RVEs with fibre orientation at (a) 0 ° (b) 90 ° (c) 45 °

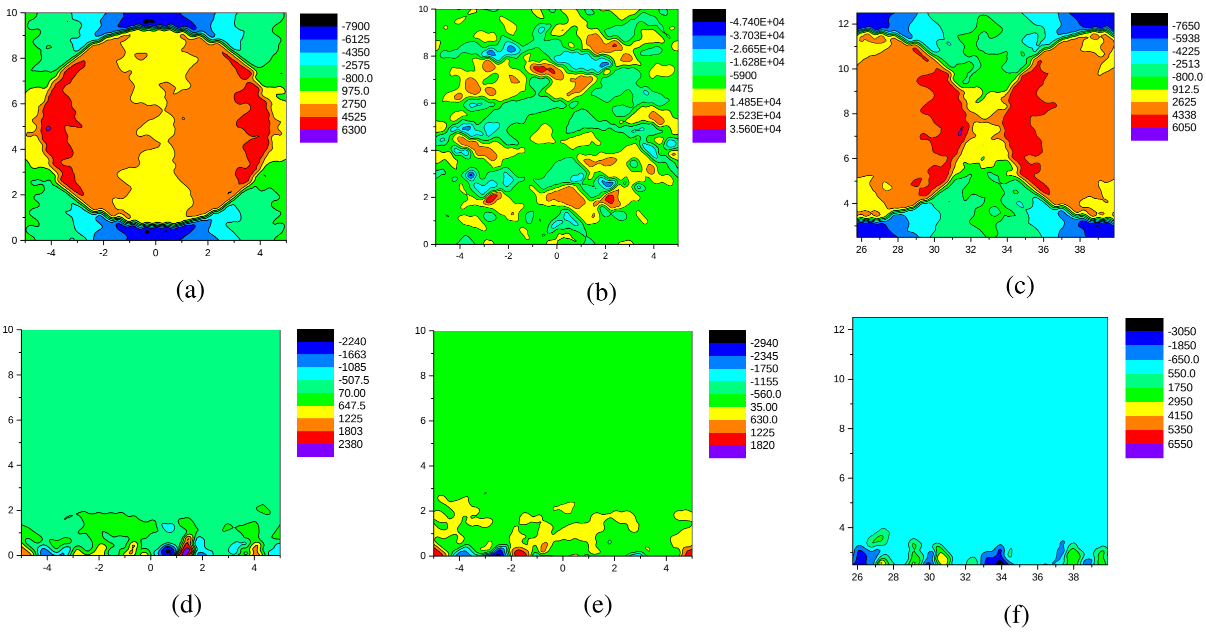


Figure 3: Distribution of micro stress fields (σ_{zz}) due to unit macro strain (ϵ_{zz}) over the RVEs of (a) 0 ° (b) 90 ° (c) 45 ° - for fully periodic problem and (d) 0 ° (e) 90 ° (f) 45 ° - for boundary layer problem

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