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JAMES JOSEPH SYLVESTER.

[(*Scientific Worthies in Nature,* XXV.); *Nature,* vol. XXXIX. (1889), pp. 217—219.]

JAMES JOSEPH SYLVESTER, born in London on September 3, 1814, is the sixth and youngest son of the late Abraham Joseph Sylvester, formerly of Liverpool*. He was educated at two private schools in London, and at the Royal Institution, Liverpool, whence he proceeded in due course of time to St John's College, Cambridge. In these early days he manifested considerable aptitude for mathematics, and so it was not matter for surprise that he came out in the Tripos Examination of 1837 as Second Wrangler; being incapacitated, by the fact of his Jewish origin, from *taking* his degree, he was not able to compete for either of the Smith's Prizes. In more enlightened times (1872), he had the degrees of B.A. and M.A., by accumulation, conferred upon him, and received therewith the honour of a Latin speech from the Public Orator. He himself says: "I am perhaps the only man in England who am a full (voting) Master of Arts for the three Universities of Dublin, Cambridge, and Oxford, having received that degree from these Universities in the order above given: from Dublin, by *ad eundem;* from Cambridge, *ob merita;* from Oxford, by decree." He is now D.C.L. of Oxford, LL.D. of Dublin and Edinburgh, and Hon. Fellow of St John's College, Cambridge. It is still open for him to receive yet higher recognition from his own *alma mater***.

Prof. Sylvester became a student of the Inner Temple, July 29, 1846, and was called to the Bar on November 22, 1850†. He has been Professor of Natural Philosophy at University College, London; of Mathematics at the University of Virginia, U.S.A.‡; then ten years later Professor at the Royal Military Academy,

* Foster's *Hand-book of Men at the Bar.*

[** The honorary degree Sc.D. was conferred upon him by Cambridge in 1890.]

† Foster, *l.c.*

‡ The late Prof. Key, of University College and School, was the first occupant of the Chair, founded by Mr Jefferson, once President of the United States, in 1824.

Woolwich; and again, after a five years' interval, Professor of Mathematics at the Johns Hopkins University, Baltimore, U.S.A., from its foundation in 1877. Finally, in December 1883, he was elected Savilian Professor of Geometry at Oxford, in succession to Prof. Henry Smith*. His first printed paper was on Fresnel's optical theory (in the *Phil. Mag.*, 1837).

We can here only briefly allude to a communication which was accompanied by many important results: we refer to the Friday evening address (January 23, 1874) to the Royal Institution, "On Recent Discoveries in Mechanical Conversion of Motion." He says:—"It would be difficult to quote any other discovery which opens out such vast and varied horizons as this of Peaucellier's,—in one direction, descending to the wants of the workshop, the simplification of the steam-engine, the revolutionising of the mill-wright's trade, the amelioration of garden-pumps, and other domestic conveniences (the sun of science glorifies all it shines upon); and in the other, soaring to the sublimest heights of the most advanced doctrines of modern analysis, lending aid to, and throwing light from a totally unexpected quarter on the researches of such men as Abel, Riemann, Clebsch, Grassmann, and Cayley. Its head towers above the clouds, while its feet plunge into the bowels of the earth."

The only works that Prof. Sylvester has published, we believe, are: (1) "A Probationary Lecture on Geometry, delivered before the Gresham Committee and the Members of the Common Council of the City of London, December 4, 1854," a slight thing which had to be written and delivered at a few hours' notice; (2) "Laws of Verse," 1870; (3) several short poems, sonnets, and translations, which have appeared in our columns and elsewhere.

Our notice would be incomplete without some record of the honours that have been conferred upon Dr Sylvester. He was elected a Fellow of the Royal Society on April 25, 1839; has received a Royal Medal (1860) and the Copley Medal (1880), this latter rarely awarded, we believe, to a pure mathematician. On this last occasion, Mr Spottiswoode accompanied the presentation with the words, "His extensive and profound researches in pure mathematics, especially his contributions to the theory of invariants and covariants, to the theory of numbers and to modern geometry, may be regarded as fully establishing Mr Sylvester's claim to the award." He is a Fellow of New College, Oxford; Foreign Associate of the United States National Academy of Sciences; Foreign Member of the Royal Academy of Sciences, Göttingen, of the Royal Academy of Sciences of Naples, and of the Academy of Sciences of Boston; Corresponding Member of the Institute of France, of the Imperial Academy of Science of St Petersburg, of the Royal Academy of Science of Berlin, of the Lyncei of Rome, of the Istituto Lombardo, and of the Société Philomathique. He has been long connected with the editorial staff of the *Quarterly Journal of Mathematics* (under

* He commences his Oxford lecture (*Nature*, vol. XXXIII. p. 222), of date December 12, 1885, with the words: "It is now two years and seven days since a message by the Atlantic cable containing the single word 'elected' reached me in Baltimore informing me that I had been appointed Savilian Professor of Geometry in Oxford, so that for three weeks I was in the unique position of filling the post and drawing the pay of Professor of Mathematics in each of two Universities."

one or another of its titles), and was the first editor of, and is a considerable contributor to, the *American Journal of Mathematics*; and he was at one time Examiner in Mathematics and Natural Philosophy in the University of London. He was not an original member of the London Mathematical Society (founded January 16, 1865), but was elected a member on June 19, 1865, Vice-President on January 15, 1866, and succeeded Prof. De Morgan as the second President on November 8, 1866. The Society showed its recognition of his great services to them and to mathematical science generally by awarding him its De Morgan Gold Medal in November 1887. Wherever Dr Sylvester goes, there is sure to be mathematical activity; and the latest proof of this is the formation, during the last term at Oxford, of a Mathematical Society, which promises, we hear without surprise, to do much for the advancement of mathematical science there.

The writings of Sylvester date from the year 1837; the number of them in the Royal Society Index up to the year 1863 is 112, in the next ten years 38, and in the volume for the next ten years 81, making 231 for the years 1837 to 1883: the number of more recent papers is also considerable. They relate chiefly to finite analysis, and cover by their subjects a large part of it: algebra, determinants, elimination, the theory of equations, partitions, tactic, the theory of forms, matrices, reciprocants, the Hamiltonian numbers, &c.; analytical and pure geometry occupy a less prominent position; and mechanics, optics, and astronomy are not absent. A leading feature is the power which is shown of originating a theory or of developing it from a small beginning; there is a breadth of treatment and determination to make the most of a subject, an appreciation of its capabilities, and real enjoyment of it. There is not unfrequently an adornment or enthusiasm of language which one admires, or is amused with: we have a motto from Milton, or Shakespeare; a memoir is a trilogy divided into three parts, each of which has its action complete within itself, but the same general cycle of ideas pervades all three, and weaves them into a sort of complex unity; the apology for an unsymmetrical solution is—symmetry, like the grace of an eastern robe, has not unfrequently to be purchased at the expense of some sacrifice of freedom and rapidity of action; and, he remarks, may not music be described as the mathematic of sense, mathematic as the music of the reason? the soul of each the same! &c. It is to be mentioned that there is always a generous and cordial recognition of the merit of others, his fellow-workers in the science.

It would be in the case of any first-rate mathematician—and certainly as much so in this as in any other case—extremely interesting to go carefully through the whole of a long list of memoirs, tracing out as well their connexion with each other, and the several leading ideas on which they depend, as also their influence on the development of the theories to which they relate; but for doing this properly, or at all, space and time, and a great amount of labour, are required. Short of doing so, one can only notice particular theorems—and there are, in the case of Sylvester, many of these, "beautiful exceedingly," which, for their own sakes, one is tempted to refer to—or one can give titles, which, to those familiar with the memoirs themselves, will recall the rich stores of investigation and theory contained therein.

A considerable number of papers, including some of the earliest ones, relate to the question of the reality of the roots of a numerical equation: in the several connexions thereof with Sturm's theorem, Newton's rule for the number of imaginary roots, and the theory of invariants. Sylvester obtained for the Sturmian functions, divested of square factors, or say for the reduced Sturmian functions, singularly elegant expressions in terms of the roots, viz. these were

$$f_2(x) = \Sigma (a-b)^2 (x-c)(x-d) \ldots, \quad f_3(x) = \Sigma (a-b)^2 (a-c)^2 (b-c)^2 (x-d) \ldots, \text{ \&c.};$$

but not only this: applying the Sturmian process of the greatest common measure (not to $f(x)$, $f'(x)$, but instead) to two independent functions $f(x)$, $\phi(x)$, he obtained for the several resulting functions expressions involving products of differences between the roots of the one and the other equation, $f(x)=0$, $\phi(x)=0$; the question then arose, what is the meaning of these functions? The answer is given by his theory of *intercalations:* they are signaletic functions, indicating in what manner (when the real roots of the two equations are arranged in order of magnitude) the roots of the one equation are intercalated among those of the other. The investigations in regard to Newton's rule (not previously demonstrated) are very important and valuable: the principle of Sturm's demonstration is applied to this wholly different question: viz. x is made to vary continuously, and the consequent gain or loss of changes of sign is inquired into. The third question is that of the determination of the character of the roots of a quintic equation by means of invariants. In connexion with it, we have the noteworthy idea of *facultative* points; viz. treating as the coordinates of a point in n-dimensional space those functions of the coefficients which serve as criteria for the reality of the roots, a point is facultative or non-facultative according as there is, or is not, corresponding thereto any equation with real coefficients: the determination of the characters of the roots depends (and, it would seem, depends only) on the bounding surface or surfaces of the facultative regions, and on a surface depending on the discriminant. Relating to these theories there are two elaborate memoirs, "On the Syzygetic Relations &c." and "Algebraical Researches &c." in the *Philosophical Transactions* for the years 1853 and 1864 respectively; but as regards Newton's rule later papers must also be consulted.

In the years 1851—54, we have various papers on homogeneous functions, the calculus of forms, &c. (*Camb. and Dub. Math. Journal*, vols. VI. to IX.), and the separate work "On Canonical Forms" (London, 1851). These contain crowds of ideas, embodied in the new words, *cogredient, contragredient, concomitant, covariant, contravariant, invariant, emanant, combinant, commutant, canonical form, plexus,* &c., ranging over and vastly extending the then so-called theories of linear transformations and hyperdeterminants. In particular, we have the introduction into the theory of the very important idea of *continuous* or *infinitesimal* variation: say that a function, which (whatever are the values of the parameters on which it depends) is invariant for an infinitesimal change of the parameters, is absolutely invariant.

There is, in 1844, in the *Philosophical Magazine*, a valuable paper, "Elementary Researches in the Analysis of Combinatorial Aggregation," and the titles of two other papers, 1865 and 1866, may be mentioned: "Astronomical Prolusions; commencing with the instantaneous proof of Lambert's and Euler's theorems, and modulating through the construction of the orbit of a heavenly body from two heliocentric distances, the subtended chord, and the periodic time, and the focal theory of Cartesian ovals, into a discussion of motion in a circle and its relation to planetary motion"; and the sequel thereto, "Note on the periodic changes of orbit under certain circumstances of a particle acted upon by a central force, and on vectorial coordinates, &c., together with a new theory of the analogues of the Cartesian ovals in space."

Many of the later papers are published in the *American Mathematical Journal*, founded, in 1878, under the auspices of the Johns Hopkins University, and for the first six volumes of which Sylvester was editor-in-chief. We have, in vol. I., a somewhat speculative paper entitled "An application of the new atomic theory to the graphical representation of the invariants and covariants of binary quantics," followed by appendices and notes relating to various special points of the theory; and in the same and subsequent volumes various memoirs on binary and ternary quantics, including papers (by himself, with the aid of Franklin) containing tables of the numerical generating functions for binary quantics of the first ten orders, and for simultaneous binary quantics of the first four orders, &c. The memoir (vols. II. and III.) on "Ternary cubic-form equations" is connected with some early papers relating to the theory of numbers. We have in it the theory of residuation on a cubic curve, and the beautiful chain-rule of rational derivation; viz. from an arbitrary point 1 on the curve it is possible to derive the singly infinite series of points $(1, 2, 4, 5, \ldots, 3p \pm 1)$ such that the chord through any two points, m, n, again meets the curve in a point $m+n$, $m \sim n$ (whichever number is not divisible by 3) of the series; moreover, the coordinates of any point m are rational and integral functions of the degree m^2 of those of the point 1.

There is in vol. V. the memoir, "A Constructive Theory of Partitions arranged in three acts, an Interact in two parts, and an Exodion," and in vol. VI. we have "Lectures on the Principles of Universal Algebra," (referring to a course of lectures on multinomial quantity, in the year 1881). The memoir is incomplete, but the general theories of nullity and vacuity, and of the corpus formed by two independent matrices of the same order, are sketched out; and there are, in the *Comptes rendus* of the French Academy, later papers containing developments of various points of the theory,—the conception of "nivellators" may be referred to.

The last-mentioned paper in the *American Mathematical Journal* was published subsequently to Sylvester's return to England on his appointment as Savilian Professor of Mathematics at Oxford. In December 1886, he gave there a public lecture containing an outline of his new theory of reciprocants (reported in *Nature*, January 7, 1887), and the lectures since delivered are published under the title, "Lectures on the Theory of Reciprocants" (reported by J. Hammond), same *Journal*, vols. VIII. to X.; thirty-three lectures actually delivered, entire or in abstract, in the course of three terms, to a class in the University, with a concluding so-called lecture 34, which is due to Hammond. The subject, as is well known, is that of the functions of a dependent variable, y, and its differential coefficients, y', y'', ..., in regard to x (or, rather, the functions of y', y'', ...), which remain unaltered by the interchange of the variables x and y: this is a less stringent condition than that imposed by Halphen ("Thèse," 1878) on his differential invariants, and the theory is accordingly a more extensive one. A passage may be quoted:—"One is surprised to reflect on the change which is come over Algebra in the last quarter of a century. It is now possible to enlarge to an almost unlimited extent on any branch of it. These thirty lectures, embracing only a fragment of the theory of reciprocants, might be compared to an unfinished epic in thirty cantos. Does it not seem as if Algebra had attained to the dignity of a fine art, in which the workman has a free hand to develop his conceptions, as in a musical theme or a subject for painting? Formerly, it consisted in detached theorems, but nowadays it has reached a point in which every properly-developed algebraical composition, like a skilful landscape, is expected to suggest the notion of an infinite distance lying beyond the limits of the canvas." And, indeed, the theory has already spread itself out far and wide, not only in these lectures by its founder, but in various papers by auditors of them, and others,—Elliott, Hammond, Leudesdorf, Rogers, Macmahon, Berry, Forsyth.

Sylvester's latest important investigations relate to the Hamiltonian numbers: there is a memoir, *Crelle*, t. C. (1887), and, by Sylvester and Hammond jointly, two memoirs in the *Philosophical Transactions.* The subject is that of the series of numbers 2, 3, 5, 11, 47, 923, calculated thus far by Sir W. R. Hamilton in his well-known Report to the British Association, on Jerrard's method. A formula for the independent calculation of any term of the series was obtained by Sylvester, but the remarkable law by means of a generating function was discovered by Hammond, viz. E_0, E_1, E_2, ..., being the series 3, 4, 6, ... of the foregoing numbers, each increased by unity; then these are calculated by the formula

$$(1-t)^{E_0} + t(1-t)^{E_1} + t^2(1-t)^{E_2} + \ldots = 1 - 2t,$$

equating the powers of t on the two sides respectively: observe the paradox, $t = \frac{1}{2}$, then the formula gives $0 =$ sum of a series of positive powers of $\frac{1}{2}$.

Enough has been said to call to mind some of Sylvester's achievements in mathematical science. Nothing further has been attempted in the foregoing very imperfect sketch.