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[NOTE ON THE THEORY OF RATIONAL TRANSFORMATION.]

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IN my paper, "Note on the Theory of the Rational Transformation between Two Planes, and on Special Systems of Points," *Proc. Lond. Math. Soc.* t. III. (1870), pp. 196—198, [450], I notice a difficulty which presents itself in the theory. The transformation is given by the equations

$$x': y': z' = X: Y: Z,$$

where X, Y, Z are functions $(*(x, y, z)^n)$, such that X = 0, Y = 0, Z = 0 are curves in the first plane passing through α_1 points each once, α_2 points each twice (that is, having each of the α_2 points for a double point), α_3 points each 3 times, and so on. We have as the condition of a single variable point of intersection,

$$\alpha_1 + 4\alpha_2 + 9\alpha_3 + \ldots = n^2 - 1,$$

and as the condition in order that each of the curves X = 0, Y = 0, Z = 0, or say the curve aX + bY + cZ = 0, may be unicursal,

$$\alpha_2 + 3\alpha_3 + \ldots = \frac{1}{2}(n-1)(n-2);$$

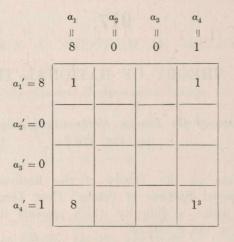
and we thence deduce

$$\alpha_1 + 3\alpha_2 + 6\alpha_3 + \ldots = \frac{1}{2}n(n+3) - 2;$$

viz. the postulation of the fixed points quoad a curve of the order n is less by 2 than the postulandum (or, as I prefer to call it, the capacity) $\frac{1}{2}n(n+3)$ of the curve of the order n; that is, there are precisely the three asyzygetic curves X=0, Y=0, Z=0. This is as it should be, assuming that the $(\alpha_1, \alpha_2, \alpha_3, ...)$ points are an ordinary system of points: but what if they form a special system having a postulation less 15-2

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than $\alpha_1 + 3\alpha_2 + 6\alpha_3 + \dots$? If, for instance, the postulation is $= \alpha_1 + 3\alpha_2 + 6\alpha_3 + \dots - 1$, then this would be $=\frac{1}{2}n(n+3)-3$, and there would be four asyzygetic curves X=0, Y=0, Z=0, W=0. I believe this to be impossible; but the only proof which I can offer rests upon a remark in regard to the form of the tables*, pp. 148, 149, of my paper "On the Rational Transformation between Two Spaces," Proc. Lond. Math. Soc., t. III. (1870), pp. 127–180, [447]. I recall that the Jacobian curve J(X, Y, Z) = 0consists of α_1' lines, α_2' conics, α_3' cubics, ..., &c., each passing a certain number of times through the $(\alpha_1, \alpha_2, \alpha_3, ...)$ points, and that the number of times of passage is shown by these tables; thus, loc. cit., n = 5, $\alpha_1 = 8$, $\alpha_4 = 1$: the Jacobian consists of eight lines and a quartic, and we have the table $(\alpha_1' = 8, \alpha_4' = 1)$,



showing that the quartic passes through the eight points α_1 , and through the point α_4 three times (has α_4 for a triple point). Imagine a new function W. Then in like manner J(X, Y, W) = 0 consists of eight lines and a quartic, and this quartic passes through the eight points α_1 and the point α_4 three times; that is, the two quartics intersect in 8+3.3, =17 points; and thus the two quartics must be one and the same curve; this implies a syzygy between X, Y, Z, W, viz. W is a mere linear function of X, Y, Z. The general remark is that, if in the tables m^p is reckoned as mp², then in the table for the several lines (exclusive of those for which the outside accented letter is = 0, and therefore the tabular numbers of the line are each = 0), i.e. for the lines which correspond to a line, a conic, a cubic, a quartic, &c., respectively, the sums of the tabular numbers are $1^2 + 1$, $2^2 + 1$, $3^2 + 1$, $4^2 + 1$, &c., respectively. This is, in fact, the case for each of the eleven tables (loc. cit.).

[* This Collection, vol. vII., pp. 208, 209.]

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