715.

NOTE ON A SYSTEM OF ALGEBRAICAL EQUATIONS.

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Assume

$$\begin{aligned} x \quad + y \ + z \ &= P, \\ yz \ + zx + xy &= Q, \\ xyz \qquad\qquad &= R, \end{aligned}$$

$$\begin{aligned} A &= x\,(nyz + Q) - w^2\,(mx + P), \\ B &= y\,(nzx + Q) - w^2\,(my + P), \\ C &= z\,\ (nxy + Q) - w^2\,(mz + P), \\ \Theta &= -\,mnR + PQ. \end{aligned}$$

Then

$$\begin{aligned} (mz + P)\,B - (my + P)\,C & \\ &= (myz + Py)\,(nzx + Q) - (myz + Pz)\,(nxy + Q) \\ &= myz\,(nzx + Q - nxy - Q) + Pnxyz + PQy - Pnxyz - PQz \\ &= mnxyz\,(z - y) - PQ\,(z - y) \\ &= (z - y)\,\{mnxyz - PQ\} = (y - z)\,\Theta\,; \end{aligned}$$

whence, identically,

$$\begin{aligned} (mz + P)\,B - (my + P)\,C &= (y - z)\,\Theta, \\ (mx + P)\,C - (mz + P)\,A &= (z - x)\,\Theta, \\ (my + P)\,A - (mx + P)\,B &= (x - y)\,\Theta. \end{aligned}$$

Hence any two of the equations $A = 0$, $B = 0$, $C = 0$ imply the third equation.

We have

$$A = x\{(n+1)yz + zx + xy\} - w^2\{(m+1)x + (y+z)\}$$
$$= (x^2 - w^2)(y+z) - x[(m+1)w^2 - (n+1)yz],$$

and similarly for B and C. The three equations therefore are

$$\frac{x}{x^2 - w^2} = \frac{y+z}{(m+1)w^2 - (n+1)yz},$$

$$\frac{y}{y^2 - w^2} = \frac{z+x}{(m+1)w^2 - (n+1)zx},$$

$$\frac{z}{z^2 - w^2} = \frac{x+y}{(m+1)w^2 - (n+1)xy};$$

and any two of these equations imply the third equation.