720.

NOTE ON ARBOGAST'S METHOD OF DERIVATIONS.

[From the *Messenger of Mathematics,* vol. VII. (1878), p. 158.]

It is an injustice to Arbogast to speak of his *first* method, as Arbogast's method*. There is really nothing in this, it is the straightforward process of expanding

$$\phi\left(a+bx+\frac{1}{1.2}cx^2+\ldots\right)$$

by the differentiation of ϕu, writing $a, b, c, d, \ldots$ in place of $u, \frac{du}{dx}, \frac{d^2u}{dx^2}, \frac{d^3u}{dx^3}$, &c. or say in place of u, u', u'', u''', &c. respectively; thus

$$\phi a,\ \phi' a\,.\,b,\ \tfrac{1}{2}\{\phi' a\,.\,c+\phi'' a\,.\,b^2\},\quad \tfrac{1}{6}\left\{\begin{matrix}\phi' a\,.\,d+\phi'' a\,.\;bc \\ +\phi'' a\,.\,2bc+\phi''' a\,.\,b^3\end{matrix}\right\}$$

$$=\tfrac{1}{6}\ \{\phi' a\,.\,d+\phi'' a\,.\,3bc+\phi''' a\,.\,b^3\},\ \&c.,$$

and in subsequent terms the number of additions necessary for obtaining the numerical coefficients increases with great rapidity.

That which is specifically Arbogast's method, is his *second* method, viz. here the coefficients of the successive powers of x in the expansion of $\phi(a+bx+cx^2+dx^3+\ldots)$, are obtained by the rule of the last and the last but one; thus we have

$$\phi a,\ \phi' a\,.\,b,\ \phi' a\,.\,c+\phi'' a\,.\,\tfrac{1}{2}b^2,\quad \phi' a\,.\,d+\phi'' a\,.\,bc+\phi''' a\,.\,\tfrac{1}{6}b^3,\ \&c.,$$

where each numerical coefficient is found directly, without an addition in any case.

* See *Messenger of Mathematics*, vol. VII. (1878), pp. 142, 143.