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NOTE ON A PAIR OF DIFFERENTIAL EQUATIONS IN THE LUNAR THEORY.

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THE equations

$$\frac{d}{dt}\frac{d\rho}{dt} - \rho\left(\frac{dv}{dt}\right)^2 + \frac{1}{\rho^2} = km^2\rho\,\{\tfrac{1}{2} + \tfrac{3}{2}\cos(2v - 2mt)\},$$

$$\frac{d}{dt}\rho^2\frac{dv}{dt} \qquad = jm^2\rho^2\,\{\;-\tfrac{3}{2}\sin(2v - 2mt)\},$$

taking therein $j = k = 1$ in effect present themselves in the Lunar Theory, and particular integrals in series have been obtained, the development being carried to a great extent; but I give the results only as far as m^4, viz., writing

$$t - mt = D,$$

we have

$$v = t + (\tfrac{11}{8}m^2 + \tfrac{59}{12}m^3 + \tfrac{893}{72}m^4)\sin 2D$$
$$+ \tfrac{201}{256}m^4 \sin 4D,$$

$$\frac{1}{\rho} = 1 + \tfrac{1}{6}m^2 \qquad - \tfrac{179}{288}m^4$$
$$+ (\;m^2 + \tfrac{19}{6}m^3 + \tfrac{131}{18}m^4)\cos 2D$$
$$+ \tfrac{7}{8}\;m^4 \cos 4D.$$

In the Lunar Theory j and k are properly each $= \dfrac{1}{1 + \dfrac{E}{m'}}$ (E the mass of the Earth, m' that of the Sun), but they are taken to be $= 1$; the numerical difference is inappreciable; but there would be a considerable theoretical advantage in retaining in the equations the coefficients j, k [regarded as each of them $=k$]: in fact, the developments could then be arranged according to the powers of k, that is according to the powers of the disturbing force; whereas, when k is taken $=1$, we have only a development in powers of m, and since m also presents itself through the coefficient $2-2m$ of t in $2v-2mt$, terms which are really of different orders in regard to the disturbing force, are united together into a single term: so that, instead of a term of the form $(Ak+Bk^2+\&c.)\,m^p$, where A, B, are numerical, we have the term $(A+B+..)\,m^p$, where of course $A+B..$ is given as a single numerical coefficient. There is no equal advantage in retaining the two coefficients k, j, as this only serves to show how a term arises from the central and tangential forces respectively; thus retaining these coefficients, the integrals as far as m^2 are

$$v = \qquad t \qquad + (\tfrac{1}{2}k+\tfrac{7}{8}j)\,m^2\sin 2D,$$

$$\frac{1}{\rho} = 1 + \tfrac{1}{6}\,m^2k + (\tfrac{1}{2}k+\tfrac{1}{2}j)\,m^2\cos 2D,$$

agreeing with the former result when $k=j=1$; but there is, nevertheless, some interest in retaining the two coefficients. I hope to develope the results somewhat further, and to communicate them to the Society.