## 948.

## REPORT OF A COMMITTEE APPOINTED FOR THE PURPOSE OF CARRYING ON THE TABLES CONNECTED WITH THE PELLIAN EQUATION FROM THE POINT WHERE THE WORK WAS LEFT BY DEGEN IN 1817.

[From the British Association Report, (1893), pp. 73-120.]
We have, on the Pellian Equation, Degen's tables, the title of which is "Canon Pellianus sive Tabula simplicissimam æquationis celebratissimæ $y^{2}=a x^{2}+1$ solutionem pro singulis numeri dati valoribus ab 1 usque ad 1000 in numeris rationalibus iisdemque integris exhibens." Autore Carolo Ferdinando Degen. Hafniæ, apud Gerhardum Bonnierum, mDcccexvii., 8vo. Introductio, pp. v-xxiv. Tabula I. Solutionem æquationis $y^{2}-a x^{2}-1=0$ exhibens, pp. 3-106. Tabula II. Solutionem æquationis $y^{2}-a x^{2}+1=0$, quotiescunque valor ipsius $a$ talem admiserit, exhibens, pp. 109-112.

The mode of calculation is explained in the Introduction, and illustrated by the examples of the numbers 209, 173.

As to the first of these, the entry in Table I. is

|  | 14, 2, 5, <br> 1, $(2)$  <br> 1, 13, 5, <br> 209 11  <br> 3220   <br>  46551  |
| :--- | :--- | :--- | :--- |

where the first line gives the expression of $\sqrt{209}$ as a continued fraction, viz. we have

$$
\sqrt{209}=14+\frac{1}{2}+\frac{1}{5}+\frac{1}{3}+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{2}+\frac{1}{28}+\frac{1}{2}+\& c .
$$

the denominators being $2,5,3,(2), 3,5,2$, then 28 , which is the double of the integer part 14, and then again. $2,5,3,(2), 3,5,2$, and so on, the parentheses of the (2) being used to indicate that this is the middle term of the period.

The second row gives auxiliary numbers occurring in the calculation of the first row and having a meaning, as will presently appear. Observe that the 11 which comes under the (2) should also be printed in parentheses (11); but this is not done.

The process for the calculation of the $x, y$ is as follows:

| 209 |  |  |  |
| :---: | ---: | ---: | ---: |
| 14 | 1 | 0 | +1 |
| 2 | 14 | 1 | -13 |
| 5 | 29 | 2 | +5 |
| 3 | 159 | 11 | -8 |
| $(2)$ | 506 | 35 | $+(11)$ |
| 3 | 1171 | 81 | -8 |
| 5 | 4019 | 278 | +5 |
| 2 | 21266 | 1471 | -13 |
| 28 | 46551 | 3220 | +1 |

viz. writing down as a first column the numbers of the first row, and beginning the second column with 1,14 ( 14 the number at the head of the first column), and the third column with 0,1 , we calculate the numbers of the second column, $29=2.14+1$, $159=5.29+14,506=3.159+29$, \&c., and the numbers of the third column in like manner, $2=2.1+0,11=5.2+1,35=3.11+2, \& c . ;$ and then writing down as a fourth column the numbers of the second row with the signs + , - alternately, we have a series of equations $y^{2}-a x^{2}= \pm A$, viz.

$$
\begin{aligned}
1^{2}-209.0^{2} & =+1 \\
14^{2}-209.1^{2} & =-13 \\
29^{2}-209.2^{2} & =+5 \\
\vdots &
\end{aligned}
$$

the last of them being

$$
(46551)^{2}-209(3220)^{2}=+1
$$

this last corresponding as above to the value +1 , and the numbers 46551 and 3220 being accordingly the $y$ and $x$ given in the fourth and third rows of the table.

As to the second of the foregoing numbers, $\mathbf{1 7 3}$, the only difference is that the period has a double middle term, viz. the entry in the Table I. is

| 173 | $13,6,(1,1)$ <br> $1,4,(13,13)$ <br>  <br>  <br> 190060 <br> 2499849 |
| :--- | :--- |

The first row gives the expression of $\sqrt{173}$, viz, that is

$$
\sqrt{173}=13+\frac{1}{6}+\frac{1}{(1)}+\frac{1}{(1)}+\frac{1}{6}+\frac{1}{26}+\& c .
$$

the denominators being $6,1,1,6$, then 26 (the double of the integer part 13 ), and then again 6, 1, 1, 6, and so on. In the second row I remark that Degen prints the parentheses $(13,13)$ for the double middle term.

The process for the calculation of the $x, y$ is similar to that in the former case, viz. we have

| 173 |  |  |  |
| :---: | ---: | ---: | ---: |
| 13 | 1 | 0 | +1 |
| 6 | 13 | 1 | -4 |
| $\binom{1}{1}$ | 79 | 6 | +13 |
| 6 | 92 | 7 | -13 |
| 26 | 171 | 13 | +4 |

where the second and third columns begin 1,13 and 0,1 respectively, and the remaining terms are calculated $79=6.13+1,92=1.79+13, \& c$. , and $6=6.1+0$, $7=1.6+1$, \&c.; and then writing down as a fourth column the terms of the second row with the signs + , - alternately, we have

$$
\begin{array}{r}
1^{2}-173.0^{2}=+1 \\
13^{2}-173.1^{2}=-4 \\
79^{2}-173.6^{2}=+13
\end{array}
$$

the last equation being

$$
(1118)^{2}-173(85)^{2}=-1,
$$

the term for the last equation being always in a case such as the present one, not +1 , but -1 . The final numbers 1118,85 are consequently entered not in Table I , but in Table II., viz. the entry in this table is

| 173 | 85 |
| :--- | :--- |
|  | 1118 |

and thence we calculate the numbers $y, x$ of Table I., viz, these are

$$
\begin{aligned}
2499849 & =2 .(1118)^{2}+1 \\
190060 & =2.1118 .85 .
\end{aligned}
$$

Generally Table II. gives for each value of $a$, comprised therein, values of $x, y$, such that $y^{2}=a x^{2}-1$, and then writing $y_{1}=2 y^{2}+1, x_{1}=2 x y$, we have

$$
y_{1}^{2}=\left(2 a x^{2}-1\right)^{2}=4 a^{2} x^{4}-4 a x^{2}+1=a \cdot 4 x^{2}\left(a x^{2}-1,+1=a x_{1}^{2}+1,\right.
$$

so that $x_{1}, y_{1}$ are for the same value of $a$ the values of $x, y$ in Table I.
It is to be remarked that the heading of Table II. is not perfectly accurate, for it purports to give for every value of $a$, for which a solution exists, a solution of the equation $y^{2}=a x^{2}-1$. What it really gives is the solution for each value of $a$ for which the period has a double middle term. But if $a=\alpha^{2}+1$, then obviously we have a solution $y=\alpha, x=1$, and for any such value of $a$ the period has a single middle term, viz. the entry in Table I. is

| $a^{2}+1$ | $a,(2 a)$ <br> 1, <br> 1,1 <br> $2 a$ <br> $2 \alpha^{2}+1$ |
| :--- | :--- |

and we, in fact, have

| $a^{2}+1$ |  |  |  |
| :---: | ---: | ---: | ---: |
| $\alpha$ | 1 | 0 | +1 |
| $(2 a)$ | $a$ | 1 | -1 |
| $2 a$ | $2 a^{2}+1$ | $2 a$ | +1 |

that is,

$$
\begin{aligned}
1^{2}-\left(\alpha^{2}+1\right) 0^{2} & =+1 \\
\alpha^{2}-\left(\alpha^{2}+1\right) 1^{2} & =-1 \\
\left(2 \alpha^{2}+1\right)^{2}-\left(\alpha^{2}+1\right)(2 \alpha)^{2} & =+1
\end{aligned}
$$

C. XIII.

The foregoing instances of the calculation of $x, y$ in the case of the numbers 209 and 173 suggest a table which may be regarded as an extended form of Degen's tables; viz. such a table, from $a=2$ to $a=99$, is as follows:

Specimen of extended Form of Table in regard to the Pellian Equation.

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | a |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 1 | 0 | + 1 | 13 | 3 | 1 | 0 | + 1 |
|  | (2) | 1 | 1 | $-1$ |  | 1 | 3 | 1 | - 4 |
|  | 2 | 3 | 2 | + 1 |  | (1) | 4 | 1 | + 3 |
|  |  |  |  |  |  | $(1)$ | 7 | 2 | - 3 |
| 3 |  |  | 0 |  |  | 1. | 11 | 3 | + 4 |
|  | (1) | 1 | 1 | - 2 |  | 6 | 18 | 5 | - 1 |
|  | 2 | 2 | 1 | +1 |  |  |  |  |  |
| 5 | 2 | 1 | 0 | $+1$ | 14 | 3 |  |  | + 1 |
|  | (4) | 2 | 1 | -1 |  | 1 | 3 | 1 | $+\quad 5$ $+\quad 2$ |
|  | 4 | 9 | 4 | +1 +1 |  | (2) | 4 11 | 1 | $+\quad 2$ <br> $-\quad 5$ |
| 6 | 2 | 1 | 0 | + 1 |  | 6 | 15 | 4 | + 1 |
|  | (2) | 2 | 1 | - 2 | 15 |  |  | 0 |  |
|  | 4 | 5 | 2 | + 1 |  | (1) | 3 | 1 | - 6 |
| 7 | 2 | 1 | 0 | + 1 |  | 6 | 4 | 1 | + 1 |
|  | 1 | 2 | 1 | $-3$ | 17 |  |  |  |  |
|  | (1) | 3 | 1 | + 2 |  | 4 |  | 0 | + 1 |
|  | 1 | 5 | 2 | - 3 |  | (8) | 4 | 1 | $-.1$ |
|  |  |  |  | + 1 |  | 8 | 33 |  |  |
| 8 |  |  |  |  | 18 | 4 | 1 | 0 | + 1 |
|  | (1) | 2 | 1 | $-4$ |  | (4) | 4 | 1 | - 2 |
|  |  |  |  | + 1 |  | 8 | 17 | 4 | + 1 |
| 10 |  |  |  |  | 19 | 4 | 1 | 0 | + 1 |
|  | (6) | 3 | $1$ | - 1 |  | 2 | 4 | 1 | - 3 |
|  | (6) 6 | $19$ | $6$ | +1 +1 |  | 1 | 9 | 2 | + 5 |
|  |  |  |  |  |  | (3) | 13 | 3 | - 2 |
| 11 | 3 | 1 | 0 | + 1 |  | 1 | 48 | 11 | + 5 |
|  | (3) | 3 | 1 | - 2 |  | 2 | 61 | 14 | - 3 |
|  | 6 | 10 | 3 | + 1 |  | 8 | 170 | 39 |  |
| 12 | 3 | 1 | 0 |  | 20 |  | 1 | 0 | + 1 |
|  | (2) | 3 | 1 | $-3$ |  | (2) | 4 | 1 | - 4 |
|  |  |  | 2 | + 1 |  |  | 9 | 2 | + 1 |

Specimen of extended Form of Pellian Equation Table-continued.

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 4 | 1 | 0 | + 1 | 29 | 5 | 1 | 0 | + 1 |
|  | 1 | 4 | 1 | - 5 |  | 2 | 5 | 1 | - 6 |
|  | 1 | 5 | 1 | + 4 |  | (1) | 11 | 2 | + 5 |
|  | (2) | 9 | 2 | - 3 |  | $(1)$ | 16 | 3 | - 3 |
|  |  | 23 | 5 | + 4 |  | 2 | 27 | 5 | + 2 |
|  | 1 | 32 | 7 | - 5 |  | 10 | 70 | 13 | - 1 |
|  | 8 | 55 | 12 | + 1 |  |  |  |  |  |
| 22 |  |  |  |  | 30 |  |  | 0 | + 1 |
|  | 4 |  | 0 |  |  | (2) | 5 | 1 | - 5 |
|  | 1 | 4 | 1 | - 6 |  | 10 | 11 | 2 | + 1 |
|  | 2 | 5 | 1 | + 3 |  |  |  |  |  |
|  | (4) | 14 | 3 | - 2 | 31 | 5 | 1 | 0 | + 1 |
|  | 2 | 61 | 13 | + 3 |  | 1 | 5 | 1 | - 6 |
|  | 1 | 136 | 29 | -6 |  | 1 | 6 | 1 | + 5 |
|  | 8 | 197 | 42 | + 1 |  | 3 | 11 | 2 | - 3 |
| 23 |  |  |  |  |  | (5) | 39 | 7 | + 2 |
|  | 1 |  | $0$ | + 1 |  | 3 | 206 | 37 | $-3$ |
|  | 1 | 4 | 1 | -7 |  | 1 | 657 | 118 | + 5 |
|  | (3) | 5 19 | 1 | $\begin{array}{r} \\ +\quad 2 \\ \hline\end{array}$ |  | 1 | 863 | 155 | -6 |
|  | 1 | $\begin{aligned} & 19 \\ & 24 \end{aligned}$ | $\begin{aligned} & 4 \\ & 5 \end{aligned}$ | +7 $+\quad 1$ |  | 10 | 1520 | 273 | + 1 |
| 24 |  |  |  |  | 32 | 5 | 1 | 0 |  |
|  | 4 | 1 | 0 | +1 $+\quad 8$ |  | 1 | 5 | 1 | - 7 |
|  | (1) | 4 5 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | +8 $+\quad 1$ |  | (1) | 6 | 1 | + 4 |
|  |  |  |  |  |  | 1 | 11 | 2 | - 7 |
| 26 | 5 | 1 | 0 | + 1 |  | 10 | 17 | 3 | + 1 |
|  | (10) | 5 | 1 | - 1 | 33 |  |  |  |  |
|  | 10 | 51 | 10 | + 1 |  | 5 1 | 1 | 1 | $\begin{aligned} & +1 \\ & +\quad 8 \end{aligned}$ |
| 27 | 5 | 1 | 0 | + 1 |  | (2) | 6 | 1 | + 3 |
|  | (5) | 5 | 1 | - 2 |  | 1 | 17 | 3 | - 8 |
|  | 10 | 26 | 5 |  |  | 10 | 23 | 4 | + 1 |
| 28 | 5 | 1 | 0 |  | 34 | 5 | 1 | 0 | + 1 |
|  | 3 | 5 | 1 | - 3 |  | 1 | 5 | 1 | - 9 |
|  | (2) | 16 | 3 | + 4 |  | (4) | 6 | 1 | + 2 |
|  | 3 | 37 | 7 | - 3 |  | 1 | 29 | 5 | - 9 |
|  | 10 | 127 | 24 | + 1 |  | 10 | 35 | 6 | + 1 |

Specimen of extended Form of Pellian Equation Table-continued.


Specimen of extended Form of Pellian Equation Table-continued.


Specimen of extended Form of Pellian Equation Table-continued.

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 1523 | 195 | + 4 |  | 3 | 25 | 3 | $+4$ |
|  | 4 | 5639 | 722 | - 3 |  | 1 | 83 | 10 | -11 |
|  | 1 | 24079 | 3083 | + 12 |  | (4) | 108 | 13 | + 3 |
|  | 14 | 29718 | 3805 | - 1 |  | 1 | 515 | 62 | $-11$ |
|  |  |  |  |  |  | 3 | 623 | 75 | + 4 |
| 62 |  |  |  |  |  | 3 | 2384 | 297 | - 5 |
|  | 1 | 7 | 1 | -13 |  | 16 | 7775 | 936 | + 1 |
|  | (6) | 8 | 1 | + 2 |  |  |  |  |  |
|  | 1 | 55 | 7 | -13 | 70 | 8 | 1 | 0 | + 1 |
|  | 14 | 63 | 8 | + 1 |  | 2 | 8 | 1 | - 6 |
|  |  |  |  |  |  | 1 | 17 | 2 | + 9 |
| 63 | 7 | 1 | 0 | + 1 |  | (2) | 25 | 3 | - 5 |
|  | (1) | 7 | 1 | - 14 |  | 1 | 67 | 8 | + 9 |
|  | 14 | 8 | 1 | + 1 |  | 2 | 92 | 11 | - 6 |
| 65 | 8 | 1 | 0 |  |  | 16 | 251 | 30 | + 1 |
|  | (16) | 8 | 1 | - 1 | 71 | 8 | 1 | 0 | + 1 |
|  | 16 | 129 | 16 | + 1 |  | 2 | 8 | 1 | - 7 |
| 66 |  |  |  |  |  | 2 | 17 | 2 | + 5 |
|  |  |  |  |  |  | 1 | 42 | 5 | --11 |
|  | (8) | 8 | 1 | - 2 |  | (7) | 59 | 7 | + 2 |
|  | 16 | 65 | 8 | + 1 |  | 1 | 455 | 54 | $-11$ |
| 67 |  |  |  |  |  | 2 | 514 | 61 | + 5 |
|  | $5$ | 8 | 1 | + |  | 2 | 1483 | 176 | - 7 |
|  | 5 | 41 | 5 | + |  | 16 | 3480 | 413 | + 1 |
|  | 1 | 90 | 11 | $-7$ | 72 | 8 | 1 | 0 | + 1 |
|  | 1 | 131 | 16 | + 9 |  | (2) | 8 | 1 | - 3 |
|  | (7) | 221 | 27 | - 2 |  | 16 | 17 | 2 | + 1 |
|  | 1 | 1678 | 205 | + 9 |  |  |  |  |  |
|  |  | 1899 | 232 | $-7$ | 73 | 8 | 1 | 0 | + 1 |
|  | 2 | 3577 | 437 | + 6 |  | 1 | 8 | 1 | - 9 |
|  | 5 | 9053 | 1106 | - 3 |  | 1 | 9 | 1 | + 8 |
|  |  |  |  |  |  | $\left({ }^{5}\right.$ ) | 17 | 2 | - 3 |
|  |  |  |  |  |  | (5) | 94 | 11 | + 3 |
| 68 |  |  | 0 | + 1 |  | 1 | 487 | 57 | - 8 |
|  | (4) | 8 | 1 | - 4 |  | 1 | 581 | 68 | + 9 |
|  | 16 | 33 | 4 | + 1 |  | 16 | 1068 | 125 | - 1 |
| 69 | 8 | 1 |  | + 1 | 74 |  | 1 | 0 |  |
|  | 3 | 8 | 1 | $-5$ |  | 1 | 8 | 1 | $-10$ |

Specimen of extended Form of Pellian Equation Table-continued.

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | ${ }^{a}$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | 9 | 1 | + 7 | 79 | 8 | 1 | 0 | + 1 |
| - | (1) | 17 | 2 | - 7 |  | 1 | 8 | 1 | -15 |
|  | 1 | 26 | 3 | + 10 |  | (7) | 9 | 1 | + 2 |
|  | 16 | 43 | 5 | - 1 |  | 1 | 71 | 8 | -15 |
|  |  |  | 0 | $+1$ |  | 16 | 80 | 9 | + 1 |
| 75 |  | 1 |  |  | 80 | 8 |  |  |  |
|  | $\begin{gathered} 1 \\ (1) \end{gathered}$ | 8 | 1 | +1 -11 |  |  | 1 | 0 | + 1 |
|  |  | 9 | 1 | + 6 |  | (1) | 8 | 1 | -16 |
|  | (1) | 17 | 2 | -11 |  | 16 | 9 | 1 | + 1 |
|  |  | 26 | 3 | + 1 | 82 | 9 | 1 | 0 | + 1 |
| 76 |  | 1 | 0 |  |  | $(18)$18 |  | $\begin{array}{r} 1 \\ 18 \end{array}$ | -1+1 |
|  | 1 | 8 | 1 | $-12$ |  |  |  |  |  |
|  | 2 | 9 | 1 | + 5 | 83 | 9 | 1 | 0 | + 1 |
|  | 1 | 26 | 3 | - 8 |  | (9) | 9 | 1 | - 2 |
|  | 1 | 35 | 4 | + 9 |  | 18 | 82 | 9 | + 1 |
|  | 1 | 61 | 7 |  |  |  |  |  |  |
|  | $\begin{gathered} 5 \\ (4) \end{gathered}$ | 340 | 39 | $\begin{array}{r} \\ +4 \\ \hline\end{array}$ | 84 | 9 $(6)$ | 1 9 | 0 1 | +1 $-\quad 3$ |
|  | (4) | 1421 7445 | 163 | -3 $-\quad 9$ |  | (6) 18 | 9 55 | 6 | +1 $+\quad 1$ |
|  | 5 1 | 7445 | 854 | + 9 |  |  |  |  |  |
|  | $\begin{array}{lr}1 & 8866 \\ 2 & 16311\end{array}$ |  | 1017 | $-8$ | 85 | 9 | 1 | 0 | + 1 |
|  |  |  | 1871 | + 5 |  | 4 | 9 | 1 | - 4 |
|  |  | $1 \begin{array}{r}1 \\ \hline 1488\end{array}$ | 4759 | -12 |  | (1) | 37 | 4 | + 9 |
|  | $16 \quad 57799$ |  | 6630 | + 1 |  | $(1)$ | 46 | 5 | - 9 |
| 77 |  |  | 0 |  |  | 4 | 83 | 9 | + 4 |
|  |  | 1 |  | + 1 |  | 18 | 378 | 41 | - 1 |
|  | 1 | 8 | 1 | -13 |  |  |  |  |  |
|  | 3 | 9 | 1 | + 4 | 86 | 9 | 1 | 0 | + 1 |
|  | (2) | 35 | 4 | $-7$ |  | 3 | 9 | 1 | - 5 |
|  | 3 | 79 | 9 | + 4 |  | 1 | 28 | 3 | + 10 |
|  | 1 | 272 | 31 | - 13 |  | 1 | 37 | 4 | - 7 |
|  | 16 | 351 | 40 | + 1 |  | 1 | 65 | 7 | + 11 |
|  |  |  |  |  |  | (8) | 102 | 11 | - 2 |
| 78 | 8 | 1 | 0 | + 1 |  | 1 | 881 | 95 | + 11 |
|  | 1 | 8 | 1 | - 14 |  | 1 | 983 | 106 | - 7 |
|  | (4) | 9 |  | + 3 |  | 1 | 1864 | 201 | + 10 |
|  | 1 | 44 | 5 | - 14 |  | 3 | 2847 | 307 | - 5 |
|  | 16 | 53 | 6 | + 1 |  | 18 | 10405 | 1122 | + 1 |

Specimen of extended Form of Pellian Equation Table-continued.

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 87 | 9 | 1 | 0 | $+1$ | 93 | 9 | 1 | 0 | + 1 |
|  | (3) | 9 | 1 | - 6 |  | 1 | 9 | 1 | -12 |
|  | 18 | 28 | 3 | + 1 |  | 1 | 10 | 1 | + 7 |
| 88 | 9 | 1 |  | + 1 |  | 1 | 19 | 2 | -11 |
|  | 2 | 9 | 1 | +7 -7 |  | 4 | 29 | 3 | + 4 |
|  | 1 | 19 | 1 | -7 +9 |  | (6) | 135 | 14 | $-3$ |
|  | (1) | 19 28 | 2 | + +8 |  | 4 | 839 | 87 | + 4 |
|  | (1) 1 | 47 | 5 | - $+\quad 9$ |  | 1 | 3491 | 362 | -11 |
|  | 2 | 75 | 8 | +7 -7 |  | 1 | 4330 | 449 | + 7 |
|  | 18 | $197$ | 21 | -7 +1 |  | 1 | 7821 | 811 | -12 |
|  |  |  |  |  |  | 18 | 12151 | 1260 | + 1 |
| 89 | 9 | 1 | 0 | $+1$ | 94 | 9 |  | 0 | $+1$ |
|  | 2 | 9 |  | $-8$ |  | , | 9 | 1 | -13 |
|  | ${ }^{3}$ ) | 19 | 2 | + 5 |  | 2 | 10 | 1 | + 6 |
|  | (3) | 66 | 7 | - 5 |  | 3 | 29 | 3 | - 5 |
|  | 2 | 217 | 23 | $+8$ |  | 1 | 97 | 10 | + 9 |
|  | 18 | 500 | 53 | - 1 |  |  | 126 | 13 | - 10 |
| 90 | 9 | 1 | 0 | + 1 |  | 5 | 223 | 23 | + 3 |
|  | (2) | 9 | 1 | - 9 |  | 1 | 1241 | 128 | -15 |
|  | 18 | 19 | 2 | + 1 |  | (8) | 1464 | 151 | + 2 |
| 91 | 9 | 1 | 0 | $+1$ |  | 1 | 12953 | 1336 | -15 |
|  | 1 | 9 | 1 | $-10$ |  | 5 | 14417 | 1487 | + 3 |
|  | 1 | 10 | 1 | + 9 |  | 1 | 85038 | 8771 | -10 |
|  | 5 | 19 | 2 | $-3$ |  | 1 | 99455 | 10258 | + 9 |
|  | (1) | 105 | 11 | + 14 |  | 3 | 184493 | 19029 | - 5 |
|  | (1) | 124 | 13 | - 3 |  | 2 | 652934 | - 67345 | + 6 |
|  | 1 | 725 | 76 | +9 |  | 1 | 1490361 | 153719 | -13 |
|  | 1 | 849 | 89 | $-10$ |  | 18 | 2143295 | 221064 | + 1 |
|  | 18 | 1574 | 165 | + 1 | 95 | 9 | 1 | 0 | + 1 |
| 92 |  | 1 |  |  |  | 1 | 9 | 1 | - 14 |
|  | 1 | 9 | 1 | -11 |  | (2) | 10 | 1 | + 5 |
|  |  | 10 | 1 | -11 $+\quad 3$ |  | 1 | 29 | 3 | - 14 |
|  | 2 | 19 | 2 | $+\quad 3$ $-\quad 7$ |  | 18 | 39 | 4 | + 1 |
|  | (4) | 48 | 5 | + 4 | 96 | 9 | 1 | 0 | + 1 |
|  | 2 | 211 | 22 | - 7 |  | 1 | 9 | 1 | -15 |
|  | 1 | 470 | 49 | + 3 |  | (3) | 10 | 1 | + 4 |
|  | 1 | 681 | 71 | - 11 |  | 1 | 39 | 4 | -15 |
|  | 18 | 1151 | 120 | $+1$ |  | 18 | 49 | 5 | + 1 |

Spectmen of extended Form of Pellian Equation Table-continued,

| $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ | $a$ |  | $y$ | $x$ | $y^{2}-a x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 97 | 9 | 1 | 0 | + 1 | 98 | 9 | 1 | 0 | + 1 |
|  | 1 | 9 | 1 | -16 |  | 1 | 9 | 1 | - 17 |
|  | 5 | 10 | 1 | + 3 |  | (8) | 10 | 1 | + 2 |
|  | 1 | 59 | 6 | - 11 |  | 1 | 89 | 9 | -17 |
|  | 1 | 69 | 7 | + 8 |  | 18 | 99 | 10 | + 1 |
|  |  | 128 | 13 | - 9 |  |  |  |  |  |
|  | (1) | 197 | 20 | + 9 | 99 | 9 | 1 | 0 | + 1 |
|  | 1 | 325 | 33 | - 8 |  | (1) | 9 | 1 | - 18 |
|  | 1 | 522 | 53 | + 11 |  | 18 | 10 | 1 | + 1 |
|  | 5 | 847 | 86 | - 3 |  |  |  |  |  |
|  | 1 | 4757 | 483 | +16 |  |  |  |  |  |
|  | 18 | 5604 | 569 | - 1 |  |  |  |  |  |

The meaning hardly requires explanation; for each number $a$, we have a series of pairs of increasing numbers, $y, x$, satisfying a series of equations $y^{2}=a x^{2} \pm b$; thus

\[

\]

The following table, calculated under the superintendence of the Committee, extends from $a=1001$ to $a=1500$ (square numbers omitted); it is (with slight typographical variations) nearly but not exactly in the form of Degen's Table I., the chief difference being that for a number $a$ having a double middle term, or of the form $a^{2}+1$ (such number being further distinguished by an asterisk), the $x, y$ entered in the table are the solutions, not of the equation $y^{2}=a x^{2}+1$, but of the equation $y^{2}=a x^{2}-1$. As remarked above, if we have $y^{2}=a x^{2}-1$, then writing $y_{1}=2 y^{2}+1$ and $x_{1}=2 x y$, we obtain $y_{1}^{2}=a x_{1}^{2}+1$.

Moreover, for each value of $a$, in the first line, the first term, which is the integer part of $\sqrt{a}$, is separated from the other by a semicolon, and the 1 , which is the corresponding first term of the second line, is omitted.
C. XIII.

The calculations were made by C. E. Bickmore, M.A., of New College, Oxford: his values for $x$ and $y$ have been revised as presently mentioned, but it has been assumed that his values for the periods and subsidiary numbers_(forming the first and second lines of each division of the table) are accurate; in fact, any error therein would cause the resulting values of $x$ and $y$ to be wildly erroneous; but (except in a single instance which was accounted for) the errors in $x$ and $y$ were in every case in a single figure or two or three figures only.

The values of $x$ and $y$ were in every case examined by substitution in the equation ( $y^{2}=a x^{2}+1$, or $y^{2}=a x^{2}-1$, as the case may be), which should be satisfied by them. These verifications were for the most part made by A. Graham, M.A., of the Observatory, Cambridge. As already mentioned, some errors were detected, and these have been, of course, corrected. The values of $x, y$ given in the table thus satisfy in every case the proper equation $y^{2}=a x^{2}+1$, or $y^{2}=a x^{2}-1$; on the ground above referred to, it is believed that the periods and subsidiary numbers are also accurate.

It may be remarked, in regard to the verification of the equation $y^{2}=a x^{2} \pm 1$ for large values of $x$ and $y$, it is in practice easier and safer to calculate $a x^{2} \pm 1$, and then to compare the square root thereof with the given value of $y$, than to further calculate the value of $y^{2}$.

The Table 1001 to 1500.

| 1001 | $\begin{array}{rrrrrr} 31 ; & 1, & 1, & 1, & 3, & 3, \\ 40, & 23, & (45) \\ 40, & 16, & 17, & 25, & (13) \end{array}$ |  |  | $\begin{array}{r} 33532 \\ 0 \quad 60905 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1002 |  |  |  | $\begin{align*} & 535248  \tag{6}\\ & 869247 \end{align*}$ |
| 1003 | $\begin{array}{rrr} 31 ; & 1, & 2, \\ 42, & 21 & (2) \end{array}$ |  |  | $\begin{array}{r} 285 \\ 9026 \end{array}$ |
| 1004 |  |  |  | $\begin{aligned} & 59730 \\ & 24199 \end{aligned}$ |
| 1005 | $\begin{array}{rrrrrrrr} 31 ; & 1, & 2, & 1, & 5, & 15, & 1, & 2, \\ 44, & 19, & 20, & 39, & 11, & 4, & 41, & 21, \end{array}$ |  |  | $\begin{aligned} & 059568 \\ & 149761 \end{aligned}$ |
| 1006 | $3 \mathrm{I} ; \mathrm{I}, 2, \mathrm{I}, \mathrm{I}, 5, \mathrm{I}, 3,2, \mathrm{I}, \mathrm{I}, \mathrm{I}, \mathrm{I}, \mathrm{I}, 9,1,20,4,5, \mathrm{I}, \mathrm{I}, \mathrm{I} 2,6, \mathrm{I},(30) 445346$ $45,18,29,33,10,43,15,22,31,27,30,25,37,6,55,3,15,1$ 1, 30, $33,5,9,53$, (2) 14125267 | $\begin{aligned} & 14025 \\ & 56378 \end{aligned}$ | $\begin{aligned} & 5574! \\ & 02146 \end{aligned}$ | $\begin{aligned} & 21748 \\ & 05455 \end{aligned}$ |
| 1007 | $\begin{array}{r} 31 ; \quad 1, \quad 2, \quad \text { (1) } \\ 46,17,(38) \end{array}$ |  |  | 15 476 |
| 1008 | $\begin{array}{rrr} 3 \mathrm{r} ; & 1,(2) \\ 47, & (16) \end{array}$ |  |  | 4 127 |
| 1009* | $\begin{array}{rrr} 31 ; & (3, & 3) \\ 48, & (15, & 15) \end{array}$ |  |  | 17 540 |
| 1010* | $\begin{array}{rrrr} 31 ; & 1, & 3, & (\mathrm{I}, \\ 49, & 14, & (31, & 3 \mathrm{I}) \end{array}$ |  |  | $\begin{array}{r} 41 \\ 1303 \end{array}$ |
| 1011 | $\begin{array}{rrrr} \hline 3 \mathrm{r} ; & 1, & 3, & 19) \\ 50, & 13, & 47, & (6) \end{array}$ |  |  | $\begin{array}{r} 265 \\ 8426 \end{array}$ |
| 1012 | $\begin{array}{rrrrrrrr} 31 ; & 1, & 4, & 3, & 6, & 1, & 3, & 8, \\ 51, & \text { (4) } \\ 51 & 19, & 9, & 43, & 16, & 7, & 48, & \text { (II) } \end{array}$ |  |  | $\begin{aligned} & 302110 \\ & \text { f } 17399 \end{aligned}$ |
| 1013* | $\begin{array}{rrrrrrrrr} 3^{I} ; & 1, & 4, & 1, & 4, & \dot{1} 5, & 1, & 2, & (2, \\ 52 & 2) \\ 52, & 11 & 44, & 13, & 4, & 43, & 19, & (23, & 23) \end{array}$ |  |  | $\begin{aligned} & 352985 \\ & 166618 \end{aligned}$ |
| 1014 | $\begin{array}{r} 3 \mathrm{I} ; \\ \mathrm{I}, \\ 53, \\ 53, \\ 10, \\ \hline \end{array} 23,1, \quad 1, \quad 1,1, \quad 1,(20)$ |  |  | $\begin{aligned} & 146266 \\ & 656965 \end{aligned}$ |
| 1015 | $\begin{array}{rrrr} 31 ; & 1, & 6, & 10, \\ 54, & (2) & 6, & (29) \end{array}$ |  |  | $\begin{array}{r} 11076 \\ 352871 \end{array}$ |
| 1016 | $\begin{array}{r} 1,(6) \\ 55,(8) \end{array}$ |  |  | 8 255 |
| 1017 |  |  | $\begin{aligned} & 09655 \\ & 0932 \end{aligned}$ | $\begin{aligned} & 584992 \\ & 297217 \end{aligned}$ |
| 1018* |  |  |  | $\begin{array}{ll} 7 & 28333 \\ 0 & 50499 \end{array}$ |
| 1019 | $\begin{array}{rrrrrrrrrrr} 3 \mathrm{I} ; & 1, & 1 \mathrm{II}, & \text { I, } & 3, & 1, & 1, & 1, & 3, & 8, & \text { I, } \\ 58, & 2, & (3 \mathrm{I}) \\ 58, & 57, & 14, & 35, & 25, & 34, & 17, & 7, & 49, & 10, & 29, \end{array}$ |  | $\begin{aligned} & 0776 \\ & 9923 \end{aligned}$ | $\begin{array}{ll} 4 & 36539 \\ 3 & 21730 \end{array}$ |
| 1020 | $\begin{array}{rcc} 3 \mathrm{I} ; & 1, & (\mathrm{I} 4) \\ 59, & (4) \end{array}$ |  |  | 16 511 |
| 1021* | 3 I ; I, 20, 3, 6, 1, 3, 2, I, I, 12, 5, 4, 15, I, 2, I, 4, I, I, 2, I, I, I, (5, 5) 9865001 $60,3,20,9,44, \mathrm{I}_{5}, 23,27,36,5, \mathrm{I} 2, \mathrm{I}_{5}, 4,45, \mathrm{I} 7,4 \mathrm{I}, \mathrm{I} 2,33,29,20,33,25,36,(\mathrm{II}, \mathrm{II}) 315217280$ | $\begin{aligned} & 29666 \\ & 37258 \end{aligned}$ | $\begin{aligned} & 6956 \\ & 4882 \end{aligned}$ | $\begin{array}{ll} 4 & 06909 \\ 5 & 15030 \end{array}$ |

Table 1001 то 1500-continued.


## Table 1001 то 1500-continued.



Table 1001 то 1500-continued.

| 1063 |  | $\begin{array}{r} 5353 \\ 174532 \end{array}$ | $\begin{aligned} & 1527412685 \\ & 4831097224 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 1064 | $\begin{array}{r} 32 ; \quad 1, \\ 40, \\ 45, \end{array} \begin{array}{rr}  & (1) \\ \hline \end{array}$ |  | $\begin{array}{r} 21 \\ 685 \end{array}$ |
| 1065 |  |  | $\begin{array}{r} 2533160 \\ 82667999 \end{array}$ |
| 1066* | $\begin{array}{rrrrrrr} 3^{2} ; & 1, & 1, & 1, & 5, & 1, & 6, \\ 42, & (2, & 2) \\ 42, & 39, & 10, & 49, & 9, & (25, & 25) \end{array}$ |  | $\begin{array}{r} 105205 \\ 3434907 \end{array}$ |
| 1067 | $\begin{array}{r} 3^{2} ; \quad 1, \quad(1) \\ 43, \end{array}$ |  | 3 98 |
| 1068 | $\begin{array}{r} \left.3^{2} ; \begin{array}{rrrr} 1, & 2, & 7, & 5, \\ 44, & 21, & 8, & 49, \\ 41 & (4) \\ \hline \end{array}\right) \end{array}$ |  | $\begin{array}{r} 353094 \\ 11539207 \end{array}$ |
| 1069* | $3_{2}$; $1,2,3,1,1,21,4,3,5,7,12,1,15,2,2,1,3,1,1,1,4,1,(4,4)$ $45,20,17,29,36,3,15,19,12,9,5,57,4,27,20,39,15,36,25,37,12,45,(13,13)$ | 1864098637841 60947759016096 | 7772615285 8572612782 |
| 1070 | $\begin{array}{rrrrrr} 32 ; & 1, & 2, & 2, & 5, & 1, \\ 46, & 1, & (12) \\ 49, & 26, & 11, & 31, & 34, & (5) \end{array}$ |  | $\begin{array}{r} 90138 \\ 2948491 \end{array}$ |
| 1071 | $\begin{array}{r} 32 ; \quad 1, \quad 2, \\ 47, \\ 48, \\ \hline 1, \\ 35, \\ \hline \end{array}$ |  | $\begin{array}{r} 880 \\ 28799 \end{array}$ |
| 1072 | $\begin{array}{r} \left.3^{2} ; \begin{array}{rrrrrrrrrrr} 1, & 2, & 1, & 1, & 1, & 8, & 1, & 4, & 1, & 1, & (3) \\ 48, & 17, & 44, & 9, & 32, & 33, & 7, & 49, & 12, & 33, & 31, \end{array}\right)\left(\begin{array}{ll} 16) \end{array}\right. \end{array}$ |  | $\begin{array}{r} 145720107 \\ 4771081927 \end{array}$ |
| 1073* | $\begin{array}{r} 3^{2} ; \quad 1, \quad 3,(9,9) \\ 49,16,(7,7) \end{array}$ |  | $\begin{array}{r} 1385 \\ 45368 \end{array}$ |
| 1074 | $\begin{array}{rlrrrrr} 3^{2} ; & 1, & 3, & 2, & 1, & 1, & 2, \\ 50, & 15, & 23, & 31, & 30, & 25, & (2) \end{array}$ |  | $\begin{array}{r} 106476 \\ 3489425 \end{array}$ |
| 1075 | $\begin{array}{r} 32 ; \end{array} \begin{array}{rrrrrrrrrrrr} 1, & 3, & 1, & 2, & 3, & 10, & 1, & 1, & 1, & 2, & 2, & 6, \\ 51, & 14, & 39, & 21, & 19, & 6, & 39, & 25, & 34, & 21, & 26, & 9, \end{array}$ | 16 | $\begin{aligned} & 5150412729 \\ & 8867574226 \end{aligned}$ |
| 1076 | $\begin{array}{rlrr} 3^{2} ; & 1, & 4, & (16) \\ 5^{2}, & 13, & (4) \end{array}$ |  | $\begin{array}{r} 410 \\ 13449 \end{array}$ |
| 1077 | $\begin{array}{llllll} 3^{2} ; & 1, & 4, & 2, & 15, & 1, \\ 53, & (20)  \tag{3}\\ 53 & 29, & 4, & 59 & (3) \end{array}$ | - | $\begin{array}{r} 716760 \\ 23522399 \end{array}$ |
| 1078 | $3^{2} ; \begin{aligned} 1, & (4) \\ 54, & \text { (II) } \end{aligned}$ |  | 6 197 |
| 1079 | $32 ;$ 1, 5, 1, 1, 2, 2, 4, 3, <br> 55, 10 $(1)$       <br> 55, 29, 22, 25, 14, 17, 35, $(26)$  |  | $\begin{array}{r} 5497325 \\ 180576876 \end{array}$ |
| 1080 |  |  | $\begin{array}{r} 161 \\ 5291 \end{array}$ |
| 1081 |  | $\begin{array}{r} 891812221 \\ 29321505610 \end{array}$ | 8728007648 1715119615 |
| 1082* | $\begin{array}{rrrrrrr} 3^{2} ; & 1, & 8, & 2, & 2, & 2, & (1, \\ 58, & 1) \\ 58, & 26, & 23, & 22, & (31, & 31) \end{array}$ |  | $\begin{array}{r} 38369 \\ 1262101 \end{array}$ |
| 1083 | $\begin{array}{rr} 3^{2} ; & 1, \\ 59 & (9) \\ 59 & (6) \end{array}$ |  | 11 362 |

Table 1001 to 1500-continued.

| 1084 | $32 ; 1,12,5,2,2,3,2,6,1,7,2,1,2,1,3,1,1,1,21,3,4,(16)$ $60,5,12,25,24,17,27,9,51,8,23,36,19,40,15,37,24,41,3,20,15$, | 817041270290926993200 $269003936420 \pm 0537919999$ |
| :---: | :---: | :---: |
| 1085 |  | $\begin{array}{r} 544 \\ 17919 \end{array}$ |
| 1086 | $\begin{gathered} 3^{2} ; \begin{array}{r} 1, \\ 62, \end{array}(30) \end{gathered}$ | 22 725 |
| 1087 | $\begin{array}{rr} 3^{2} ; & 1, \\ 63, & (2 \mathrm{I}) \\ 63 \end{array}$ | $\begin{array}{r} 33 \\ 1088 \end{array}$ |
| 1088 | $3^{2} ; \begin{gathered} (1) \\ \left(6_{4}\right) \end{gathered}$ | $\begin{array}{r} 1 \\ 33 \end{array}$ |
| 1090* | $33 ; \underset{(\mathrm{I})}{(66)}$ | $\begin{array}{r} 1 \\ 33 \end{array}$ |
| 1091 | $\begin{array}{r} 33 ;(33) \\ (2) \end{array}$ | 33 1090 |
| 1092 | $33 ;(22)$ (3) | 22 727 |
| 1093* | $\begin{array}{rrr} 33 ; & 16, & (1, \\ 4, & (33, & 33) \end{array}$ | $\begin{array}{r} 545 \\ 18018 \end{array}$ |
| 1094 | 33 ; $13,4,1,1,1,5,2,1,2,3,9,6,1,1,(32)$ <br> $5,14,37,25,38,11,23,35,22,19,7,10,31,35$, (2) | 450438017467914 14898540581538085 |
| 1095 | $\begin{array}{r} 33 ;\left(\begin{array}{rl} (11) \\ (6) \end{array}\right. \end{array}$ | 11 364 |
| 1096 | $\begin{array}{rlrrr} 33 ; & 9, & 2, & 3, & 1, \\ 7, & (15) \\ 7, & 28, & 15, & 49, & (4) \end{array}$ | $\begin{array}{r} 119595 \\ 3959299 \end{array}$ |
| 1097* | $\left.\begin{array}{rlrrrrrr} 33 ; & 8, & 3, & 1, & 3, & 2, & 1, & (1, \\ 8, & 17 \end{array}\right)$ | $\begin{array}{r} 634621 \\ 21019276 \end{array}$ |
| 1098 | $\begin{array}{rlrr} \hline 33 ; & 7, & 2, & 1, \\ & 9, & 22, & 4 \mathrm{I}, \\ \hline \end{array}$ | $\begin{array}{r} 3564 \\ 118097 \end{array}$ |
| 1099 | 33 ; 6, $1,1,1,1,2,21,1,2,1,1,6,1,(3)$ 10, $37,27,30,31,25,3,46,19,30,35,9,47$, (14) | $\begin{array}{r} 48057545715 \\ 1593163815326 \end{array}$ |
| 1100 | $\begin{aligned} & 33 ;(6) \\ &(\mathrm{II}) \end{aligned}$ | 6 199 |
| 1101 | $\begin{array}{r} 33 ; \\ 5,1,1,16, \\ 12,3 \mathrm{I}, 35, \\ 1, \end{array}$ | $\begin{array}{r} 732732 \\ 24313015 \end{array}$ |
| 1102 | $\begin{array}{r} 33 ; 5, \\ \text { 10, } \\ \text { 13, } \\ \hline \end{array}$ | $\begin{array}{r} 342882 \\ 11382443 \end{array}$ |
| 1103 | $\left.\begin{array}{r} 33 ; 4,1,2,1,2,3,1,1,45,2,9, \end{array} \begin{array}{r} (33) \\ 14,41, \\ 49 \end{array}\right) 37,22,17,31,34,11,29,77,(2)$ | 70645611145 2346242745024 |
| 1104 | $\left.\begin{array}{r} 33 ; \\ 4, \\ 15, \\ 15 \end{array}\right)(25,(23)$ | 234 7775 |
| 1105* | $\begin{array}{r} 33 ; \quad 4,(7,7) \\ 16,(9,9) \end{array}$ | $\begin{array}{r} 857 \\ 28488 \end{array}$ |

Table 1001 то 1500-continued.

| 1106 | $\begin{array}{r} 33 ; 3,1,(8) \\ 17,46,(7) \end{array}$ |  | $\begin{array}{r} 152 \\ 5055 \end{array}$ |
| :---: | :---: | :---: | :---: |
| 1107 |  |  | $\begin{array}{r} 218295 \\ 7263026 \end{array}$ |
| 1108 | $33 ; 3,2,21,1,3,4,1,6,1,1,2,2,1,3,2,5,9,3,(16)$ $19,28,3,49,16,13,48,9,36,29,23,21,39,16,27,12,7,21$, (4) | $\begin{array}{r} 478120058 \\ 15915007379 \end{array}$ | $\begin{aligned} & 6939013510 \\ & 8980475849 \end{aligned}$ |
| 1109* | $\begin{array}{r} 33 ; \end{array} \begin{array}{rrrrrrrrr} 3 & 3, & 1, & 13, & 16,1, & 1, & 2, & 1, & 4, \\ 20, & (2, & 2) \\ 19, & 11, & 44, & 17, & 5, & 4, & 37, & 29, & 20, \\ 41 & 13, & (25, & 25) \end{array}$ | $\begin{array}{r} 1832 \\ 61020 \end{array}$ | 3595738617 6001542610 |
| 1110 | $33 ; \begin{array}{rr} 3, & (6) \\ 2 \mathrm{I}, & (\mathrm{IO}) \end{array}$ |  | $\begin{array}{r} 60 \\ 1999 \end{array}$ |
| 1111 | $33 ; \underset{(22)}{(3)}$ |  | 3 100 |
| 1112 | $\begin{array}{rccc} 33 ; & 2, & 1, & (7) \\ 23, & 41 & (8) \end{array}$ |  | $\begin{array}{r} 75 \\ 2501 \end{array}$ |
| 1113 | $\left.\begin{array}{r} 33 ; \\ 24, \\ 24, \\ 37, \\ 17, \end{array}\right)$ |  | $\begin{array}{r} 21056 \\ 702463 \end{array}$ |
| 1114* | $\begin{array}{r} 33 ; \quad 2,1,1,1,8,1,10,4,2,1,3,1,3,7,6,(1,1) \\ 25,33,26,39,7,55,6,15,22,39,15,42, \\ 17,9, \end{array}$ | $\begin{array}{r} 18311 \\ 611178 \end{array}$ | 5747156745 8103202293 |
| 1115 | $\begin{array}{rrrrrrr} 33 ; & 2, & 1, & 1, & 4, & 5, & 1, \\ 26, & \text { (5) } \\ & 29, & 35, & 14, & 1 \text { I }, & 49, & \text { (10) } \end{array}$ |  | $\begin{array}{r} 136565 \\ 4560126 \end{array}$ |
| 1116 | $\begin{array}{rlrrr} 33 ; & 2, & 2, & 5, & 1, \\ 27, & 25, & (16) \\ 21 & 40, & 23, & (4) \end{array}$ |  | $\begin{array}{rr} 138320 \\ 46 & 20799 \end{array}$ |
| 1117* |  | $\begin{array}{r} 627259559 \\ 20963986690 \end{array}$ | $\begin{aligned} & 5385543645 \\ & 7824541118 \end{aligned}$ |
| 1118 | $\begin{array}{rrrr} 33 ; & 2, & 3, & (2) \\ 29, & 17, & (26) \end{array}$ |  | $\begin{array}{r} 126 \\ 4213 \end{array}$ |
| 1119 | $\begin{array}{rrrrrr} \hline 33 ; & 2, & 4, & 1, & 1, & 1, \\ 30, & 6, & (22) \\ 30 & 3^{8}, & 25, & 39, & 10, & (3) \end{array}$ |  | $\begin{array}{r} 946364 \\ 31657255 \end{array}$ |
| 1120 | $\begin{array}{rrr} 33 ; & 2,6,1, & (15) \\ 31, & 9, & 55, \end{array}$ |  | $\begin{array}{r} 3765 \\ 126001 \end{array}$ |
| 1121 | $\left.\begin{array}{r} 33 ; \end{array} \begin{array}{r} 2, \\ 32, \\ 32, \\ 3 \end{array}\right)$ |  | $\begin{aligned} & 9397558620 \\ & 51043 \quad 64951 \end{aligned}$ |
| 1122 | $33 ; \begin{array}{r} (2) \\ (33) \end{array}$ |  | 2 67 |
| 1123 | $\begin{array}{r} \left.33 ; \begin{array}{rrrrrrrrrrr} 1, & 21, & 1, & 5, & 3, & 1, & 1, & 2, & 2, & 10, & 1, \\ 34, & 33 & 3, & 54, & 11, & 9, & 18, & 33, & 31, & 22, & 27 \end{array}\right) 6,47, \\ 17 \end{array}$ | $\begin{array}{r} 49257 \\ 16 \quad 50664 \end{array}$ | $\begin{aligned} & 1123214799 \\ & 5562632482 \end{aligned}$ |
| 1124 |  |  | 7475787740 2200630049 |
| 1125 | 33 ; $1,1,5,1,1,2,7,16,1,1,1,2,1,(6)$ $36,3 \mathrm{I}, 11,36,29,25,9,4,4 \mathrm{I}, 25,36,19,44, \text { (9). }$ |  | $\begin{aligned} & 1600291864 \\ & 0726404001 \end{aligned}$ |
| 1126 | $\begin{array}{rrrrr} 33 ; & 1, & 1, & 3, & \text { (32) } \\ 37, & 30, & 15, & 51 & (2) \end{array}$ |  | $\begin{array}{r} 2718 \\ 91205 \end{array}$ |

Table 1001 то 1500 -continued.

C. XIII.

Table 1001 то 1500 -continued.

| 1147 | $33 ; \mathrm{I}, 6, \mathrm{I}, \mathrm{I}, 5, \mathrm{I}, \mathrm{I}, \mathrm{I}, \mathrm{I}, \mathrm{I}, 2 \mathrm{I},(\mathrm{I})$ <br> $58,9,34,33,1 \mathrm{I}, 38,27,33,26,4 \mathrm{I}, 3$, (62) | $\begin{array}{rr} 278945403 \\ 94471 & 52318 \end{array}$ |
| :---: | :---: | :---: |
| 1148 | 33; 1, 7, 2, (16) | 4896 |
|  | 59, 8, 31, (4) | 165887 |
| 1149 | $33 ; 1,8,1,2,3,(22)$ | 212624 |
|  | 60, 7, 44, 21, 20, (3) | 7207295 |
| 1150 | 33; 1, 10, 3, 7, 4, 1, 2, (2) | 34925592 |
| 1150 | 6I, 6, 21, 9, 14, 41, 2I, (25) | 1184384449 |
| 1151 | $33 ; \begin{aligned} & 1,12,1,1,2,2,3,6,2,(33) \\ & 62,\end{aligned}$ | 1942607807 |
|  | 33; 1, (15) | 17 |
| 1152 | ${ }^{3} 63,(4)$ | 577 |
| 1153* | 33 ; $1,21,1,1,1,6,1,7,1,1,1,1,1,2,4,1,5,2,1,3,(\mathrm{I}, 1)$ | 3017890256875073 |
|  |  |  |
| 1154 | 33; 1, (32) | 34 |
| 1154 | 65 , (2) | 1155 |
| 1155 | 33; (1) | - 1 |
| 1155 | (66) | 34 |
| 1157* | 34; (68) | 1 |
| 1157 | (1) | 34 |
| 1158 | 34; (34) | 34 |
| 1158 | (2) | 1157 |
| 1159 | $34 ; 22,1,2,7,4,2,2,13,4,1,3,1,(2)$ | 49028938575180 |
| 1159 | $3,45,22,9,15,25,27,5,14,45,15,42$, (19) | 1669145551424551 |
| 1160 | 34; (17) | 17 |
| 1160 | (4) | 579 |
| 1161 | $34 ; 13,1,1,1,1,2,8,7,2,4,1,3,12)$ | 562121440972 |
|  | $5,40,27,31,32,25,8,9,29,13,45,16,(27)$ | 19153416854935 |
| 1162 | 34; II, 2, I, 6, 1, (8) | 663462 |
| 1162 | $6,23,42,9,54,(7)$ | 22616173 |
| 1163 | $34 ; 9,1,2,1,2,4,1,1,(33)$ | 36956541 |
|  | $7,46,19,38,23,14,3 \mathrm{r}, 37$, (2) | 1260321002 |
| 1164 | $34 ; 8,1,1$, (16) | 4930 |
| 1164 | $8,33,35$, (4) | 168199 |
| 1165* | $34 ; 7,1,1,3,16,1,3,1,(\mathrm{I}, ~ \mathrm{I})$ | 86720773 |
|  | 9, 36, 31, 19, 4, 51, 15, 36, (29, 29) | 2959961778 |
| 1166 | $34 ; 6,1,4,2,1,1,9,{ }^{(6)}$ | 19340870 |
| 1166 | 10, 49, I3, 25, 29, $3^{8,7}$, (II) | 660427701 |
| 1167 |  | 10943 |
|  |  | 373828 |
| 1168 |  | 66750 |
|  |  | 2281249 |

Table 1001 то 1500 -continued.


Table 1001 то 1500 -continued.

| 1190 | $34 ;(2)$ | 2 69 |
| :---: | :---: | :---: |
| 1191 | $\begin{array}{rrr} 34 ; & 1, & 1, \\ 35, & (22) \\ 34, & (3) \end{array}$ | 92 3175 |
| 1192 | $\begin{array}{rrrrrrrrrr} 34 ; & 1, & 9, & 2, & \text { I, } & 3, & 2, & 1, & 1, & 1, \\ \mathbf{3}, & 7, & (17) \\ 33, & 7, & 24, & 39, & 17, & 24, & 33, & 3 & \mathbf{1}, & 28, \\ 39 & 9, & (4) \end{array}$ | $\begin{array}{r} 9716739825 \\ 5473872499 \end{array}$ |
| 1193* |  | $\begin{array}{r} 24753805 \\ 854992268 \end{array}$ |
| 1194 |  | $\begin{array}{r} 626476750  \tag{6}\\ 21647468749 \end{array}$ |
| 1195 | $\begin{array}{rrrrrrr} 34 ; & 1, & 1, & 3,7, & 2, & 1, & 1, \\ 39, & 10, & 1, & (12) \\ 30, & 19, & 9, & 26, & 29,39 & 6, & 59, \end{array}$ | $\begin{array}{r} 1345 \\ 46500 \quad 60959 \end{array}$ |
| 1196 | $\begin{array}{rrrr} 34 ; & 1, & 1, & (2) \\ 40, & 29, & (23) \end{array}$ | 12 415 |
| 1197 | $\begin{array}{rrrrr} \hline 34 ; & \mathbf{1}, & \mathbf{I}, & \mathbf{2}, & 16, \\ 4 \mathrm{I}, & 28, & 27, & 4, & 59, \end{array}$ | $\begin{array}{r} 74820 \\ 25 \quad 88599 \end{array}$ |
| 1198 |  | $\begin{array}{lll}8 & 36463 & 16460 \\ 23592 & 51599\end{array}$ |
| 1199 | $\begin{array}{r} 34 ; \\ 4,1,1, \\ 43, \\ 46,35, \\ 25, \\ \hline \end{array}$ | $\begin{array}{r} 14375599 \\ 497777820 \end{array}$ |
| 1200 | $\begin{array}{rrrrr} \hline 34 ; & 1, & 1, & 1, & (3) \\ 44, & 25, & 39, & (16) \end{array}$ | $\begin{array}{r} 39 \\ 1351 \end{array}$ |
| 1201* |  | $\begin{array}{lll} 9 & 44230 & 77385 \\ 6 & 50571 & 44832 \end{array}$ |
| 1202 | $\begin{array}{rrrr} 34 ; & 1, & 2, & (34) \\ 46, & 23 & (2) \end{array}$ | $\begin{array}{r} 312 \\ 10817 \end{array}$ |
| 1203 | $\begin{array}{rrrrr} 34 ; & 1, & 2, & 5, & 1, \\ 47, & 22, & 11, & 57 & (2) \end{array}$ | $\begin{array}{r} 12521 \\ 434282 \end{array}$ |
| 1204 |  | 9556554116 3392537695 |
| 1205 | $\begin{array}{r} 34 ; \quad 1,2, \quad 2,16,1, \\ 49,20,29,4, \\ 4 \mathrm{I}, \end{array}$ | $\begin{array}{r} 206668 \\ 7174089 \end{array}$ |
| 1206 | $\begin{array}{rlrr} 34 ; & 1, & 2, & 1, \\ 50, & 2, & (34) \\ 50 & 19, & 38, & 25, \end{array}$ | $\begin{array}{r} 4202 \\ 145925 \end{array}$ |
| 1207 | $\begin{array}{r} 34 ; \\ \mathrm{I}, \\ 5 \mathrm{I}, \\ 28, \\ \hline \end{array}$ | $\begin{array}{r} 33387 \\ 1159928 \end{array}$ |
| 1208 |  | $\begin{array}{rr} 1 & 23046 \\ 42 & 76623 \end{array}$ |
| 1209 | $\left.\begin{array}{r} 34 ; \\ 1, \\ 53, \\ 53, \\ 16, \\ 23, \\ 40 \end{array}\right)$ | $\begin{array}{r} 1640156 \\ 57029335 \end{array}$ |

Table 1001 то 1500-continued.

| 1210 | $\left.\begin{array}{r} 34 ; \\ \mathrm{I}, \\ 54, \\ 54 \end{array}\right)$ |  |  |  | $\begin{aligned} & 99524 \\ & 19281 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1211 | $34 ; \begin{array}{rr} 1, & (3) \\ 55, & (14) \end{array}$ |  |  |  | 5 |
| 1212 | $34 ; \begin{array}{rrrrrr} 1, & 4, & 2, & 1, & 2, & 2, \\ 56, & 13, & (16) \\ 54, & 24, & 37, & 23, & 21, & 47, \end{array}$ |  |  |  | $65980$ |
| 1213* | $\left.\begin{array}{rrrrrrrr} 34 ; & 1, & 4, & 1, & 4, & 1, & 1, & 9, \\ 57, & (2, & 2) \\ 54 & 12, & 49, & 13, & 33, & 36, & 7, & (27, \\ 27 \end{array}\right)$ |  |  |  | $\begin{aligned} & 27117 \\ & 00734 \end{aligned}$ |
| 1214 |  |  | $\begin{array}{r} 209 \\ 7315 \tag{2} \end{array}$ | $\begin{aligned} & 94702 \\ & 07983 \end{aligned}$ | $74975$ |
| 1215 | $\begin{aligned} & 34 ; \text { I, } \\ & 59,(5) \\ & 59 \end{aligned}$ |  |  |  | 7 244 |
| 1216 | $\left.\begin{array}{r} 34 ; \\ \mathbf{1}, 6,1,  \tag{4}\\ 60, \\ 6, \\ 48, \\ 17 \end{array}\right)$ |  |  | $\begin{array}{r} 1916 \\ 66814 \end{array}$ | $\begin{aligned} & 03685 \\ & 48801 \end{aligned}$ |
| $1217^{*}$ |  |  |  |  | $\begin{aligned} & 91969 \\ & 28256 \end{aligned}$ |
| 1218 | $\begin{array}{r} 34 ; \quad 1,(8) \\ 62,(7) \end{array}$ |  |  |  | $\begin{array}{r} 10 \\ 349 \end{array}$ |
| 1219 | $\left.\begin{array}{r} 1,10,1,1,1,7,9,1,5,2,4,5,6,1, \\ \hline 63,6,43,25,42,9,7,54, \\ 64, \end{array}\right)$ | 26 | $\begin{aligned} & 76791 \\ & 81111 \end{aligned}$ | $\begin{aligned} & 47002 \\ & 24548 \end{aligned}$ | $\begin{aligned} & 95135 \\ & 55326 \end{aligned}$ |
| 1220 | $\begin{array}{rrr} \hline 34 ; & (12) \\ 64, & (5) \end{array}$ |  |  |  | 14 489 |
| 1221 | $\begin{array}{rrrr} 34 ; & \text { ェ, } 16, & (2) \\ 65 . & 4, & (33) \end{array}$ |  |  |  | $\begin{array}{r} 612 \\ 21385 \end{array}$ |
| 1222 |  |  |  | $\begin{aligned} & 12495 \\ & 42454 \end{aligned}$ | $\begin{aligned} & 24122 \\ & 09507 \end{aligned}$ |
| 1223 | $34 ; \begin{array}{rr} 1, & (33) \\ 67 & (2) \end{array}$ |  |  |  | 35 1224 |
| 1224 | $\begin{array}{cc} \hline 34 ; & (\mathrm{I}) \\ (68) \end{array}$ |  |  |  | 1 35 |
| 1226* | 35 ; (70) <br> (I) |  |  |  | 1 35 |
| 1227 | 35 ; (35) <br> (2) |  |  |  | 35 1226 |
| 1228 | $\left.\begin{array}{r} 35 ; 23,2,1,7,8,1,4,1, \\ 3,24,43,9,8,41,27,36,23,24, \\ 34, \end{array}\right)$ |  | $\begin{aligned} & 44730 \\ & 67486 \end{aligned}$ | $\begin{aligned} & 59716 \\ & 75428 \end{aligned}$ | $\begin{aligned} & 699506 \\ & 871047 \end{aligned}$ |
| 1229* | $\begin{array}{r} 35 ; \\ 17, \\ 4, \\ 4, \\ (35, \\ \hline \end{array}$ |  |  |  | $\begin{array}{r} 613 \\ 21490 \end{array}$ |
| 1230 | $35 ;(14)$ <br> (5) |  |  |  | $\begin{array}{r}14 \\ 491 \\ \hline\end{array}$ |
| 1231 | 35 ; II, $1,2,7,2,4,1,13,4,1,1,1,1,6,2,2,4,3,1,2,23,(35)$ $6,45,23,9,30,13,54,5,15,37,30,29,39,10,27,26,15,18,39,25,3$, (2) | $\begin{array}{r} 58479034 \\ 2051772574 \end{array}$ | $\begin{array}{r} 452350 \\ +24010 \end{array}$ | $\begin{aligned} & 15227 \\ & 98134 \end{aligned}$ | $\begin{aligned} & 51177 \\ & 87200 \end{aligned}$ |

Table 1001 то 1500-continued.


Table 1001 то 1500-continued.


Table 1001 то 1500 -continued.


Table 1001 то 1500-continued.

C. XIII.

## Table 1001 то 1500-continued.



Table 1001 то 1500-continued.


Table 1001 то 1500-continued.


Table 1001 то 1500-continued.

| 1375 | $\begin{array}{rrrrrrrrr} 37 ; & 12, & 2, & 1,7, & 1, & 1, & 3, & 2, & 1, \\ 6, & 25, & (6) \\ 46, & 9, & 39 & 34, & 19, & 25, & 39, & 26, & \text { (11) } \end{array}$ | $\begin{array}{r} 2409073932 \\ 89330852249 \end{array}$ |
| :---: | :---: | :---: |
| 1376 | $\begin{array}{rrrrrrr} \hline 37 ; & 10, & 1, & 1, & 2, & 2, & 3, \\ 7, & 11, & (17) \\ 32, & 25, & 28, & 17, & 55, & (4) \end{array}$ | $\begin{array}{r} 5837205 \\ 2165 \quad 28049 \end{array}$ |
| 1377 | $\begin{array}{rrrrrrrrr} \hline 37 ; & 9, & 3, & 1, & 3, & 1, & 7, & 2, & 5, \\ 8, & 19, & 47, & 16, & 53, & 9, & 32, & 13, & (17) \end{array}$ | $\begin{array}{r} 1112162480 \\ 41270070401 \end{array}$ |
| 1378* | $\begin{array}{rrr} 37 & ; 8, & (4, \\ 9, & 4) \\ & (17, & 17) \end{array}$ | $\begin{array}{r} 1153 \\ 42801 \end{array}$ |
| 1379 | $\begin{array}{r} 37 ; \\ 7, \\ 10, \\ 10 \end{array} 2,2,26,2, \quad 1, \quad 1,3,3,(10)$ | $\begin{array}{r} 67806016 \\ 2517968895 \end{array}$ |
| 1380 | $\begin{array}{rrrr} 37 ; & 6, & 1, & (2) \\ & 11, & 49, & (20) \end{array}$ | $\begin{array}{r} 182 \\ 6761 \end{array}$ |
| 1381* | $\begin{gathered} 37 ; \\ 6,5,1,1,4,2,2,3,3,4,14,1,1,1,2,1,1,2,1,24, \\ 12,13,37,36,15,28,27,20,21,17,5,45,28,39,23,35,36,21,52,3 \\ 18,1,1,5,1,2,7,1,9,1, ~ 2, ~ 1, ~(4,4) \end{gathered}$ <br> 5767949505076394 <br> 214347434092377992 | 4676092612487905 503819770876282 |
| 1382 | $\begin{array}{rrrrrr} 37 ; & 5, & 1, & 2, & 2, & 1, \\ & 1, & (36) \\ & 13, & 46, & 23, & 26, & 31, \\ 43 & & (2) \end{array}$ | $\begin{array}{r} 349782 \\ 13003237 \end{array}$ |
| 1383 | $\begin{array}{rrrr} 37 ; & 5, & 2, & 1, \\ \hline & (11) \\ 14, & 2 \mathrm{r} & 23, & 49, \end{array}$ | $\begin{array}{r} 34821 \\ 1294948 \end{array}$ |
| 1384 | $\begin{aligned} 37 ; & 4, & 1, & (17) \\ & 15, & 57, & (4) \end{aligned}$ | $\begin{array}{r} 465 \\ 17299 \end{array}$ |
| 1385* | $\begin{array}{rlrrrr} 37 ; & 4, & 1, & 1, & 1, & (3, \\ 16, & 41 & 29, & 40, & (19, & 19) \end{array}$ | $\begin{array}{r} 2797 \\ 104092 \end{array}$ |
| 1386 | $\begin{array}{rrrrr} 37 & 4, & 2, & 1, & (2) \\ & 17, & 25, & 41, & (22) \end{array}$ | $\begin{array}{r} 572 \\ 21295 \end{array}$ |
| 1387 | $\begin{array}{rrr} 37 ; & 4, & 8, \\ 18, & (37) & (2) \end{array}$ | $\begin{array}{r} 40557 \\ 15 \quad 10442 \end{array}$ |
| 1388 |  | $\begin{array}{r} 22098697686 \\ 823306252807 \end{array}$ |
| 1389 | $\begin{array}{rrrr} 37 ; & 3, & 1, & 2, \\ 20, & (24) \\ 23, & 23, & 31, & (3) \end{array}$ | $\begin{array}{r} 16796 \\ 625975 \end{array}$ |
| 1390 | 37 ; 3, $1,1,6,4,1,4,1,1,11,1,7,2,1,2,1,(6)$ 21, 34, 39, 11, 15, 51, 14, 35, 39, 6, 61, 9, 26, 41, 21, 49, (10) | $\begin{array}{r} 61653851607582 \\ 2298622285432981 \end{array}$ |
| 1391 |  | $\begin{array}{r} 243492 \\ 9081305 \end{array}$ |
| 1392 | $\begin{array}{r} 37 ; \quad 3, \\ 23, \end{array}$ | $\begin{array}{r} 42 \\ 1567 \end{array}$ |
| 1393 | $\begin{array}{r} 37 ; \\ 34, \\ 24,(7) \end{array}$ | $\begin{array}{r} 96 \\ 3583 \end{array}$ |
| 1394 | $\begin{array}{rrrr} 37 ; & 2, & 1, & (36) \\ 25, & 49, & (2) \end{array}$ | $\begin{array}{r} 336 \\ 12545 \end{array}$ |

Table 1001 то 1500-continued.


Table 1001 то 1500 -continued.


Table 1001 то 1500-continued.

| 1437 | $\left.\begin{array}{r} 37 ; \\ \mathbf{1}, \\ 68, \\ 68, \\ 7, \\ 59, \\ 12, \\ 12 \end{array}\right)$ |  | $\begin{aligned} & 83205 \\ & 93382 \end{aligned}$ | $\begin{aligned} & 64040 \\ & 10399 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1438 |  |  | $\begin{aligned} & 08945 \\ & 07281 \end{aligned}$ | $\begin{aligned} & 06064 \\ & 27743 \end{aligned}$ |
| 1439 |  | $\begin{aligned} & 95537 \\ & 98517 \end{aligned}$ | $\begin{aligned} & 37610 \\ & 00303 \end{aligned}$ | $\begin{aligned} & 21521 \\ & 64960 \end{aligned}$ |
| 1440 | $\begin{array}{r} 37 ; ~ \\ 71,(17) \\ 7,(4) \end{array}$ |  |  | 19 721 |
| 1441 | $\begin{array}{r} 37 ; ~ 1,24,3,8,9,2,1,2,2,1,3,1,3,4,1,3,1,14,2,1,1,4,2, \text { (6) } \\ 72,3,24,9,8,27,40,25,24,45,17,48,19,15,51,16,57,5,29,33,40,15,32, \text { (11) } \\ 741014472 \end{array}$ | $\begin{aligned} & 95953 \\ & 84306 \end{aligned}$ | $\begin{aligned} & 31104 \\ & 85240 \end{aligned}$ | $\begin{aligned} & 82700 \\ & 54999 \end{aligned}$ |
| 1442 | $\begin{array}{rrr} 37 ; & 1, & (36) \\ 73, & (2) \end{array}$ |  |  | 38 1443 |
| 1443 | $37 \text {; ( } \mathrm{I} \text { ) }$ |  |  | 1 38 |
| 1445* | $38 ;(76)$ <br> (I) |  |  | 1 38 |
| 1446 | $38 ;(38)$ <br> (2) |  |  | 38 1445 |
| 1447 | $38 ;$ 25 2, 1,7, 1, 3, 1, 1, 2, 3, 1, 5, <br> 3 26,4, 12 $(37)$ 3        | $\begin{aligned} & 42829 \\ & 41033 \end{aligned}$ | $\begin{aligned} & 14389 \\ & 17574 \end{aligned}$ | $\begin{aligned} & 10093 \\ & 21848 \end{aligned}$ |
| 1448 | $\begin{equation*} 3^{8 ;} \text { (19) } \tag{4} \end{equation*}$ |  |  | 19 723 |
| 1449 | $\begin{array}{rrrrr} 38 ; & 15, & 4, & 1, & 2, \\ 5, & 16, & (8) \\ 55, & 25, & 17, & (9) \end{array}$ |  |  | $\begin{aligned} & 84800 \\ & 01249 \end{aligned}$ |
| 1450* | $\begin{array}{r} 38 ; 12, \quad 1, \quad 2,(8,8) \\ 6,49,25, \end{array}$ |  |  | $\begin{aligned} & 01933 \\ & 81493 \end{aligned}$ |
| 1451 | $\begin{array}{r} 38 ; 10, ~ 1, ~ 6, ~ 1, ~ 2, ~ 2, ~ 3, ~ 1, ~ 1, ~ 2, ~ 2, ~ 14, ~ 1, ~ 4, ~ 1, ~ \\ 7, \end{array}$ |  | $\begin{aligned} & 31566 \\ & 78443 \end{aligned}$ | $\begin{aligned} & 20707 \\ & 28530 \end{aligned}$ |
| 1452 | $\begin{array}{rlll} 38 ; & 9, & \text { I, } & \text { I, } \\ 8, & (18) \\ 87, & 39, & (4) \end{array}$ |  |  | $\begin{array}{r} 6878 \\ 62087 \end{array}$ |
| 1453* |  | $\begin{aligned} & 54563 \\ & 34442 \end{aligned}$ | $\begin{aligned} & 77143 \\ & 96990 \end{aligned}$ | $\begin{aligned} & 54805 \\ & 99982 \end{aligned}$ |
| 1454 | $\begin{array}{r} 38 ; \\ 7, \\ 10, \\ 13, \\ 43, \\ \hline 1, \end{array}$ | 36 | $\begin{aligned} & 96169 \\ & 67077 \end{aligned}$ | $\begin{aligned} & 61884 \\ & 58095 \end{aligned}$ |
| 1455 |  |  |  | 623 23764 |
| 1456 | $\begin{array}{r} 38 ; \\ \\ 12, \\ 12, \\ 25, \\ \hline \end{array}$ |  |  | $\begin{aligned} & 29855 \\ & 54951 \end{aligned}$ |
| 1457 | $\begin{array}{rrrrrrrrr} 38 ; & 5, & 1, & 6, & 9, & 2, & 1, & 1, & 10, \\ 13 & 56, & 11 & 8, & 29, & 32, & 43, & 7, & 23 \\ & 16, & (31) \end{array}$ |  | $\begin{aligned} & 94953 \\ & 31628 \end{aligned}$ | $\begin{aligned} & 10312 \\ & 36703 \end{aligned}$ |

Table 1001 то 1500-continued.

C. XIII.

Table 1001 то 1500-continued.


In connexion with the subject we have a paper, "A Table of the Square Roots of Prime Numbers of the form $4 m+1$ less than 10000 expanded as Periodic Continued Fractions," by C. A. Roberts, with Introduction and Explanation by Artemas Martin, the Mathematical Magazine, vol. II. (No. 7, for October, 1892), pp. 105-120. This extends, in fact, to numbers up to 10501, but only the denominators of the continued fractions (that is, the first lines of Degen's and the present table) are given: thus the entry for 1009 is $31 ; 1,(3,3)$.

The paper just referred to notices errors in Degen's tables for the numbers 853 and 929 . For 853 the first line should be

$$
29,4,1,5,1,2,4,1,1,15,19,(2,2)
$$

(15 instead of Degen's 14). For 929 the first and second lines should be

$$
\begin{array}{r}
30,2,11,1,2,3,2,7, \\
1,
\end{array} 29, \quad 5,40,19,16,25,8,11,(23,23) .
$$

The values of $x, y$ in Table I. and those in Table II. (for the solution of $y^{2}=a x^{2}-1$ ) are correct for each of the numbers 853 and 929 .

