NUMERICAL TREATMENT OF MATERIAL PARAMETER IDENTIFICATION USING FINITE ELEMENTS

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1. Introduction

There are many materials having different properties and behaviors which are characterized in solid mechanics by constitutive models. These constitutive models have material parameters, which have to be calibrated to experiemental data in order to predict the material behavior within the range of application. This calibration is called *material parameter identification* or briefly *parameter identification*, see [2, 4].

Depending on the homogeneity of the deformation, the procedure of material parameter identification varies. If the deformation is inhomogeneous, the entire boundary-value problem need to be solved, commonly by using finite element method. The material parameter identification for inelastic materials using finite elements were discussed in [1, 5, 6]. In order to identify the material parameters, the sensitivities of the parameter needs to be calculated. The most common method to calculate these sensitivites is by using external numerical differentiation, see, for the terminology, [7]. However, for a constitutive model with inhomogeneous deformation having large number of parameters, the computational cost is high. In order to circumvent this issue, internal numerical differentiation is performed which reduces the computational time by a considerable amount.

In this presentation, initially the underlying problem is explained. This is followed by a brief description of the internal and external differentiation procedures. A couple of examples showing the difference between external and internal numerical differentiation is also provided to round off the presentation.

2. Problem statement

The most commonly used basis for parameter identification process is the least-square problem. The simulated results, $\boldsymbol{s} \in \mathbb{R}^{n_s^e}$ (n_s^e is the number of simulated data), from the finite-element program are compared with the experimental data, $\boldsymbol{d} \in \mathbb{R}^{n_d^e}$ (n_d^e is the number of experimental data from one or more experiments). The residual vector, $\tilde{\boldsymbol{r}}(\boldsymbol{\kappa}) = \boldsymbol{W}_j \{\boldsymbol{s}(\boldsymbol{\kappa}) - \boldsymbol{d}\}$, describes the difference between the experimental and the simulated data. In order to consider the different order of physical values (forces, displacements), weighting matrices, $[\boldsymbol{W}_j]$, are introduced. The so-called *objective function*, $f(\boldsymbol{\kappa}) = \frac{1}{2}\tilde{\boldsymbol{r}}^T\tilde{\boldsymbol{r}}$, is then minimized in a least-square sense using methods outlined in [3]. A trust-region reflective algorithm, provided in Matlab routine `lsqnonlin.m` is used to solve the problem. A necessary condition for a minimum at $\boldsymbol{\kappa} = \boldsymbol{\kappa}^*$ requires the derivative to be zero

$$\boldsymbol{F}(\boldsymbol{\kappa}^*) = \left.\frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{\kappa}}\right|_{\boldsymbol{\kappa}=\boldsymbol{\kappa}^*} = \boldsymbol{D}^T(\boldsymbol{\kappa}^*)\{\boldsymbol{s}(\boldsymbol{\kappa}^*) - \boldsymbol{d}\} = \boldsymbol{0}, \tag{1}$$

with $\boldsymbol{D}(\boldsymbol{\kappa}) := \mathrm{d}\tilde{\boldsymbol{r}}(\boldsymbol{\kappa})/\mathrm{d}\boldsymbol{\kappa} = \mathrm{d}\boldsymbol{s}(\boldsymbol{\kappa})/\mathrm{d}\boldsymbol{\kappa}$, $\boldsymbol{D} \in \mathbb{R}^{n_\mathrm{D} \times n_\kappa}$. $\boldsymbol{D}(\boldsymbol{\kappa})$ represent the sensitivites and are calculated either by external differentiation or internal differentiation.

3. External Numerical Differentiation

It is more common to apply external differentiation schemes to approximate the sensitivites. Here, the entire finite element program has to be run $(n_\kappa + 1)$-times, for each iteration of the least-square solver. For each iteration, the material parameters are perturbed in order to obtain the sensitivites using a forward difference scheme. Therefore, the computational cost to calculate the sensitivities using external numerical differentiation is very high when the number of parameters to be identified are high. The details will be shown in the presentation.

4. Internal Numerical Differentiation

In the case of internal numerical differentiation, the entire time integration step is assumed to be dependent on the parameters $\boldsymbol{\kappa}$. The sensitivities are calculated based on Multilevel-Newton algorithm and are provided from the finite element program itself. These calculated sensitivities are then provided to the Matlab routine `lsqnonlin.m` to identify the material parameters. Since the sensivities are calculated internally using Acegen, the entire finite element program has to be run only once per iteration of the least-square solver. This reduces the computational cost considerably. However, there are storage requirements that need to be fulfilled. More details will be outlined in the presentation.

5. Examples

Examples showing the difference between external and internal differentiation will also be shown in the presentation. A particular focus lies on the storage requirements and the computational costs, where use is made of both models of hyperelasticity and models with internal variables.

References

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