

# RESPONSE SURFACES IN THE NUMERICAL HOMOGENIZATION OF NON-LINEAR POROUS MATERIALS

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## 1. Introduction

Numerical homogenization methods are widely applied to the calculation of the equivalent properties of microscopically inhomogeneous media. Their direct application to non-linear materials usually leads to very high computational effort. Nowadays, the solution of such a task requires the use of computational clusters or supercomputers and cannot be effectively used in engineering simulation.

One of techniques which allows reducing computational time is the response surfaces attitude. The use of the response surfaces allows rapid approximate determination of homogenised material properties.

The aim of the paper is to obtain a reliable range of the equivalent material properties values of a non-linear porous material by means of the numerical homogenization and response surfaces methodology. The boundary-value problem has been solved by means of the finite element method (FEM) ANSYS software in both considered scales.

## 2. Numerical homogenization of non-linear materials

Numerical homogenization, belonging to the group of homogenization methods, is an efficient numerical procedure which allows obtaining a medium macroscopically equivalent to a medium micro-scale inhomogeneous one. It is usually assumed that a representative volume element (RVE) represents the whole structure (global periodicity) or its part (local periodicity) [1]. If the material is non-linear, the number of the required RVE calculations is usually extremely high as it is necessary to solve the boundary-value problem(s) for each integration point in each iteration for the global model. Different attitudes to non-linearities in multi-scale analysis of heterogeneous media are presented e.g. in [2] and [3].

In the present paper an attitude with the use of response surfaces is proposed. The response surface methodology allows obtaining an approximate input-output relation if the true relationship is not known or hard to obtain [4]. The advantages of response surface approach are almost instantaneous output parameters evaluation and several design point calculations requirement to obtain high accuracy (for most cases).

In the present paper the response surface methodology with an artificial neural network (ANN) implemented in the ANSYS Workbench software has been used.

## 3. Numerical example

A RVE made of porous Ti-6Al-4V alloy with 64 uniformly distributed spherical pores has been created (Fig. 1a). It is assumed that the material is an elastoplastic with bilinear isotropic hardening one with the parameters collected in Table 1.

Parameter	Young modulus $E$	Poisson's ratio $\nu$	Density $\rho$	Yield strength $R_e$	Tangent modulus $E_t$
value	105.66 GPa	0.342	4430 kg/m <sup>3</sup>	1.09 GPa	0.85 GPa

Table 1: Parameters of Ti-6Al-4V elastoplastic alloy.

Tensile tests for 9 different porosity values  $p=0.01-0.8$  regulated by pores' diameter has been performed to calculate the dependence between equivalent stresses and strains. The results have been used to construct the response surface by means of the Kriging method (Fig. 1 b).

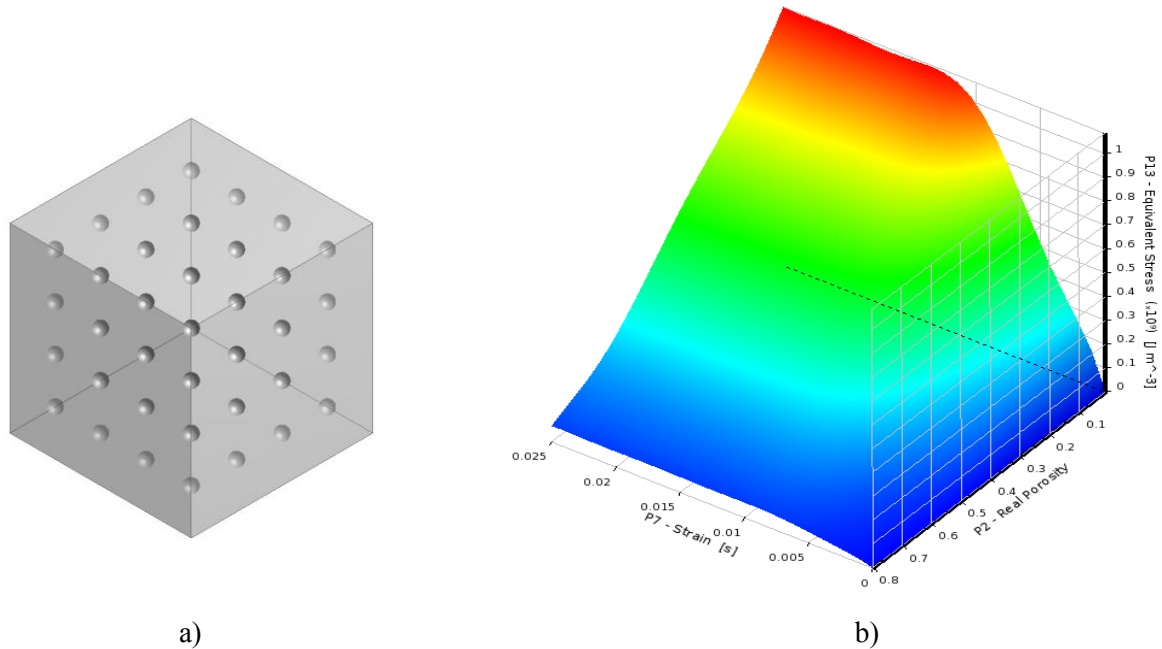


Figure 1: Non-linear homogenization: a) RVE and pores location b) obtained response surface.

The response surface has been used to calculate stresses and strains of the two-point supported beam of the same non-linear material with porosity  $p=0.2$ . The beam has been divided into 1250 hexahedral finite elements, which represent RVE models. A feed-forward ANN of 2-5-1 topology has been used to determine non-linear behaviour of each finite element. The obtained results have been compared to a macroscopic model with  $1250 \times 64$  pores and discretized by fine tetrahedral mesh. The maximum deflection evaluation error between applied approaches was lower than 4%. A calculation time in the response surface approach was about 1350 times lower than in the macroscopic model approach.

#### 4. Conclusions

The application of the response surfaces idea to the numerical homogenization of non-linear porous materials has been presented. Such attitude allows obtaining satisfactory results dramatically reducing the number of RVE calculations, comparing with classical attitude. The presented methodology may be applied to different non-homogeneous media with material nonlinearities, e.g. composites.

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#### References

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