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Natural oscillations of subsonic gas low near a cascade and a bip lane

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SOME problems of natural oscillations of the potential subsonic gas flow are considered. In case of the cascade of plates, the corresponding mathematical problem is solved by the method of sticking together and in case of the biplane, the same problem is solved approximately by means of Wiener-Hopf method. The natural frequencies dependencies on the cascade or the biplane parameters and parameters of flow have been obtained. Examples and comparisons with the experiment are given.

THE POSSIBILITY of gas to make natural oscillations in open regions was known in acoustics long ago. The theory of open resonators has recently been intensively developed in view of their extensive application in radio engineering [1, 2].

Certain attention has recently been paid to the problem of natural oscillations of a gas flow near open regions due to the phenomenon of acoustic resonance in turbo-machines. In this case, problems of acoustics are substantially complicated by the necessity of taking into consideration the windage losses.

Natural oscillations of gas flow near a flat cascade of plates and a biplane are considered in this paper.

1. A corresponding mathematical problem is formulated as follows. Determine in the region of gas flow (Fig. 1) the function $\varphi(x, y)$—the amplitude of the unsteady constituent function of the velocity potential

$$\Phi(x, y, t) = \Phi_0(x, y)+\varphi(x, y)e^{i\omega t}.$$

The function φ is to satisfy the equation:

$$(1-\mathsf{M}^2)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}-2k\mathsf{M}_i\frac{\partial\varphi}{\partial x}+k^2\varphi = 0, \tag{1}$$

where M is Mach number of mainstream, $k = \omega b/a$, ω—frequency (rad/sec) of gas oscillations, b—the chord of a plate, a—the sound velocity, and satisfy boundary conditions as well:

1. $\dfrac{\partial\varphi}{\partial y} = 0$ at $(x, y) \in L_n$,
2. $[p] = 0$, $\left[\dfrac{\partial\varphi}{\partial y}\right] = 0$ at $(x, y) \in Z_n$,
3. $[p] = 0$ at $(x, y) \in L_n$, $x \to b+nh\sin\beta$,
4. $\varphi = 0$ at $\xi \to -\infty$,

where p is the unsteady constituent of pressure.

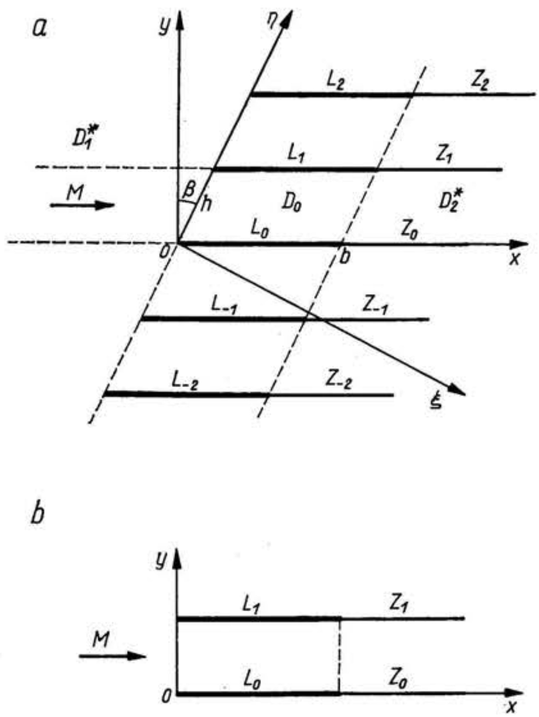

FIG. 1.

Moreover, for the case of cascade, let us introduce a supplementary periodicity condition:

(3) $$e^{in\mu}\varphi(\xi,\eta) = \varphi(\xi,\eta+nh),$$

where $\mu = 2\pi m/Nh$, h—is the cascade gap, N—the natural number.

2. In the case of a cascade, the problem formulated is solved as follows. The whole region of the solution is divided into region D_1^* in front of the cascade, region D_2^* behind the cascade and regions D_n between plates. In consequence of periodicity condition (3), the solution in regions D_1^* and D_2^* can be represented by Fourier series of the form:

$$\varphi_1^* = \sum_{n=0}^{\infty} a_n f_n, \quad \varphi_2 = \sum_{n=0}^{\infty} b_n f_n + c\varphi_0^*,$$

where a_n, b_n are arbitrary constants, and the term $c\varphi_0^*$ contains in itself a wake information.

In the region D_0 the solution can be represented by the Green-Stokes formula:

(4) $$\varphi_0 = \int_s \left[G\frac{\partial\varphi_0}{\partial\nu} - \frac{\partial G}{\partial\nu}\varphi_0 - 2ik\mathrm{M}G\varphi_0\cos\alpha \right] ds,$$

where s is the contour of the region D_0, G—Levy function of the Eq. (1), ν is the conormal to the contour s, α—the angle between the external normal vector and the flow velocity vector.

Let us take as G the Green function of the Neumann problem for the strip region $0 < y < h\cos\beta$—i.e., $\partial G/\partial\nu = 0$ with $y = 0, h$.

Then, taking into account that on the left and the right boundaries of the region D_0 the function φ_0 must continuously turn into the function φ_1^* and φ_2^*, respectively, we can obtain from formula (4) the relations:

$$\frac{\varphi_1^*(0,\eta)}{2} = \int_0^h F[G(0,\eta;0,\eta_0),\varphi_1^*(0,\eta_0)]d\eta_0 + \int_{b\sin\beta}^{h+b\sin\beta} F[G(0,\eta;\ b\cos\beta,\eta_0),\varphi_2^*(b\cos\beta,\eta_0]d\eta_0,$$

$$(5)\qquad \frac{\varphi_2^*(b\cos\beta,\eta)}{2} = \int_0^h F[G(b\cos\beta,\eta;0,\eta_0)\varphi_1^*(0,\eta_0)]d\eta_0 + \int_{b\sin\beta}^{h+b\sin\beta} F[G(b\cos\beta,\eta;b\cos\beta,\eta_0),\varphi_2^*(b\cos\beta,\eta_0)]d\eta_0,$$

$$F[G,\varphi] = G\frac{\partial\varphi}{\partial\nu} - \frac{\partial G}{\partial\nu}\varphi - 2ik\mathrm{M}G\varphi\cos\alpha.$$

From the relations (5), an infinite system can be obtained of homogeneous algebraic equations for determination of constants a_n and b_n, the system being closed for the constant c by third boundary condition. Eigenvalues are determined from the nontriviality condition of the solution of this system.

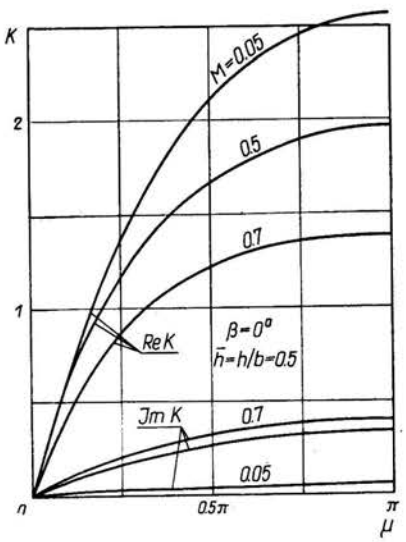

FIG. 2.

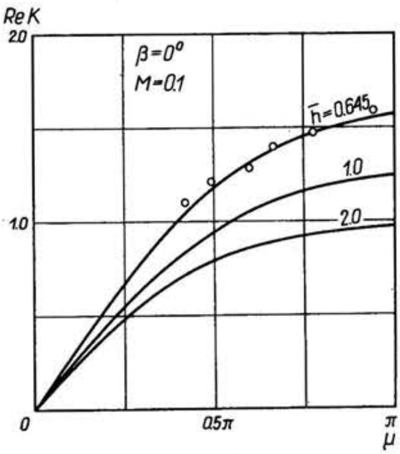

FIG. 3.

Note that natural frequencies of gas oscillations are complex ones due to the windage losses. Some of their typical dependencies on the cascade and the flow parameters are shown in Fig. 2. Given in Fig. 3 is a comparison of calculated dependencies of natural frequencies with the PARKER [3] experiment, the results of which are denoted by dots on the calculated curve $\overline{h} = 0.645$.

If an external source of oscillations exists in a flow, then, at frequencies of oscillations equal to natural ones, the corresponding disturbances will resonate with natural frequencies oscillations of gas. As an example, Fig. 4a shows the dependence of hydrodynamic reactions upon plates on the frequency of forced oscillations of plates. During acoustic resonance when the frequency of oscillations plates coincides with the natural frequency of gas oscillations, this dependence is close in character to analogous dependence for a simple oscillator. This analogy also occurs in the shear dependence γ phase between a hydrodynamic reaction and the plate displacement (Fig. 4b).

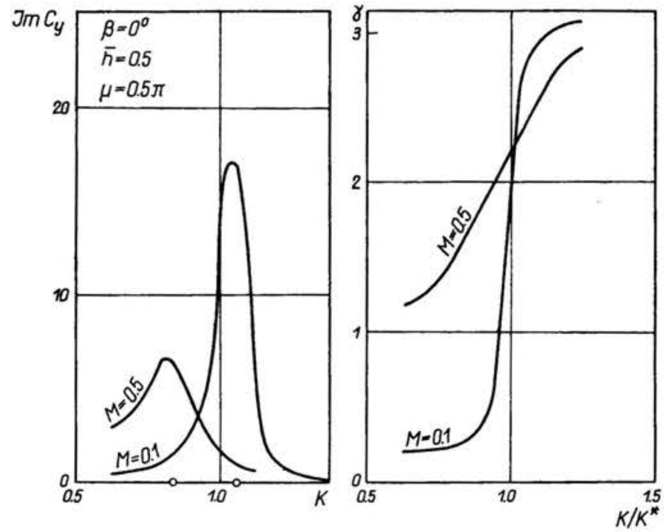

FIG. 4.

3. In the case of biplane, let us represent the general solution of the problem in the region confined between the plates, by Fourier series:

$$\varphi = e^{-i\frac{kMx}{1-M^2}} \sum_{m=1}^{\infty} [c_m e^{\lambda_m(x-b)} + b_m e^{-\lambda_m x}] \cos\frac{m\pi}{h} y, \tag{6}$$

where

$$\lambda_m = \frac{1}{1-M^2}\sqrt{\left(\frac{m\pi}{h}\right)^2(1-M^2)-k^2}, \quad \lambda_0 = \frac{ik}{1-M^2}.$$

If we assume the additional condition $\lambda_1 b \gg 1$, then the expression (6) with $x = 0$ can approximately be represented as:

$$\varphi(0, y) = c_0 e^{-\lambda_0 b} + \sum_{m=0}^{\infty} d_m \cos\frac{m\pi}{h} y,$$

and with $x = b$

$$\varphi(b, y) = d_0 e^{-\lambda_0 b} + \sum_{m=0}^{\infty} c_m \cos\frac{m\pi}{h} y. \tag{7}$$

The first terms of these expressions may be considered as the amplitudes of waves coming from opposite ends of the biplane, and the other terms under summation sign may be considered as amplitudes of diffraction back waves from open ends.

With such interpretation of the expression (7), the following relations hold:

$$d_0 = c_0 e^{-\lambda_0 b} R_0, \quad c_0 = d_0 e^{-\lambda_0 b} R_b, \tag{8}$$

where R_0 and R_b are reflection coefficients for waves coming to the left and to the right ends of the biplane, respectively. In approximation (7), these coefficients can be determined as for a semi-infinite biplane by the Wiener-Hopf method.

The relation (8) comprises a homogeneous system of algebraic equations with respect to constants c_0 and d_0. The equality of its determinant to zero gives a condition for determination of natural frequencies of gas flow oscillations near the biplane. It is of the form:

$$\frac{1-\mathsf{M}}{1+\mathsf{M}} \exp\left[\frac{4\pi i\bar{f}}{\bar{h}\sqrt{1-\mathsf{M}^2}} + 4if\left(1+i\frac{\pi}{2}-c+\ln\frac{2}{f}\right)\right] \times \prod_{n=1}^{\infty}\left(\frac{\sqrt{n^2-f^2}-if}{\sqrt{n^2-f^2}+if}\right)^2 \exp\left(\frac{i4f}{n}\right) - 1 = 0$$

where

$$\bar{h} = \frac{h}{b}, \quad f = \frac{k'\bar{h}}{2\pi\sqrt{1-\mathsf{M}^2}}, \quad \bar{f} = \frac{k\bar{h}}{2\pi\sqrt{1-\mathsf{M}^2}}, \quad k = k'+ik'', \quad c = 0.577216.$$

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