Archives of Mechanics • Archiwum Mechaniki Stosowanej • Brief Notes • 26, 3, pp. 529–533 Warszawa 1974

The flow in the supersonic exit of turbine cascades

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A SEMI-EMPIRICAL formula correlating the velocity changes in the vicinity of the sonic line and the geometry of this line and the streamline has been derived. This relation was then used for the assessment of the effect of the sonic line shape on the flow in the exit region of a supersonic cascade. The flow field was solved by the method of characteristics and, in addition, the trailing edge shock waves were also considered. The paper includes a comparison of this solution with interferometric measurements.

1. Fenomenological analysis

THE FENOMENOLOGICAL analysis of the development of transonic flow in cascades of airfoils as described by DVOŘÁK in the preceding paper [1] has shown discrepancies between what has been measured and what has been assumed in theoretical calculations of the supersonic flow in the exit of transonic cascades. For example, LAWACZECK [2], CURTIS, HUTTON and WILKINSON [3] and others presume the sonic line to be identical with the geometrica throat and solve the supersonic region by the classical method of characteristics.

Let us estimate the consequences of this assumption using as an example the flow past a cascade mentioned by DVOŘÁK [1] at $\mathsf{M}_1 = 0.863$ (Fig. 1). The experimental results (subscript I) are compared with results of the theoretical solution using the actual sonic line shape (II) and with the theoretical solution assuming a straight sonic line (III).

The following consequences can be noted:

1. *A difference in mass flow.* In this case, the difference is very small, $Q_{\text{II}}/Q_{\text{III}} = 0.998$, though at highly curved channels it is much greater.

2. *A difference in distribution of pressure on the blade surface.* The pressure coefficient $C_{p\text{III}}$ (for theoretical solution with a straight sonic line) is on the lower blade of the channel evidently higher than in cases I and II (Fig. 2). The pressure coefficient $C_{p\text{III}}$ is on the upper blade comparable to the corresponding value $C_{p\text{II}}$ (for the suggested theoretical solution) (Fig. 2b).

A distinct difference of pressure coefficients $C_{p\text{II}}$ and $C_{p\text{I}}$ is caused by the change of effective airfoil geometry due to the boundary layer displacement effect, especially after its interaction with a shock wave at the trailing edge of the lower airfoil. An additional consequence of this effect can be noticed in the middle part of the upper airfoil, where compressive characteristics from the exit part of the lower airfoil increase the $C_{p\text{I}}$ and decrease the shock strength.

3. *A difference in losses.* This has not been measured directly; it can only be inferred from differences in pressure distribution on airfoil surfaces.

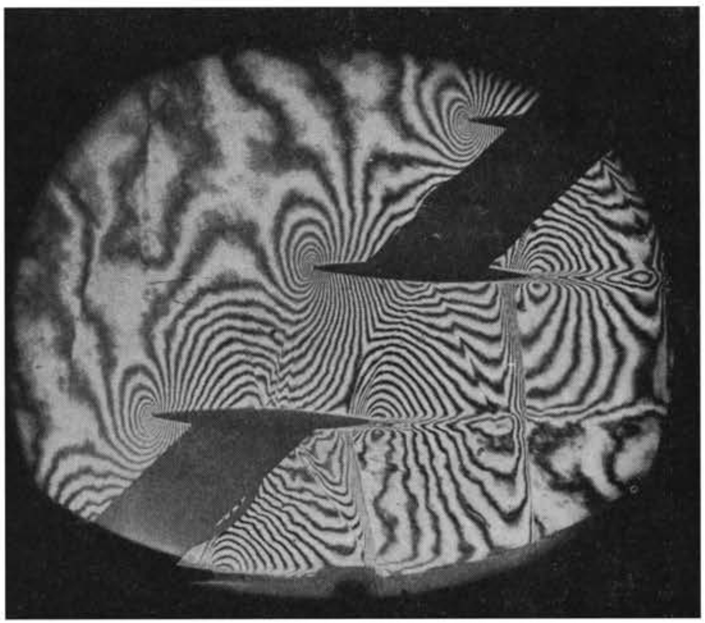

FIG. 1. Interferogramme of flow past a cascade (air, $M_1 = 0.863$).

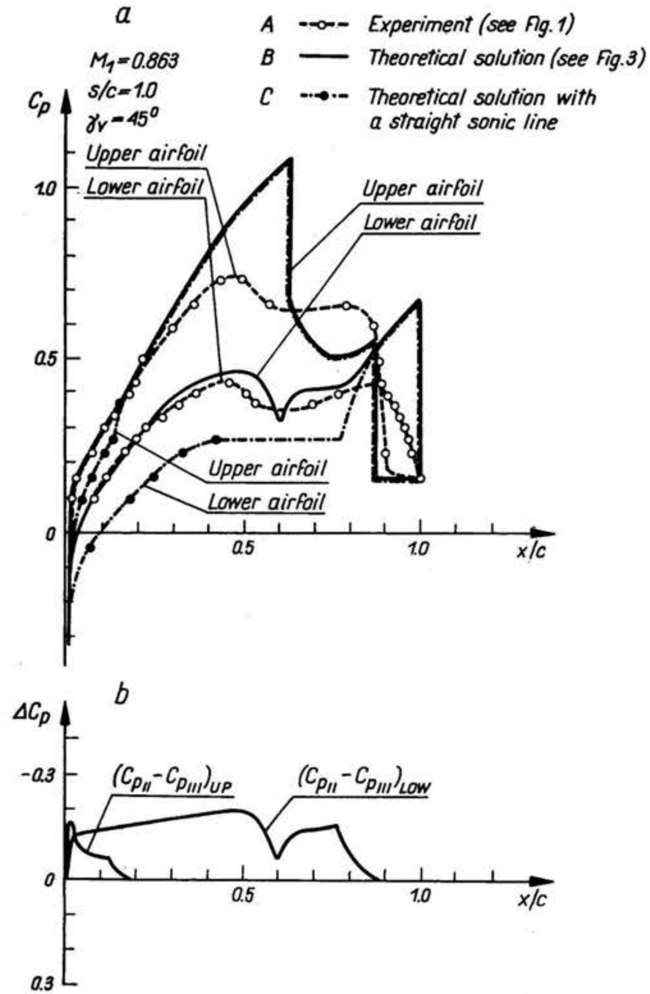

FIG. 2. Distribution of pressure coefficient along the blade chord.

4. *A difference in aerodynamic forces on the airfoil.* This difference is evident from Fig. 2b. Lift coefficients were determined from Fig. 2a. This value is for the suggested solution $C_{L_{II}} = -0.1782$. For the evaluated experiment, $C_{L_I} = -0.1919$, which gives $(C_{L_I}-C_{L_{II}})/C_{L_{II}} = 7.1\%$. For theoretical solution with a straight sonic line in the geometrical throat $C_{L_{III}} = -0.2842$, which yields the relative difference as high as $(C_{L_{III}}-C_{L_{II}})/C_{L_{II}} = 87.6\%$.

2. Theoretical solution of the supersonic exit of transonic cascades of airfoils

The theoretical solution based on the experimentally stated sonic line is given in Fig. 3. The supersonic region was solved by the method of characterictics. Parameters near the

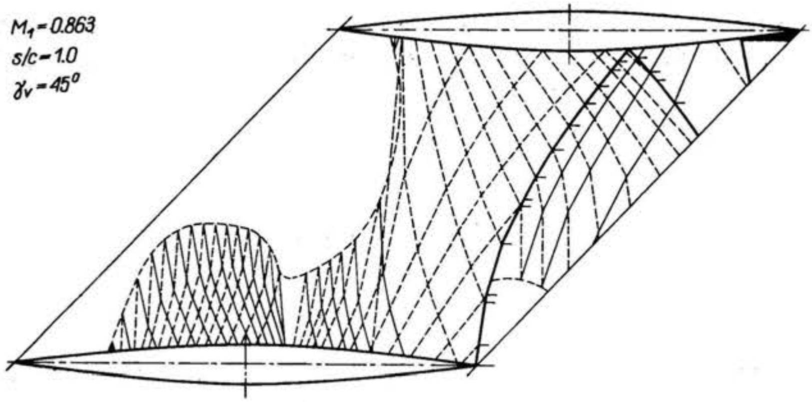

FIG. 3. Solution of the supersonic exit of the cascade.

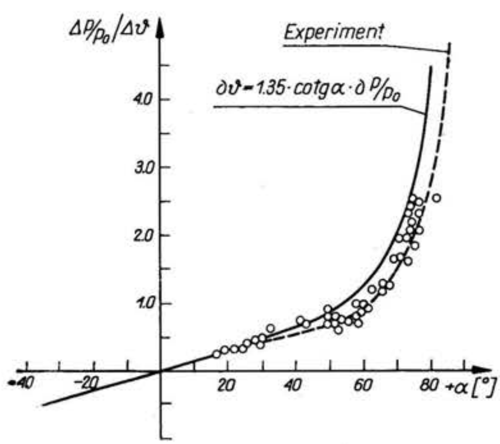

FIG. 4. Semi-empirical relation $\Delta \frac{p}{p_0} \Big/ \Delta\vartheta = f(\alpha)$ (Ref. [4]).

sonic line were determined using a semi-empirical formula $\Delta \frac{p}{p_0} \Big/ \Delta\vartheta = f(\alpha)$ (Fig. 4 and Ref. [4]), which correlates changes of velocity vector near the sonic line, the change of pressure along the streamline, and the geometry of both the sonic line and streamline.

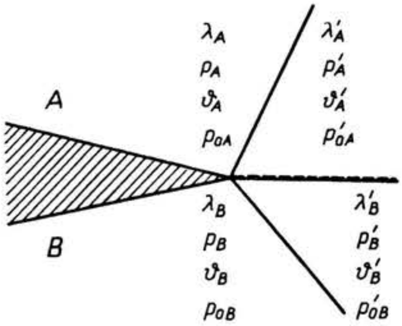

FIG. 5 . The flow round sharp supersonic trailing edge.

The flow at the supersonic trailing edge constitutes considerable research problem. Let us consider only a sharp trailing edge (Fig. 5), when the flow field is completely described by the following set of equations:

$$p'_A = p'_B, \tag{1}$$

$$\vartheta'_A = \vartheta'_B, \tag{2}$$

$$F\left(K\frac{p'}{p_0}, \quad \vartheta' - \vartheta, \lambda\right) = 0, \tag{3}$$

$$K = \frac{p_0}{p_{0\,\mathrm{relative}}}. \tag{4}$$

The boundary conditions for the shear layer are given by the expressions (1) and (2). The expression (3) is a function between parameters on either side of an oblique shock

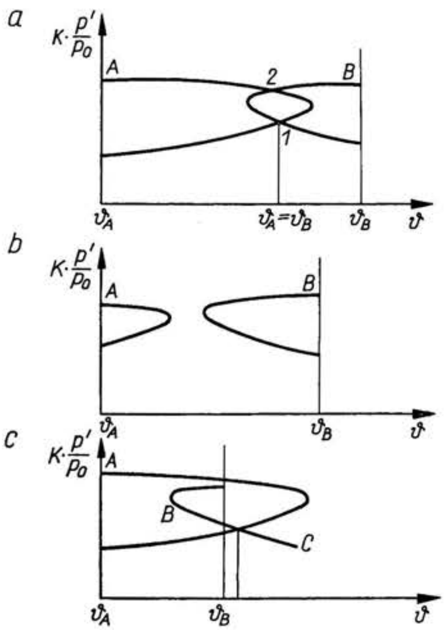

FIG. 6. The solution of the flow round sharp supersonic trailing edge.

wave. The solution of the closed system of equations is possible only by numerical or graphical means. The latter has been chosen in following discussion.

The transition through the exit shock waves on either side of the airfoil can be represented by two shock polars (relation [3]). They intersect at points which are solutions of the above system (Fig. 6a). The solution given by point 2 is unstable. If there are no intersections (Fig. 6b), at least one of the shock becomes detached from the trailing edge and is followed by flow separation. The polars do not intersect even in the case shown in Fig. 6c (one is inside the other) which means that instead of one shock wave an expansion arises. Also a detachment may appear, followed by boundary layer separation.

In the numerical solution, consideration was given to the interaction of these exit shock waves with the field of characteristics.

3. Conclusion

It is shown that the consequences of disregarding the real shape of the sonic line may lead to substantial differences in aerodynamic forces on the airfoil. The evaluated examples proved the applicability of the method which takes into account the shape of sonic line. Acceptable agreement with results of interferometric measurements has been obtained, but it is also shown that the method requires consideration of the viscosity effects.

4. References

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