74.

ON A GEOMETRICAL TREATMENT OF A THEOREM IN NUMBERS.

[*Johns Hopkins University Circulars*, I. (1882), p. 209.]

THE author made some remarks additional to those made on the same subject at the preceding meeting of the seminarium. In a plane reticulation four cases present themselves, namely, a line may be drawn through a line of nodes, or through a solitary node, or parallel to a line of nodes, or so as neither to pass through any node nor to be parallel to a line of nodes. In the third case the distance of the nodes of nearest approach is constant: in the second and fourth cases it approximates continually to zero. So in a solid reticulation eight cases present themselves, namely, four in addition to those last detailed: for without lying in a nodal plane, the line of flight may (α) pass through a single node, or (β) it may be parallel to a line of nodes, or (γ) it may be parallel to a nodal plane but not to a nodal line, or (δ) it may not pass through any node. In case (β) the distance of the nodes of nearest approach is constant; in case (γ) it approximates to a constant finite limit: in cases (α) and (δ) it approximates to zero.

There are thus four cases in all for which the distance from the nodes of nearest approach is a continually decreasing infinitesimal, namely: two for which the line of flight does not pass through any node, and two for which it does pass through a node—these latter two being those which serve to establish the theorem relating to the non-existence of trebly periodic functions.

The author further drew attention to the singular metamorphosis undergone by the geometrical setting forth of this theorem. It may be put under the form of asserting that a trilateral whose three sides are conditioned to be exact multiples of, and parallel to, three given straight lines lying in a plane may either be made to form a closed triangle or else such that the line closing the trilateral shall be less than any assigned quantity. On the other hand, the very same fact lends itself to, and is absolutely equivalent in substance to the statement that an arrow let fly from a node of a solid reticulation whether it speed along a nodal plane or be shot miscellaneously at the stars must (the law of gravity being supposed to be suspended) pass *indefinitely near* an infinite number of nodes in the course of its flight. The corresponding theorem for space of five dimensions serves to show that Quaternion Functions cannot have a higher than a quadruple periodicity.