89.

ON FAREY SERIES.

[*Johns Hopkins University Circulars*, II. (1883), p. 143.]

THE ordinary Farey Series is a succession of proper vulgar fractions arranged in order of magnitude, whereof the denominator does not exceed a given amount. The theory may be generalized and simplified by considering the terms of each fraction as the coordinates of a node in a "réseau." If a *simple* and *anautotomic* closed boundary drawn on a tessellation be called a scroll, and any node within it be assumed as origin, a radius of indefinite length rotating about that point as centre, will pass through a series of nearest nodes to it in succession, all lying within the scroll—and the coupled coordinates to those successive points, say (p, q), (p', q'), ... will form a certain series which in general but not universally will satisfy the equation $pq' - p'q = 1$ or $= - 1$ according as the order of magnitude is descending or ascending. The character of the series may be termed Farey if this law is satisfied throughout the entire succession and otherwise Non-Farey. The character obviously can only depend on the form, magnitude, position, and aspect of the scroll and the position of the assumed centre. The author of the paper showed that the position of the centre was indifferent except that it must be taken at some node within the scroll, and that the scroll might undergo uniform expansion and contraction about any internal node (and consequently also translation along any line of nodes) without change of character. Application of these principles was made to a triangular or rectangular scroll (including Mr Glaisher's extension of the theory of ordinary Farey Series to a two-fold constant limit), to the case potentially indicated by Dirichlet where the scroll is formed by two asymptotes to, and the branch of a hyperbola, and two other cases: the theory is deduced without the use of continued fractions or indeterminate linear equations or any other algebraical process whatever, from the well-known fact that all elementary triangles in a reticulation are of equal area and from the not very recondite theorem that a triangle may be divided into four equal and similar triangles by straight lines joining the bisections of its three sides; and with the aid of a solid reticulation may be extended to triplets and so on indefinitely. It will be found fully set forth in Note H, interact, part 2, Vol. v., No. 4, *American Journal of Mathematics* [Vol. IV. of this Reprint].

CAMBRIDGE: PRINTED BY JOHN CLAY, M.A. AT THE UNIVERSITY PRESS.