ANALYSIS OF STATIC AND DYNAMICAL THREE-DIMENSIONAL MODELS OF THERMOELASTIC PIEZOELECTRIC SOLIDS

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Modern complex engineering constructions and technological processes are controlled by using sensors and actuators, which gather information and facilitate the adequate adjustment of construction or process. The need of construction of sensors and actuators with the appropriate physical characteristics stimulate the analysis of interaction between various physical fields, such as elastic, thermal, electric and magnetic. Mathematical models of continuum mechanics, in which processes of interaction of several physical fields are considered, are described by boundary or initial-boundary value problems for quite complicated systems of partial differential equations on three-dimensional domains. Consequently, investigation of the problems of continuum mechanics of this type is of crucial importance from practical as well as theoretical point of view. Piezoelectric materials are one of the most common materials currently being used and investigated for smart structures applications due to their direct and converse piezoelectric effects, which permit them to be utilized as both actuators and sensors. The wide area of application of them is aerospace engineering, where most structures operate in changing thermal environments. Therefore, due to their possible applications in the fabrication of smart and adaptive material systems, the study of mechanics and physics of thermo-electro-magneto-elastic materials has attracted increasing attention.

One of the first rigorous theoretical model of piezoelectricity describing the interaction between elastic, electric and thermal properties of thermoelastic body was constructed by W. Voigt [1]. Later on, W. Cady [2] treated the physical properties of piezoelectric crystals as well as their practical applications. H. Tiersten [3] studied problems of vibration of piezoelectric plates. A three-dimensional model of piezoelectric body taking into account thermal properties of the constituting material was derived by R. Mindlin [4] applying variational principle. Further, W. Nowacki [5] developed some general theorems for thermoelastic piezoelectric materials. R. Dhaliwal and J. Wang [6] proved uniqueness theorem for linear three-dimensional model of the theory of thermo-piezoelectricity, which was generalized by J. Li in the paper [7], where a generalization of the reciprocity theorem of W. Nowacki [8] was also obtained. Applying the potential method and the theory of integral equations D. Natroshvili [9] studied the problems of statics and pseudo-oscillations with basic and crack type boundary conditions.

Note that the three-dimensional boundary and initial-boundary value problems with general mixed boundary conditions for mechanical displacement, electric and magnetic potentials, and temperature corresponding to the linear static and dynamical models of inhomogeneous anisotropic thermoelastic piezoelectric bodies with regard to the magnetic field have not been well investigated. The well-posedness results are mainly obtained for thermoelastic piezoelectric bodies consisting of homogeneous materials.

The present paper is devoted to the investigation of the linear three-dimensional boundary and initial-boundary value problems with mixed boundary conditions corresponding to the linear static and dynamical three-dimensional models of piezoelectric solid taking into account magnetic and thermal properties of the material. We consider thermoelastic piezoelectric body consisting of inhomogeneous anisotropic material, when on certain parts of the boundary density of surface force, and normal components of electric displacement, magnetic induction and heat flux vectors are given, and on the remaining parts of the boundary mechanical displacement, temperature, electric and magnetic potentials vanish. We investigate boundary and initial-boundary value problems for coupled systems of partial differential equations corresponding to the static and dynamical three-dimensional models of thermoelastic piezoelectric bodies with regard to magnetic field, which consist of the linearized equations of motion or static equilibrium, equation of the entropy balance and quasi-static equations for electro-magnetic fields, where the rate of change of magnetic field is small, i.e. electric field is curl free, and there is no electric current, i.e. magnetic field is curl free. In the case of thermoelastic piezoelectric bodies, which consist of several subdomains with piecewise continuous parameters characterizing elastic, thermal, electric and magnetic properties of the body, the partial differential equations corresponding to the static or dynamical three-dimensional models are given in the subdomains and along the common interfaces between the subdomains rigid contact conditions are fulfilled, i.e. mechanical displacement and stress vectors, temperature, electric and magnetic potentials, and normal components of the heat flux, electric displacement and magnetic induction are continuous. From the differential formulations of the boundary and initial-boundary value problems we obtain integral equations, which are equivalent to the original problems in the spaces of smooth enough functions, but require less regularity of the unknown functions than in the differential formulations. On the basis of the obtained integral equations we present variational formulations of the static and dynamical three-dimensional problems in the corresponding function spaces, which are Sobolev spaces for static problem, and spaces of vector-valued distributions with respect to the time variable with values in Sobolev spaces for dynamical problem. In the static case we determine the structure of the set of solutions of the homogeneous boundary value problem, when the density of surface force, and normal components of electric displacement, magnetic induction and heat flux vectors vanish. In the dynamical case we consider the set of solutions of the homogeneous equations corresponding to electric and magnetic fields, when the density of electric charges, normal components of electric displacement and magnetic induction, mechanical displacement and temperature vanish. By applying the sets of solutions of the corresponding homogeneous problems we define the factor spaces of suitable Sobolev spaces, which we use for investigation of the well-posedness of the three-dimensional problems. By using variational formulation and Lax-Milgram lemma [10] with non-symmetric bilinear form we obtain the well-posedness result for the boundary value problem corresponding to the linear static three-dimensional model of thermoelastic piezoelectric bodies in suitable factor-spaces of Sobolev spaces. By applying Faedo-Galerkin method [11], suitable a-priori estimates and compactness arguments we prove new existence result for the initial-boundary value problem corresponding to the linear dynamical three-dimensional model of thermoelastic piezoelectric solids in the corresponding spaces of vector-valued distributions with values in suitable factor spaces of Sobolev spaces. Furthermore, an energy equality is obtained, which permits us to prove the uniqueness result and continuous dependence of the solution on the given data in suitable function spaces. If mechanical displacement, temperature, electric and magnetic potentials vanish on the parts of the boundary with positive areas, then the corresponding homogeneous boundary value problems possess only trivial solutions and we obtain existence and uniqueness results in suitable Sobolev spaces. Note, that the proof of the existence of solution of the initial-boundary value problem gives an algorithm for approximation of the solution of the three-dimensional dynamical problem by a sequence of solutions of linear finite-dimensional systems of ordinary differential equations. The methodology outlined in this paper can be used for investigation of various coupled problems in the continuum mechanics and the construction of algorithms of their solution.

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