NATANSON'S NONLINEAR EXTENDED THERMODYNAMICS

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1. Introduction

Here we present the previously underrepresented contributions of Ladislavus Natanson to the field of thermodynamics. We focus on the mathematical reconstruction of a few of his principal ideas that until now have been neglected by the literature. To set these ideas in proper epistemological order, we thought it would be valuable to first revalue and reconstruct some missing parts of the proceedings process by Natanson constructed their thermodynamics. We also aimed to present Natanson's achievements against the background of modern continuum mechanics, exemplifying old but still relevant approaches. We propose that Natanson's ideas were ahead of their time by about one century. Give that scientist was educated in the scientific royal way: chemistry, through mechanic of solid and fluid, thermodynamics, electro-chemistry, electrodynamics, early quantum and relativistic mechanics, we can closely compare their conceptions and solutions. Natanson was in strong opposition with Newtonian mechanisms, the Maupertuis least action principle formed the basis of his activities, which they were developing as a sum of elementary quantum actions.

2. Natanson's nonlinear extended thermodynamics

Ladislavus Natanson initial interests were focused on the Maxwell kinetic theory of gases, which was the subject of his doctoral theses [1] prepared at Dorpat under the supervisor of professor Arthur von Oettingen. In 1890, Natanson (now 26-years-old) and living in Warsaw produced his first book, entitled "Introduction to Theoretical Physics" [2]. This book was wholly original and the few last chapters were completely novel, containing an introduction to extended thermodynamics in a fully three-dimensional framework. According to Maxwell, Natanson introduced the use of two kinds of velocity vectors: molar $\mathbf{u}$ and molecular $\mathbf{c}$ [3]:

(1) $$\mathbf{u}+\mathbf{c}=\left(u\,\mathbf{e}_x+v\,\mathbf{e}_y+w\,\mathbf{e}_z\right)+\left(\xi\,\mathbf{e}_x+\eta\,\mathbf{e}_y+\zeta\,\mathbf{e}_z\right)$$

Taking into account the body force $\mathbf{f}$ Natanson, repeating Maxwell's original reasoning [4], was able to extended Maxwell fundamental equation [2]:

(2) $$\frac{d}{dt}\left(\overline{Q}n\right)+\operatorname{div}\left(n\,\overline{\mathbf{c}\otimes Q}\right)+\operatorname{div}(\mathbf{u})\overline{Q}n=\frac{\delta}{\delta t}\left(\overline{Q}n\right)+n\overline{\left(\mathbf{f}\cdot\frac{\delta Q}{\delta\mathbf{u}}\right)}$$

That is now the well-known starting point for the kinetic theory of gases. By repeating extending Maxwell's reasoning, step-wise, Natanson obtained a set of evolution equations for different balanced quantities Q (topological charges). By firstly taking $Q=m$ and the following identities [5] :

(3) $$\overline{\mathbf{c}\otimes Q}=\overline{m\,\mathbf{c}}=0,\quad \overline{Q}n=\rho,\quad \overline{Q}=m,\quad \overline{\frac{\delta Q}{\delta\mathbf{u}}}=0$$

Next, by setting (3) into (2), a non-conservative form of the fundamental equation was obtained [6,§1,eq.5]:

(4) $$\rho\frac{d}{dt}\overline{Q}+\operatorname{div}\left(\rho\,\overline{\mathbf{c}\otimes Q}\right)=\rho\frac{\delta}{\delta t}\,\overline{Q}+\rho\overline{\left(\mathbf{f}\cdot\frac{\delta Q}{\delta\mathbf{u}}\right)}$$

where the d'Alembert-Euler material derivative is defined as: $\frac{d}{dt}(\cdot)_{X=\mathrm{const}}=\frac{\partial}{\partial t}(\cdot)_{x=\mathrm{const}}+\operatorname{grad}(\cdot)\mathbf{u}$

In order to identify a source of irreversibility at Nature, Natanson introduced the concept of Coertia, which is similar to inertia. Natanson's Coertia is a fundamental property of space that is responsible for every irreversible phenomena in matter, as well as in the electromagnetic and gravitational fields. Owing to this concept, the irreversible changes proposed in the Maxwell procedure can be described with appropriate relaxation times as [7]:

(5) $$\frac{\delta}{\delta t}\mathbf{q} = -\frac{\mathbf{q}}{\tau_q} \quad , \quad \frac{\delta}{\delta t}\mathbf{p} = -\frac{\mathbf{p}}{\tau_P} \quad , \quad \frac{\delta}{\delta t}\mathbf{j} = -\frac{\mathbf{j}}{\tau_j}$$

where τ_q, τ_p, τ_j are relaxation times for heat, momentum and mass fluxes.

By looking at the Maxwell procedure of finding moments of the fundamental equation, Natanson quickly realized the necessity for cutting of the moment, hereby setting appropriate closure equations. He proposed the following logical structure: taking Q as a balanced quantity and the $\mathbf{f}_Q$ flux of Q and $\mathbf{F}_Q$ as a super-flux of $\mathbf{f}_Q$, the set of equations were determined [6]:

- Balance equation

(6) $$\frac{\partial}{\partial t}\overline{Q} + \operatorname{div}\mathbf{f}_Q = 0$$

- Evolution equation $\mathbf{f}_Q$

(7) $$\frac{\partial}{\partial t}\mathbf{f}_Q + \frac{1}{\tau_{\mathbf{f}}}\mathbf{f}_Q + \operatorname{div}\mathbf{F}_Q = 0$$

- Algebraic clousure for $\mathbf{F}_Q$

(8) $$\mathbf{F}_Q = a^2 \operatorname{grad}\overline{Q}$$

- Resulting equation for Q

(9) $$\frac{\partial^2}{\partial t^2}\overline{Q} + \frac{1}{\tau_{\mathbf{f}}}\frac{\partial}{\partial t}\overline{Q} - a^2 \operatorname{div}\left(\operatorname{grad}\overline{Q}\right) = 0$$

In Natanson's opinion, the above governing equation describes a whole real phenomena of nature, where reversibility is entangled with irreversibility by the relaxation time only. Thus, if $\tau_{\mathbf{f}} = \infty$ (inertia), it is a case of reversible, while if $\tau_{\mathbf{f}} = 0$ (coertia) it is an irreversible phenomenon. More information about Natanson's nonlinear extended thermodynamics can be found in works [8-10].

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