NOTES AND REFERENCES.

173. I ATTACH some value to this analysis and development of Laplace's Method, showing how it leads to the actual expression for the Potential of an Ellipsoid upon an exterior point in a series of terms of the form $\left(\alpha^2 \frac{d^2}{da^2} + \beta^2 \frac{d^2}{db^2} + \gamma^2 \frac{d^2}{dc^2}\right)^i \frac{1}{\sqrt{a^2+b^2+c^2}}$, being in fact the series deduced by me in the year 1842 from a result of Lagrange's; see vol. I., Notes and References 2 and 3.

191. The theorem obtained at the end of the paper is a very peculiar one; the only paper that I know of in anywise relating to the theory is Donkin, "On an application of the Calculus of Operations to the transformation of Trigonometric Series," *Quart. Math. Jour.* t. III. (1860), pp. 1—15, where (pp. 13—15) my theorem is referred to and a more general theorem involving two arbitrary functions ϕ, F, is arrived at.

194. In connexion herewith see the Memoir, Donkin, "On the Analytical Theory of the Attraction of Solids bounded by Surfaces of a hypothetical Class including the Ellipsoid," *Phil. Trans.* t. 150 (1860), pp 1—11. The author referring to my Note remarks that I there showed that if *two* of the principal theorems of attraction (in the case of the ellipsoid) be given the rest follow very simply and are common to all the surfaces of which these two can be predicated: but that the demonstration of the two assumed theorems constitute the most essential part of the analytical problem, and that it was his present object to show that they and the others connected with them are implied in the two partial differential equations

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 2\left(\frac{1}{a^2+h} + \frac{1}{b^2+h} + \frac{1}{c^2+h}\right)$$

and

$$\left(\frac{du}{dx}\right)^2 + \left(\frac{du}{dy}\right)^2 + \left(\frac{du}{dz}\right)^2 + 4\frac{du}{dh} = 0,$$

satisfied by the function $\frac{x^2}{a+h} + \frac{y^2}{b+h} + \frac{z^2}{c+h}$: and he accordingly derives the whole theory, and in particular the theorems V. and VI. (equivalent to my assumed theorems) from these two partial differential equations.

221. It is well known that Plana, developing the explanation given by Laplace for the secular variation of the moon's mean motion, obtained in the expression of the true longitude the terms $-(\frac{3}{2}m^2 - \frac{2187}{128}m^4)\int(e'^2 - E'^2)\,ndt$, and that Prof. Adams in his memoir "On the Secular Variation of the Moon's Mean Motion," *Phil. Trans.* t. 143 (1853), pp. 397—406, corrected this into $-(\frac{3}{2}m^2 - \frac{3771}{64}m^4)\int(e'^2 - E'^2)\,ndt$. The validity of the correction was a good deal discussed, and it was interesting to establish the result by an entirely independent method.

END OF VOL. III.

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