XII

ON SOME EXTENSIONS OF QUATERNIONS

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Sir W. R. Hamilton read a paper on some extensions of quaternions:

Besides some general remarks on associative polynomes, and on some extensions of the modular property, Sir W. R. Hamilton remarked that if, in the quadrinomial expression

$$\mathrm{Q} = w + \iota x + \kappa y + \lambda z,$$

the laws of the symbols $\iota\kappa\lambda$ be determined by the following formula of vector-multiplication,

$$\begin{aligned}(\mathrm{A}) \ldots (\iota x + \kappa y + \lambda z)(\iota x' + \kappa y' + \lambda z') &= (m_1^2 - l_2 l_3) x x' + (l_1 m_1 - m_2 m_3)(yz' + zy') \\ &+ (m_2^2 - l_3 l_1) yy' + (l_2 m_2 - m_3 m_1)(zx' + xz') + (m_3^2 - l_1 l_2) zz' + (l_3 m_3 - m_1 m_2)(xy' + yx') \\ &+ (\iota l_1 + \kappa m_3 + \lambda m_2)(yz' - zy') + (\kappa l_2 + \lambda m_1 + \iota m_3)(zx' - xz') + (\lambda l_3 + \iota m_2 + \kappa m_1)(xy' - yx'),\end{aligned}$$

then this expression, which he proposes to call a QUADRINOME, has many properties (associative, modular, and others), analogous to the quaternions; which latter are indeed only that *case* of such quadrinomes, for which,

$$l_1 = l_2 = l_3 = 1, \qquad m_1 = m_2 = m_3 = 0, \qquad \iota = i, \quad \kappa = j, \quad \lambda = k.$$

He has, however, found another distinct sort of associative quadrinomial expression, which has also several analogous properties, and for which he suggests the name of TETRADS; the product of two vectors being in it,

$$\begin{aligned}(\mathrm{B}) \ldots (lx + my + nz)(lx' + my' + nz') + (\kappa n - \lambda m)(yz' - zy') \\ + (\lambda l - \iota n)(zx' - xz') + (\iota m - \kappa l)(xy' - yx').\end{aligned}$$