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A THEOREM CONCERNING POLYGONIC SYNGRAPHY

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Professor Sir William Rowan Hamilton exhibited the following Theorem, to which he had been conducted by that theory of geometrical *syngraphy* of which he had lately submitted to the Academy a verbal and hitherto unreported sketch, and on which he hopes to return in a future communication.*

Theorem. Let $A_1, A_2, \ldots, A_n$ be any n points (in number odd or even) assumed at pleasure on the n successive sides of a closed polygon $BB_1B_2 \ldots B_{n-1}$ (plane or gauche), inscribed in any given surface of the second order. Take any three points, P, Q, R, on that surface, as initial points, and draw from each a system of n successive chords, passing in order through the n assumed points (A), and terminating in three other superficial and final points, P', Q', R'. Then there will be (in general) *another* inscribed and closed polygon, $CC_1C_2 \ldots C_{n-1}$, of which the n sides shall pass successively, in the same order, through the same n points (A): and of which the initial point C shall also be connected with the point B of the former polygon, by the relations

$$\frac{ael}{bc}\frac{\beta\gamma}{\alpha\epsilon\lambda}=\frac{a'e'l'}{b'c'}\frac{\beta'\gamma'}{\alpha'\epsilon'\lambda'}, \quad \frac{bfm}{ca}\frac{\gamma\alpha}{\beta\zeta\mu}=\frac{b'f'm'}{c'a'}\frac{\gamma'\alpha'}{\beta'\zeta'\mu'}, \quad \frac{cgn}{ab}\frac{\alpha\beta}{\gamma\eta\nu}=\frac{c'g'n'}{a'b'}\frac{\alpha'\beta'}{\gamma'\eta'\nu'};$$

where

$$a=QR, \quad b=RP, \quad c=PQ,$$
$$e=BP, \quad f=BQ, \quad g=BR,$$
$$l=CP, \quad m=CQ, \quad n=CR,$$
$$a'=Q'R', \quad b'=R'P', \quad c'=P'Q',$$
$$e'=BP', \quad f'=BQ', \quad g'=BR',$$
$$l'=CP', \quad m'=CQ', \quad n'=CR';$$

while $\alpha\beta\gamma\epsilon\zeta\eta\lambda\mu\nu$, and $\alpha'\beta'\gamma'\epsilon'\zeta'\eta'\lambda'\mu'\nu'$, denote the semidiameters of the surface, respectively parallel to the chords $abcefglmn$, $a'b'c'e'f'g'l'm'n'$.

As a very particular *case* of this theorem, we may suppose that $PQ'RP'QR'$ is a plane hexagon in a conic, and BC its Pascal's line.

* [See *Lectures*, Appendix B, footnote on p. 728, and Appendix C.]