ANSWERS TO EXAMPLES AND PROBLEMS.

VOLUME II.

CHAPTER XXIII.

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2. $\frac{2^8 . 3}{5 . 7}(a^{\frac{5}{3}} \sim b^{\frac{5}{3}})(c^{\frac{7}{3}} - d^{\frac{7}{3}})$.

6. $I=\int_0^a \int_{u\sqrt{2}}^{\infty} V' \frac{2uv\,du\,dv}{\sqrt{v^4-4u^4}}$.

7. $-2a\frac{\cos(m+n)a}{n(m+n)}+2\frac{\sin(m+n)a}{n(m+n)^2}-\frac{\sin(m-n)a}{n^2(m-n)}+\frac{\sin(m+n)a}{n^2(m+n)}$.

8. $I=\frac{1}{a^2}\int_0^a \int_0^{\sqrt{2\xi(a-\xi)}} d\xi\,d\eta$.

9. $I=\int_0^{\frac{a}{4}} \int_{\frac{a}{2}-\sqrt{\frac{a^2}{4}-ay}}^{\sqrt{ay}} V\,dy\,dx$.

10. $I=\frac{1}{2}\int_0^1 \int_0^{\sqrt{1-\eta}} V' \frac{d\eta\,d\xi}{\sqrt{\eta^2+4\xi^2}}+\frac{1}{2}\int_{-1}^0 \int_0^{\sqrt{1+\eta}} V' \frac{d\eta\,d\xi}{\sqrt{\eta^2+4\xi^2}}$.

1. $I=\int_0^{\frac{hk}{h+k}} \int_0^{\frac{c\sqrt{y}}{\sqrt{k-y}}} V\,dy\,dx+\int_{\frac{hk}{h+h}}^h \int_0^{\frac{c\sqrt{h-y}}{\sqrt{y}}} V\,dy\,dx$; $I=-\frac{c}{2}\iint\frac{d\xi\,d\eta}{\xi^2+c^2}$.

13. $I=\int_0^{\frac{a}{2}} \int_{\frac{y^2}{a}}^{\frac{a}{2}-\sqrt{\frac{a^2}{4}-y^3}} f(x, y)dy\,dx+\int_0^{\frac{a}{2}} \int_{\frac{a}{2}+\sqrt{\frac{a^2}{4}-y^2}}^a f(x, y)dy\,dx$

$$+\int_{\frac{a}{2}}^a \int_{\frac{y^2}{a}}^a f(x, y)dy\,dx.$$

14. $I=\int_0^{\frac{ab}{\sqrt{a^2+b^2}}} \int_0^{\frac{ab}{\sqrt{a^2+b^2}}} f(x, y)dx\,dy+\int_{\frac{ab}{\sqrt{a^2+b^2}}}^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} f(x, y)dx\,dy$.

15. $I=\int_a^{\frac{4a}{3}} \int_0^{2\cos^{-1}\sqrt{\frac{a}{r}}} f(r, \theta)dr\,d\theta+\int_{\frac{4a}{3}}^{\frac{8a}{3}} \int_0^{\cos^{-1}\frac{3r}{8a}} f(r, \theta)dr\,d\theta$.

16. $I=\int_0^{\frac{a^2}{2}} \int_{2v}^a (V_P+V_Q)\frac{dv\,du}{2\sqrt{u^2-4v^2}}$, V_P, V_Q being the values of V at P and Q, the intersections of $x^2+y^2=u$, $xy=v$ in the first quadrant.

17. $I=\int_{\frac{b}{a}\sqrt{a^2-c^2}}^{b} dy\int_c^a dx\,V+\int_0^{\frac{b}{a}\sqrt{a^2-c^2}} dy\int_{\frac{a}{b}\sqrt{b^2-y^2}}^{a} dx\,V.$

18. $I=\int_0^{\frac{a}{2}} dy\int_{\sqrt{a^2-4y^2}}^{\sqrt{a^2-y^2}} dx\,U+\int_{\frac{a}{2}}^{a} dy\int_0^{\sqrt{a^2-y^2}} dx\,U=\int_0^{\frac{\pi}{2}} d\theta\int_{a/\sqrt{\cos^2\theta+4\sin^2\theta}}^{a} dr\,U.$

19. $I=\int_0^a d\xi\int_a^{a+\xi} d\eta\left(\frac{2x}{y}V\right)=\int_a^{2a} d\eta\int_{\eta-a}^{a} d\xi\left(\frac{2x}{y}V\right).$ **20.** One.

21. $I=\int_0^{\frac{1}{4}}\int_{2v}^{1-2v}\frac{F(u,v)}{2\sqrt{u^2-4v^2}}dv\,du,\quad F(u,v)\equiv\phi(x,y).$ **22.** Art. 832.

26. $I=-c^2\iint(\sin^2\xi+\sinh^2\eta)\,d\xi\,d\eta.$

28. $I=\frac{\pi}{2}\cot\alpha+\sinh^{-1}\cot\alpha.$

29. $S=abc\iint\sin\theta\sqrt{\frac{\cos^2\theta}{c^2}+\sin^2\theta\left(\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}\right)}\,d\theta\,d\phi.$

CHAPTER XXIV.

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21. $\frac{1}{n}\left\{\Gamma\left(\frac{1}{n}\right)\right\}^2\Big/\Gamma\left(\frac{2}{n}\right).$ **22.** $A=a\frac{\Gamma(c+1)}{(\log c)^{c+1}},\quad V=\pi a^2\frac{\Gamma(2c+1)}{(2\log c)^{2c+1}}.$

33. Art. 902, $\sqrt{\pi}\operatorname{sech}\pi a.$

CHAPTER XXV.

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1. $\mu a^{p+q+2}\frac{\Gamma(p+1)\Gamma(q+1)}{\Gamma(p+q+3)}.$ **2.** $\frac{1}{120}\mu a^2b^2c^2.$

3. $\frac{\bar{x}}{p+1}=\frac{\bar{y}}{q+1}=\frac{1}{p+q+3}\,\frac{h_1^{\,p+q+3}-h_2^{\,p+q+3}}{h_1^{\,p+q+2}-h_2^{\,p+q+2}}.$

4. (i) $\frac{\bar{x}}{a}=\frac{\bar{y}}{b}=\frac{\bar{z}}{c}=\frac{4}{15}\,\frac{\delta_1^{\,5}-\delta_2^{\,5}}{\delta_1^{\,4}-\delta_2^{\,4}};$

(ii) $\frac{\bar{x}}{(p+1)a}=\text{etc.}=\frac{1}{p+q+r+4}\,\frac{\delta_1^{\,p+q+r+4}-\delta_2^{\,p+q+r+4}}{\delta_1^{\,p+q+r+3}-\delta_2^{\,p+q+r+3}};$

(iii) $\bar{x}=\frac{a}{6}\,\frac{3a^2+b^2+c^2}{a^2+b^2+c^2}\,\frac{\delta_1^{\,6}-\delta_2^{\,6}}{\delta_1^{\,5}-\delta_2^{\,5}}$, etc.

5. $\pi\mu a^3b^3c^3/1890.$ **6.** $Ma^2/\sqrt{2},\quad M(b^2+c^2)/4.$

7. $M=\mu R^5\{\pi(a+b+c)+4(f+g+h)\}/30.$

8. $M=\pi\mu abc(a^2+b^2+c^2)/30,\quad \bar{x}=5a(2a^2+b^2+c^2)/16(a^2+b^2+c^2).$
$A=M[2b^2c^2+c^2a^2+a^2b^2+3b^4+3c^4]/7(a^2+b^2+c^2).$

11. $\frac{\bar{x}}{a}=\Gamma\left(\frac{p+2}{2n}\right)\Gamma\left(\frac{p+q+r+3}{2n}+1\right)\Big/\Gamma\left(\frac{p+1}{2n}\right)\Gamma\left(\frac{p+q+r+4}{2n}+1\right).$

13. $\frac{1}{4a^lb^m}\,\frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}\int_0^{c^2}v^{l+m-1}f(v)\,dv.$ **18.** $\frac{2abc}{3n^2}\left\{\Gamma\left(\frac{1}{2n}\right)\right\}^3\Big/\Gamma\left(\frac{3}{2n}\right).$

CHAPTER XXVI.

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4. $\pi/2$.

5. A system of discontinuous lines and points, the origin being the centre of the system,

$$(-\infty < x < -1),\quad x=-1,\quad (-1<x<-\tfrac{1}{3}),\quad x=-\tfrac{1}{3},\quad (-\tfrac{1}{3}<x<0),$$
$$y=-\frac{\pi}{4},\qquad y=-\frac{\pi}{16},\qquad y=\frac{\pi}{8},\qquad y=\frac{\pi}{16},\qquad y=0,\qquad \text{etc.}$$

6. The part of the plane $z=1$ between $y=\pm x$ which contains (1, 0, 1).

The part of the plane $z=-1$ between $y=\pm x$ which contains $(-1, 0, -1)$.

The parts of the plane $z=0$ between $y=\pm x$ which contain the y-axis.

The portions of the lines $x/1=y/1=(z-\frac{1}{2})/0$, $x/1=y/(-1)=(z-\frac{1}{2})/0$, for which x is positive.

The portions of the lines $x/1=y/1=(z+\frac{1}{2})/0$, $x/1=y/(-1)=(z+\frac{1}{2})/0$, for which x is negative.

9. A staircase of "treads and risers," the former consisting of lines, the latter marked by points.

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4. $\sqrt{\frac{\pi}{a}}\,e^{\frac{b^2-4ac}{4a}}$.

6. $\frac{1}{2^n m}\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\,d\theta=\text{etc.}$

20. (*a*) 0, (*b*) $\frac{1}{4}$, (*c*) ∞, (*d*) $\frac{1}{2}$.

23. $\frac{2\pi}{\sqrt{3}}e^{-m\frac{\sqrt{3}}{2}}\cos\frac{m}{2}$.

33. $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-y^2}\left\{1-e^{-\frac{1}{y^2}\left(\log\frac{1}{\sqrt{x}}\right)^2}\right\}^{-1}dy$.

42. $\sqrt{\pi/2e}$.

CHAPTER XXVII.

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27. (i) $\log\frac{n+2}{n}$; (ii) $(n+4)\log(n+4)-2(n+2)\log(n+2)+n\log n$;

(iii) $\frac{1}{2}\{(n+6)^2\log(n+6)-3(n+4)^2\log(n+4)+3(n+2)^2\log(n+2)-n^2\log n\}$.

CHAPTER XXVIII.

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2. $\frac{k^n.k!}{(k+n)(k+n-1)\dots n}$.

14. $\pi a^n/2n$.

23. $\beta-\beta'=\gamma-\gamma'$; $\alpha'\gamma'+\alpha\gamma'+\alpha'\beta+\beta\gamma'=\alpha\gamma+\alpha'\gamma+\alpha\beta'+\beta'\gamma$;
$\alpha'\gamma'(\alpha+\beta)=\alpha\gamma(\alpha'+\beta')$.

30. $\pi/4$.

33. $\frac{\pi}{a\sqrt{1+a}}\left(\frac{1}{\sqrt{1-a}}-\frac{1}{\sqrt{1+a}}\right)$.

57. $\frac{\pi}{4}\log\frac{a+b}{a-b}$.

CHAPTER XXIX.

Page 415.

1. (i) $(x^2+y^2)^{\frac{n}{2}}$, $n\tan^{-1}\dfrac{y}{x}$;

(ii) $\sqrt{(\log\sqrt{x^2+y^2})^2+(\tan^{-1}y/x)^2}$, $\tan^{-1}\dfrac{\tan^{-1}y/x}{\log\sqrt{x^2+y^2}}$; (iii) a^x, $y\log a$.

(iv) $e^{a\log\sqrt{x^2+y^2}-b\tan^{-1}y/x}$, $b\log\sqrt{x^2+y^2}+a\tan^{-1}y/x$.

(v) $\sqrt{\cosh^2 y-\cos^2 x}$, $\tan^{-1}\dfrac{\tanh y}{\tan x}$;

(vi) $\sqrt{\cosh^2 y-\sin^2 x}$, $-\tan^{-1}(\tan x\tanh y)$;

(vii) $\dfrac{2\sqrt{\sinh^2 y+\cos^2 x}}{\cos 2x+\cosh 2y}$, $\tan^{-1}(\tan x\tanh y)$.

(viii) $\frac{1}{2}\left[\left(\tan^{-1}\dfrac{2x}{1-x^2-y^2}\right)^2+\left(\tanh^{-1}\dfrac{2y}{1+x^2+y^2}\right)^2\right]^{\frac{1}{2}}$,

$\tan^{-1}\left\{\tanh^{-1}\dfrac{2y}{1+x^2+y^2}\Big/\tan^{-1}\dfrac{2x}{1-x^2-y^2}\right\}$.

4. (i) $-1\pm\iota$, $2\pm\iota\sqrt{2}$; (ii) $1\pm\iota$, $-2\pm\iota\sqrt{2}$;
(iii) $-1\pm\iota$, $-2\pm\iota\sqrt{2}$; (iv) $1\pm\iota$, $2\pm\iota\sqrt{2}$.

5. (i) One in each quadrant; (ii) n in each quadrant;
(iii) One in each quadrant and one on negative part of x-axis;
(iv) and (v) n in each quad. and one on $-^{ve}$ part of x-axis;
(vi) n in each quad. and one on each part of y-axis.

6. (i) $\pm\iota, \pm 2\iota, -1\pm\iota$; (ii) $\pm\iota, \pm 2\iota, 6$.

7. (i) Cassinian, (ii) Two st. lines, (iii) Rect. Hyp.

8. $(X^2-a^2\cos^4 c)^{\frac{3}{2}}/a^2\sin c\cos^4 c$. **9.** $\rho=a^3/4r^2$. **11.** A diameter.

15. (ii) $X_s=ae^{-\frac{Y_m}{a}}\cos\dfrac{X_m}{a}$, $Y_s=ae^{-\frac{Y_m}{a}}\sin\dfrac{X_m}{a}$;

(v) (a) Concurrent lines, Meridians; (b) Conc. circles, Parallels of lat.; (c) Equi. spirals, Rhumb lines.

16. $(a_1{}^n-a_2{}^n)(b_1{}^n-b_2{}^n)/n^2$.

CHAPTER XXX.

Page 479.

1. (i) $\frac{2}{3}(z_1-1)^{\frac{3}{2}}+\frac{2}{3}\iota$, (ii) $\frac{2}{3}\iota-\frac{2}{3}(z_1-1)^{\frac{3}{2}}$. **2.** $2\pi\iota\sin a, 2\pi\iota\cos a, -\pi\iota\sin a$

3. $2\pi\iota a, 4\pi\iota a, 2\pi\iota, 0$. **12.** $z/\sqrt{a^2-z^2}$.

17. $\dfrac{2\pi\iota}{3a^2}\left(\sin a+\sin\dfrac{a}{2}\cosh\dfrac{a\sqrt{3}}{2}-\sqrt{3}\cos\dfrac{a}{2}\sinh\dfrac{a\sqrt{3}}{2}\right)$, if $a<1$; 0 i $a>1$.

18. 0 if $a>1$, $2\pi\iota\log(1-a)-2\pi^2$ if $a<1$. **19.** $\pi, 2\pi, 2\pi$.

CHAPTER XXXI.

Page 520.

6. $a=-\frac{1}{2}$, $\alpha=\frac{5}{12}\pi$.

22. (i) $\frac{1}{k}\sin^{-1}(k\,\mathrm{sn}\,u)$; (ii) $\frac{1}{k'}\sinh^{-1}\left(\frac{k'}{k\,\mathrm{cn}\,u}\right)$; (iii) $\mathrm{tn}\,u-\mathrm{am}\,u$.

31. (i) $\mathrm{am}\,u$; (ii) $-\frac{1}{k'}\tan^{-1}\left(\frac{1}{k'}\mathrm{ctn}\,u\right)$; (iii) $-\mathrm{sech}^{-1}(k\,\mathrm{sn}\,u)$.

62. $\{(x^2+y^2)(1-x^2y^2)-c^2(1+x^2y^2)^2\}^2=4x^2y^2(1-x^4)(1-y^4)$.

63. Put $y=(1+k)x/(1+kx^2)$. Multiplier $1/(1+k)$, Mod $2\sqrt{k}/(1+k)$.

CHAPTER XXXII.

Page 561.

11. $\wp^{n+1}(u)/(n+1)$; $\log\wp(u)$; $e^{\wp(u)}$; $2\sqrt{4\wp^3(u)-I\wp(u)-J}$.

12. $\frac{1}{6}\wp'(u)+\frac{1}{12}Iu$; $\frac{1}{120}\wp'''(u)-\frac{3}{20}I\zeta(u)+\frac{1}{10}Ju$; $AP_5+BP_1-C\zeta(u)+Du$ (Art. 1432); $\frac{1}{\wp'(v)}\left[\log e^{2u\zeta v}\frac{\sigma(u-v)}{\sigma(u+v)}+C\right]$, where $\wp(v)=0$;

$$\frac{1}{\{\wp'(v)\}^2}\left[-\zeta(u-v)-\zeta(u+v)-2u\wp(v)-\wp'(v)\int\frac{du}{\wp(u)}\right];$$

$$\frac{1}{2\{\wp'(v)\}^3}\left[-\wp(u-v)-\wp(u+v)-2u\wp'(v)-\wp'''(v)\int\frac{du}{\wp(u)}-3\wp'(v)\wp''(v)\int\frac{du}{\{\wp(u)\}^2}\right].$$

19. $y=c_1\phi(u,v)+c_2\phi(u,-v)$.

32. (i) $\frac{1}{6}\wp'u+\left\{(\wp v)^2+\frac{I}{12}\right\}u+2\wp(v)\zeta(u)+C$;

(ii) $\frac{1}{(\wp'v)^2}\left[-\zeta(u-v)-\zeta(u+v)-2u\wp(v)-\frac{\wp''(v)}{\wp'(v)}\left\{\log e^{2u\zeta(v)}\frac{\sigma(u-v)}{\sigma(u+v)}\right\}\right]+C.$

39. $x=\left\{\wp\left(\frac{\omega_1}{2}\right)-\wp(\omega_1)\right\}\Big/\left\{\wp\left(\frac{\omega_3}{2}\right)-\wp(\omega_1)\right\}$.

CHAPTER XXXIII.

Page 598.

1. $I=1+3\lambda^2$, $J=\lambda^3-\lambda$, $H=-144[\lambda(x^4+y^4)-(1-3\lambda^2)x^2y^2]$,
$\Delta=(9\lambda^2-1)^2$.

8. $z=\wp(u,39,25)$, $x=z/(z-1)$. 10. $z=-3+6/x^2$.

12. $\sin^{-1}u$, $\cos^{-1}u$, 1, $\tan u$, for $k=0$; $\tanh^{-1}u$, $\mathrm{sech}^{-1}u$, $\mathrm{sech}\,u$, $\sinh u$, for $k=1$.

14. $\frac{1}{\sqrt{e_1-e_2}}\tan^{-1}\sqrt{\frac{e_1-e_2}{z-e_1}}$; $\frac{1}{\sqrt{e_1-e_2}}\tanh^{-1}\sqrt{\frac{e_1-e_3}{z-e_3}}$; $(z-e_1)^{-\frac{1}{2}}$.

15. $u=\wp^{-1}(y,0,36)$, $y=1+t^2$; or $u=\frac{1}{\sqrt{6}}\mathrm{sn}^{-1}\sqrt{\frac{6}{t^2+4}}$, mod $\frac{1}{\sqrt{2}}$ $\left(t^2=-4+\frac{6}{x^2}\right)$.

16. $-2^{\frac{4}{3}}u=\wp^{-1}(z,0,\frac{1}{16})$, $t=1/4z$.

22. (i) $2[\zeta(\alpha)-\zeta(u)]$, where $z=\wp(u, 0, 4)$ and $\wp(\alpha)=\alpha$;

(ii) $-\frac{1}{\sqrt{7}}\log e^{2u\zeta(\alpha)}\frac{\sigma(\alpha-u)}{\sigma(\alpha+u)}$, where $z=\wp(u, 0, 4)$, $\alpha=\wp^{-1}(2, 0, 4)$;

(iii) $2u-\frac{25}{7\sqrt{7}}\log e^{2u\zeta(\alpha)}\frac{\sigma(\alpha-u)}{\sigma(\alpha+u)}-\frac{2}{7}[\zeta(u-\alpha)+\zeta(u+\alpha)-4u]$, $\alpha=\wp^{-1}(2,0,4)$;

(iv) $-\frac{3\sqrt{3}}{74}\log e^{2u\zeta(\alpha)}\frac{\sigma(\alpha-u)}{\sigma(\alpha+u)}+\frac{3}{\sqrt{2}}\log e^{2u\zeta(\beta)}\frac{\sigma(\beta-u)}{\sigma(\beta+u)}$,

where $x=\frac{5}{3}+\wp(u, \frac{52}{3}, -\frac{368}{27})$, $\wp(\alpha)=2$, $\wp(\beta)=1$

(v) $u+\log e^{2u\zeta(\alpha)}\frac{\sigma(\alpha-u)}{\sigma(\alpha+u)}$, where $x=\frac{\wp(u, 0, -4)-8}{\wp(u, 0, -4)-2}$ and $\wp(\alpha)=2$.

23. $I=\frac{\sqrt{e_1-e_3}(e_2-e_3)}{e_1e_2+2e_3^2}\left[e_1u+\zeta(u)+\frac{1}{2}\frac{\wp'(u)}{\wp(u)-e_3}\right]$.

24. $I=\sqrt{e_1-e_3}\,u+\frac{(e_1-e_3)^{\frac{3}{2}}a^2}{\wp'(v)}\log\left\{e^{2u\zeta(v)}\frac{\sigma(v-u)}{\sigma(v+u)}\right\}$,

where $v=\wp\{e_3+(e_1-e_2)a^2, I, J\}$

27. $u\sqrt{3}=K-\text{am}\,u$, $x\sqrt{3}=\text{sn}\,u\sqrt{3}/\text{dn}\,u\sqrt{3}$; mod $\sqrt{2/3}$;

or $y=\wp(\omega_1-u)$ where $y=z+\left(\frac{9}{16}\right)\frac{6}{6z+1}$ and $x=(12z-7)/(12z+11)$.

28. $u=\frac{2}{\sqrt{(a_4-a_2)(a_1-a_3)}}\text{sn}^{-1}\left(\sqrt{\frac{a_2-a_4}{a_2-a_1}\cdot\frac{x-a_1}{x-a_4}}, \sqrt{\frac{a_2-a_1}{a_3-a_1}\cdot\frac{a_3-a_4}{a_2-a_4}}\right)$

(Art. 1339).

CHAPTER XXXIV. Section I.

Page 650.

1. The points are opp. extremities of a diam. of a circle, centre at origin diam. $=a$.

2. $y=\sinh nx/\sinh na$. **4.** $r^m\sin m\theta=a^m$, where $(n+1)m=n$.

6.

	i	ii	iii	iv	v	vi
Force/u^2=	y/a^2	$a/2y^2$	$a^2y/(a^2+y^2)^2$	a^2/y^3	y/a^2	$a/2y^2$
	rep.	att.	rep.	att.	att.	rep.
Line	$y=0$	$y=\infty$	$y=0$	$y=\infty$	$y=a$	$y=a$

	vii	viii	ix	x
Force/u^2=	a^2/y^3	$1/3a^{\frac{2}{3}}y^{\frac{1}{3}}$	$\frac{2a^4y^3}{(a^4+y^4)^2}$	$\frac{a^2b^4}{\{b^4+(a^2-b^2)y^2\}^2}$
	rep.	att.	att.	rep.
Line	$y=a$	$y=a$	$y=\infty$	$y=0$

7.

	i	ii	iii	iv	v	vi	vii	viii
Force ∝	const.	r	r^2	r^3	r^5	r^{-3}	r^{2n+1}	r
	rep.	rep.	rep.	rep.	rep.	att.	rep. $n > -1$ att. $n < -1$	rep.
Circle	$r=0$	$r=0$	$r=0$	$r=0$	$r=0$	$r=\infty$	$r=0$ $r=\infty$	$r=a$

	ix	x	xi	xii
Force ∝	$p^2=Ar^2+B$ r	$\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{a^2}$ $r/(r^2+a^2)^2$	$\frac{a^2b^2}{p^2}=a^2+b^2-r^2$ $r/(a^2+b^2-r^2)^2$	$\frac{b^2}{p^2}=\frac{2a}{r}-1$ $1/(2a-r)^2$
	rep. A + att. A −ᵛᵉ	rep.	rep.	rep.
Circle	$r=\sqrt{\frac{-B}{A}}$	$r=0$	$r=\infty$	$r=0$

9. The parabola $11(y-1)+3x(x+4)=0$ satisfies the conditions.

10. Two straight lines equally inclined in opp. directions to the x-axis.

11. Rect. Hyp.

12 and 13. Circular arc. Discont. solutions as in Art. 1505 (1).

14. A central conic. **16.** $y=a\sin\frac{\pi x}{l}$, where a is known.

19. Ellipse. Centre on initial line. Action a min. Free path under att. radial force to focus.

22. A circle. **25.** A. catenary.

28. A circle. Max. area for given length $[p=A+B\cos(\psi+\alpha)]$.

31. Parabolic arc wrapped on a cone. Focus at vertex. Axis along a generator.

CHAPTER XXXIV. Section II.

Page 692.

1. $y=a\cosh n(x-b)$. Minimum.

3. Taking c +ᵛᵉ and

$x_0 > -a$, $(x_1 > x_0 > -a$, min.), $(x_0 > x_1 > a$, max.), $(x_0 > -a > x_1$, neither);
$x_0 < -a$, $(x_1 < x_0$, max.), $(x_0 < x_1 < -a$, min.), $(x_0 < -a < x_1$, neither).

CHAPTER XXXV. Section I.

Page 717.

1. If a cosine curve $y=\cos x$ be drawn from $x=0$ to $x=\pi$ and a point placed at the origin, the total graph consists of this portion with repetitions from π to 2π, 2π to 3π, etc.

10. $\phi(x)=\sum_1^\infty A_n\sin\frac{n\pi x}{a_3}$, where

$$A_n=\frac{2}{n\pi}\left[c_1\left(1-\cos n\pi\frac{a_1}{a_3}\right)+c_2\left(\cos n\pi\frac{a_1}{a_3}-\cos n\pi\frac{a_2}{a_3}\right)+c_3\left(\cos n\pi\frac{a_2}{a_3}-\cos n\pi\right)\right].$$

11. $\phi(x)=A_0+\sum_1^\infty A_n\cos 2n\pi x/a_3+\sum_1^\infty B_n\sin 2n\pi x/a_3$, where

$$A_0=\{c_1a_3+c_2(a_2-a_1)+c_3(a_3-a_2)\}/a_3,$$

$$A_n=\frac{1}{n\pi}\{c_1\sin 2n\pi a_1/a_3+c_2(\sin 2n\pi a_2/a_3-\sin 2n\pi a_1/a_3)-c_3\sin 2n\pi a_2/a_3\},$$

$$B_n=\frac{1}{n\pi}\{c_1(1-\cos 2n\pi a_1/a_3)+c_2(\cos 2n\pi a_1/a_3-\cos 2\pi na_2/a_3)+c_3(1-\cos 2n\pi a_2/a_3)\}.$$

14. Repetitions of the portion of $y=x(\pi^2-x^2)/12$ which lies between $x=\pm\pi$.

CHAPTER XXXV. Section II.

Page 737.

1. $\frac{8a}{\pi^2}\sum_0^\infty\frac{1}{(2r+1)^2}\cos\frac{(2r+1)\pi x}{2a}$.

2. $\frac{\pi^2}{3}+4\sum_1^\infty(-1)^r\frac{1}{r^2}\cos rx\ (-\pi<x<\pi)$. A series of equal parabolic arcs.

3. $\frac{4kl}{\pi^2}\sum_0^\infty(-1)^r\frac{1}{(2r+1)^2}\sin\frac{(2r+1)\pi x}{l}$; $\frac{kl}{4}-\frac{2kl}{\pi^2}\sum_0^\infty\frac{1}{(2r+1)^2}\cos\frac{(2r+1)2\pi x}{l}$.

4. $\frac{8}{\pi}\sum_0^\infty\frac{1}{(2r+1)^3}\sin(2r+1)x$; 0 to π inclusive.

5. $\frac{2nk}{\pi^2}\sum_1^\infty\frac{1}{p^2}(1-\cos p\pi)\sin\frac{p\pi}{n}\sin\frac{p\pi x}{l}$.

6. $y=-\frac{a\pi^2}{8c}x$ $(0$ to $2c-a)$; $y=-\frac{2c-a}{8c}\pi^2(2c-x)$, $(2c-a$ to $2c)$.

7. $\sum_1^\infty A_n\sin\frac{n\pi x}{l}$, $A_n=\left(-\frac{l^2}{2n\pi}+\frac{4l^2}{n^3\pi^3}\right)\cos\frac{n\pi}{2}+\frac{2l^2}{n^2\pi^2}\sin\frac{n\pi}{2}-\frac{4l^2}{n^3\pi^3}$,

$B_0+\sum_1^\infty B_n\cos\frac{n\pi x}{l}$, $B_n=\left(\frac{l^2}{2n\pi}-\frac{4l^2}{n^3\pi^3}\right)\sin\frac{n\pi}{2}+\frac{2l^2}{n^2\pi^2}\cos\frac{n\pi}{2}$, $B_0=\frac{l^2}{24}$.

10. $\frac{l^2}{48a}+\frac{l^2}{a\pi^2}\sum_1^\infty\frac{1}{r^2}\cos\frac{r\pi}{2}\cos\frac{r\pi x}{l}$; repetitions of the part between $x=0$ and $x=l$.

13. If $f(x)$ changes to $\phi(x)$ and $f'(x)$ to $\phi'(x)$ at $x=\alpha$,

$$A_n\frac{l}{2}=\int_0^\alpha f(x)\sin\frac{n\pi x}{l}dx+\int_\alpha^l\phi(x)\sin\frac{n\pi x}{l}dx,$$

$$B_n\frac{l}{2}=\frac{n\pi}{l}A_n+\frac{2}{l}\left[f(\alpha)\cos\frac{n\pi\alpha}{l}-f(0)\right]+\frac{2}{l}\left[\phi(l)(-1)^n-\phi(\alpha)\cos\frac{n\pi\alpha}{l}\right].$$

16. $u=\frac{1}{\pi}\sum_1^\infty\frac{a^nb^n}{a^{2n}-b^{2n}}\left(\frac{r^n}{b^n}-\frac{b^n}{r^n}\right)\int_0^{2\pi}f(\phi)\cos n(\phi-\theta)\,d\phi$.

19. $\frac{4}{\pi}\sum_0^\infty\frac{1}{2r+1}\sin(2r+1)\frac{\pi x}{a}$.

27. $C=\frac{1}{2}\tan^{-1}\frac{2m\cos\theta}{1-m^2}$. Arc of a circle, centre at the origin, and radius $\frac{1}{4}\pi a$ symmetrically placed about the initial line, and subtending an angle $\pi-2\alpha$ at the origin; together with the origin itself.

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2. $\left(\frac{\pi}{3}+\frac{1}{\pi}\right)(\text{rad.})^2$. **7.** $3a^2/2$, $5a^3/2$, axis $=2a$. **8.** Art. 1650 (12).

10. Edges $2a$, $2b$; (i) $(a^2+b^2)/3$; (ii) and (iii) $r^2+(a^2+b^2)/3$ for a point dist. r from centre.

11. (i) a; (ii) $4a/3$. **12.** $10a/7$, a (axis $2a$).

13. (i) sides a, b, c, $(a^2+b^2+c^2)/6$; (ii) $a^2/3$ (side $=a$); (iii) $6a^2/5$ (rad. $=a$); (iv) $a^2/2$ (edge $=a$).

14. $2/a$, $3/2a$, $a=$ semi. maj. ax. **36.** $17a^2/16$.

39. $2(\sqrt{b^2+c^2}-c)/b^2$, where $b=$ rad. of disc, $c=$ dist. between centres.

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5. ${}^{m+n}C_n\left(\frac{r^2}{ab}\right)^m\left(1-\frac{r^2}{ab}\right)^n$. **6.** (a) $\frac{1}{10}$; (b) $\frac{3}{10}$; (c) $\frac{1}{15}$.

11. $1/7$. **12.** $280/1287$.

23. π^2a^2. **24.** $\frac{c^2}{a^2}+\frac{2}{\pi}\left(1-\frac{c^2}{a^2}\right)\sin^{-1}\frac{c}{2a}-\frac{1}{\pi}\frac{c}{a}\left(1+\frac{c^2}{2a^2}\right)\sqrt{1-\frac{c^2}{4a^2}}$. **25.** $b/2a$.

27.
$$\frac{(\Sigma p)!\,(\Sigma q)!}{p_1!\,p_2!\dots p_n!\,q_1!\,q_2!\dots q_n!}\;\frac{(p_1+q_1)!\,(p_2+q_2)!\dots(p_n+q_n)!}{(\Sigma p+\Sigma q)!}\times\frac{(\Sigma p+1)(\Sigma p+2)\dots(\Sigma p+n-1)}{(\Sigma\overline{p+q}+1)(\Sigma\overline{p+q}+2)\dots(\Sigma\overline{p+q}+n-1)}.$$

28. $128/45\pi^2$. **39.** $5c^2/6$.

CHAPTER XXXIX.

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4. $5+(7x+2y+3z)+(-x^2+z^2+7yz+8zx+9xy)+\{10x^3-6x(x^2+y^2+z^2)\}+11xyz$.

15. If $\sin^3\theta$ could be expressed in a finite series of P's, it could be expressed in a finite series of cosines.

19. Art. 1806.

CHAPTER XL.

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1. $\frac{x}{1+x+x^5}$, $\frac{x}{1+x^8+x^9}$, $\frac{-x}{1+x+x^6}+\log(1+x+x^6)$.

2. $\frac{-x}{1+x+x^5}$. **4.** $-\frac{1}{a'}\frac{ax+b}{a'x^2+2b'x+c'}$.

5. (a) conv. (b) $n \not< 1$ conv.; $0<n<1$ conv.; $n \not> 0$ div. $\to +\infty$.
(c) conv. (d) $0<n<1$ conv.; $n \leqslant 0$ div.; $n>1$ div.

8. All conv. if $m>1$, div. if $m \not> 1$.