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3.3.11. – maszyny

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5/1992

WARSZAWA 1992

ISSN 0208-5658

Praca wpłynęła do Redakcji dnia 20 marca 1992 r.

IPPT PAN

Na prawach rękopisu

Instytut Podstawowych Problemów Techniki PAN
Nakład 100 egz. Ark.wyd.1,2 Ark.druk. 1,5
Oddano do drukarni w kwietniu 1992 r.

Wydawnictwo Spółdzielcze sp. z o.o.
Warszawa, ul.Jasna 1

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ON CERTAIN METHODS OF DETERMINING THE ELLIPSES AND ELLIPSOIDS OF THE POSITIONING ACCURACY OF ROBOT MANIPULATORS

Abstract

When analysing the problem of the positioning accuracy of robot manipulators it is important to know how large are the random deviations of the hand from the desired position if the joint positioning errors possess a normal distribution. Two simple methods of determining the ellipses end ellipsoids of probability concentration are proposed. One of them consists in finding at first the polygon or polyhedron of the positioning accuracy, and then in finding the ellipse or ellipsoid of the principal axes and second order moments coinciding with those of the polygon or polyhedron respectively. In the second of proposed methods a computer generates random Gaussian deviations from the desired joint positions. The calculated numerous positioning errors are forming elliptical or ellipsoidal pattern demonstrating good agreement with theoretically obtained ellipses or ellipsoids of probability concentration.

1. Introduction

Small changes of position of the hand of a manipulator are caused among others by small random deviations Δq_i from the desired (nominal) joint co-ordinate q_i^o In the case of a revolute joint the deviation Δq_i corresponds to a small angular deviation $\Delta\Theta_i$ from the desired angle Θ_i^o. In the case of a prismatic joint Δq_i corresponds to a small linear deviation Δl_i from the desired distance l_i^o.

The problem of the positioning accuracy of robot manipulators has been analysed in several papers and books. Basic notions are discussed in the book by R. P. Paul [1]. A mathematical model of random positioning errors has been developed by A. Kumar and K.J. Waldron [2]. In the paper by A.Antshev et all [3] the problem of calculating the ellipsoids of the positioning accuracy has been shortly mentioned.

Following Ref.[2] three sources of positioning errors may be distinguished:

1. Errors in positioning the joints accurately.
2. Dynamic errors due to elastic deflections of individual members of the manipulator.
3. Mechanical clearance in the system.

In the present paper only errors in positioning the hand accurately due to random Gaussian errors in positioning the joints will be analysed. A particular positioning error of the hand may be represented by a displacement vector whose components represent deviations from the desired (nominal) co-ordinates of the hand. Since the joint positioning errors have random magnitudes in each of the repeated cycles, the end point of such a vector will have random co-ordinates. Analysing a large number of realizations of the movement of the manipulator we have to deal with the problem of the probability concentration of the distribution of the end points of all displacement vectors.

For manipulators operating in two dimensions the probability concentration may be represented by an ellipse of equal probability. For a general case when the manipulator operates in three dimensions the probability concentration may be represented by a certain ellipsoid. We shall present below two approximate methods of determining such ellipses and ellipsoids for robot manipulators.

Let us assume that hand positioning errors are the result of random errors in joint positions. The errors Δq_i are assumed to be distributed according to a normal distribution. Thus the probability that the joint positioning error is of a magnitude Δq_i is given by the formula

$$\varphi(\Delta q_i) = \frac{1}{\sigma_i \sqrt{2\Pi}} \exp\left[- \frac{1}{2} \left(\frac{\Delta q_i}{\sigma_i} \right)^2 \right] , \qquad (1)$$

where σ_i is the standard deviation, whose value depends on the accuracy of the joint labelled by the index i.

In the present work an approximate theoretical procedure for determining the probability concentration ellipses and ellipsoids has been used (cf.Cramer's book [4]). At first we shall assume that instead of the normal distribution (1) of joint positioning errors, this distribution is uniform with the same standard deviation σ_1.

Then we shall find such uniform distribution the regions within which all the possible error displacements of the hand will be located. For manipulators operating in two dimensions such regions take the form of certain polygons, while for manipulators operating in three dimensions they have the form of certain polyhedrons. As the final step of calculations we shall find the orientation and dimensions of the ellipses or ellipsoids of the same principal axes and second order moments as those of the corresponding polygons or polyhedrons respectively.

2. Ellipses of positioning accuracy concentration

We shall now analyse the positioning accuracy of manipulators operating in two dimensions. Any position of a chosen reference point of the hand is defined by its two Cartesian co-ordinates X, Y. Each co-ordinate is a certain function of the joint position parameters $q_i = q_i^0 + \Delta q_i$

$$X = X\,(q_1,\ q_2, \ldots\ldots,\ q_n), \qquad (2)$$
$$Y = Y\,(q_1,\ q_2, \ldots\ldots,\ q_n).$$

To analyse the hand positioning errors we shall use a local co-ordinate system x, y with the axes parallel to the corresponding axes of the basic system X, Y and the origin at the desired position of the reference point on the hand. The positioning error will be represented by a displacement vector with the components

$$x = X - X^0, \qquad y = Y - Y^0\ , \qquad (3)$$

where X^0, Y^0 define the desired position of the hand an X, Y are the actual co-ordinates of the hand position.

We shall assume that the concentration of the two-dimensional distribution of the hand positioning error may be represented in the reference system x,y by a certain ellipse

$$\left(\frac{x}{\sigma x}\right)^2 + \left(\frac{y}{\sigma y}\right)^2 - 2\rho_{xy}\frac{x}{\sigma x}\frac{y}{\sigma y} = \text{const}, \qquad (4)$$

where σx, σy are the standard deviations and ρ_{xy} is the correlation coefficient.

According to the approximate procedure used in this paper we shall at first assume that the distribution of joint positioning errors is uniform with the same standard deviation as that of the original Gaussian distribution. Note that the two standard deviations are equal if the errors in the uniform distribution are limited by two extreme values $\pm\sigma_i\sqrt{3}$.

It has been demonstrated in [5] that when joint positioning errors vary within two extreme values, then the end point of any error displacement vector of the hand will lie inside a certain polygon bounded by several pairs of parallel straight lines. The equations of these lines are

$$\frac{\partial Y}{\partial q_r}x - \frac{\partial X}{\partial q_r}y = \sum_{i=1}^{n}\left|\begin{matrix}\frac{\partial X}{\partial q_i} & \frac{\partial X}{\partial q_r}\\ \frac{\partial Y}{\partial q_i} & \frac{\partial Y}{\partial q_r}\end{matrix}\right|\Delta q_i. \qquad (5)$$

Writing consecutively such equations for all joint positioning parameters q_r we obtain the equations of several families of parallel straight lines. Their extreme positions constitute the edges of the polygon of the positioning accuracy.

Now having found the polygon we can calculate its second order (inertia) moments and then find the orientation of its principal axes 1,2 and second order principal moments J_1, J_2. The principal radii a and b of the ellipse of probability concentration can be calculated by solving the following system of equations

$$\frac{1}{4}\Pi a^3 b = J_1, \qquad \frac{1}{4}\Pi a b^3 = J_2 . \qquad (6)$$

2.1. Examples

As the working examples we shall determine the ellipses of the probability concentration of the positioning accuracy for a simple manipulator with two revolute joints shown schematically in Fig. 1. The position of the hand (point 0) is determined by

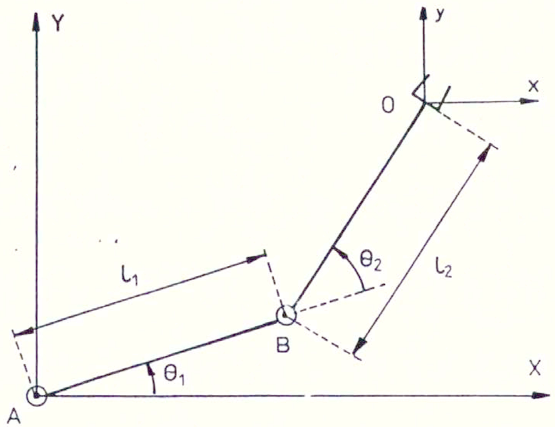

Fig. 1.

the functions of two independent variables Θ_1 end Θ_2

$$X = l_1 \cos \Theta_1 + l_2 \cos (\Theta_1 + \Theta_2) ,$$
$$Y = l_1 \sin \Theta_1 + l_2 \sin (\Theta_1 + \Theta_2) , \qquad (7)$$

In the particular examples we assume

$$l_1 = l_2 = 1000 \text{ mm} .$$

Standard deviations for the normal distribution of joint positioning errors are taken to be

$$\sigma_{\Theta_1} = \sigma_{\Theta_2} = 0{,}01 \text{ rad} .$$

The polygon of the positioning accuracy will be bounded by extreme positions of straight lines determined by the equations [cf. Eqn. (5)].

$$\frac{\partial Y}{\partial \Theta_1} x - \frac{\partial X}{\partial \Theta_1} y = \begin{vmatrix} \frac{\partial X}{\partial \Theta_2} & \frac{\partial X}{\partial \Theta_1} \\ \frac{\partial Y}{\partial \Theta_2} & \frac{\partial Y}{\partial \Theta_1} \end{vmatrix} \Delta\Theta_2 , \qquad (8a)$$

for the first family of lines and

$$\frac{\partial Y}{\partial \Theta_2} x - \frac{\partial X}{\partial \Theta_2} y = \begin{vmatrix} \frac{\partial X}{\partial \Theta_1} & \frac{\partial X}{\partial \Theta_2} \\ \frac{\partial Y}{\partial \Theta_1} & \frac{\partial Y}{\partial \Theta_2} \end{vmatrix} \Delta\Theta_1 \qquad (8b)$$

for the second family of lines, Here $\Delta\Theta_1 = \Delta\Theta_2 = \pm 0{,}01 \sqrt{3}$ rad.

Example 1.

Desired position of the hand is determined by the following joint positioning angles

$$\Theta_1 = 0\ , \qquad \Theta_2 = \frac{1}{2}\,\Pi\ . \qquad (9)$$

Making use of relations (7) and equations (8) we find that the polygon of the positioning accuracy is bounded by two pairs of straight lines

$$x + y = \pm 17.3\ \text{mm}\ ,$$

$$y = \pm 17.3\ \text{mm}\ .$$

The polygon is shown in Fig. 2.

The second order (inertia) moments of the polygon with respect to the reference axes x and y are

$$J_x = \frac{1}{12}\, d\, h^3 = 12\ .\ 10^4\ \text{mm}^4, \qquad J_y = \frac{1}{6}\, d^3 h = 24\ .\ 10^4\ \text{mm}^4$$

and the mixed second order moment is

$$J_{xy} = \frac{1}{12}\, d^2 h^2 = 12\ .\ 10^4\ \text{mm}^4$$

Now the principal second order moments of the polygon can be found by constructing the Mohr circle. These principal moments are

$$J_1 = 31.4\ .\ 10^4\ \text{mm}^4, \qquad J_2 = 4.6\ .\ 10^4\ \text{mm}^4.$$

The principal axis 1 makes the angle of $31^\circ 30'$ with the x-axis as shown in the figure.

The principal radii of the ellipse of probability concentration can now be calculated by solving the system of equations (6). Finally we obtain

$$a = 31.97 \text{ mm}, \qquad b = 12.23 \text{ mm}.$$

The ellipse is shown in Fig. 2.

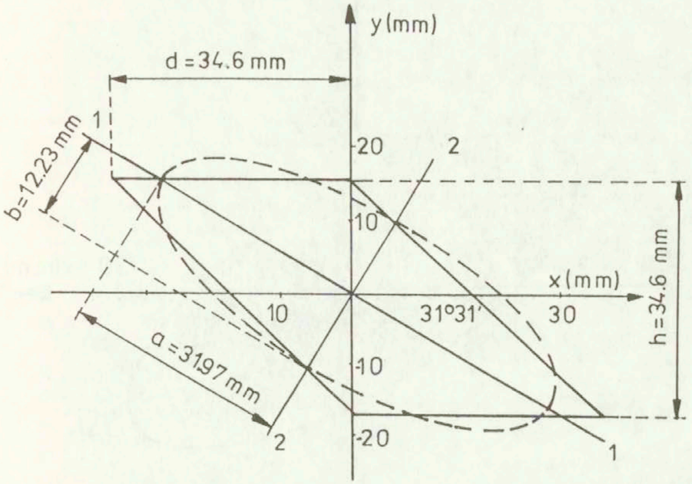

Fig.2

The theoretically calculated ellipse from Fig.2 has been compared with the results of a numerical experiment. Random small Gaussian deviations from the desired nominal joint positions (9) have been numerically generated by a program for a personal computer calculating the displacement of the hand from its nominal position. Calculated displaced positions are shown in Fig.3 as the corresponding points. Altogether five thousand repeated cycles of the movement have been numerically simulated with randomly generated joint positioning errors. Theoretical ellipse from Fig.2 is also shown in the figure for ready comparison. It can be seen that the theoretical ellipse coincides well with the assembly of points obtained in the numerical experiment.

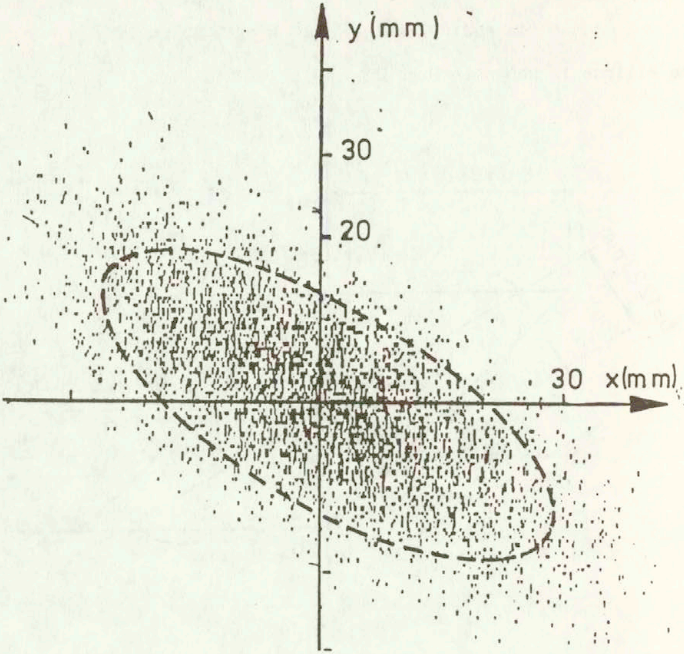

Fig.3

Example 2.

Desired position of the hand of the manipulator shown in Fig.1 is determined by the following values of the joint positioning angles

$$\Theta_1 = 0\,, \qquad \Theta_2 = \frac{3}{4}\Pi\,.$$

Repeating the procedure described in Example 1 we find the theoretical ellipse of the probability concentration of the positioning accuracy. The ellipse is compared in Fig.4 with the results of a numerical experiment analogous to that described in the previous example.

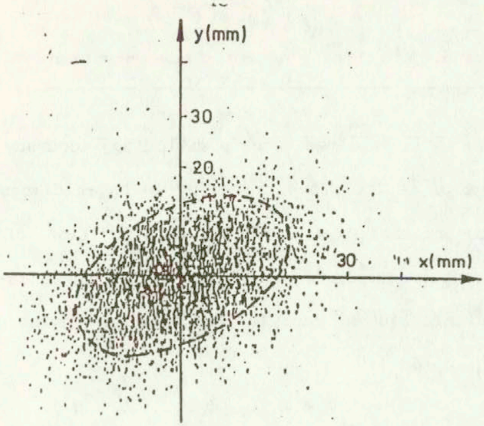

Fig. 4

Example 3.

Desired position of the hand is now determined by the joint positioning angles

$$\Theta_1 = 0 , \qquad \Theta_2 = \frac{1}{4}\pi .$$

The theoretical ellipse is compared in Fig. 5 with the result of numerical experiment as in the previous examples.

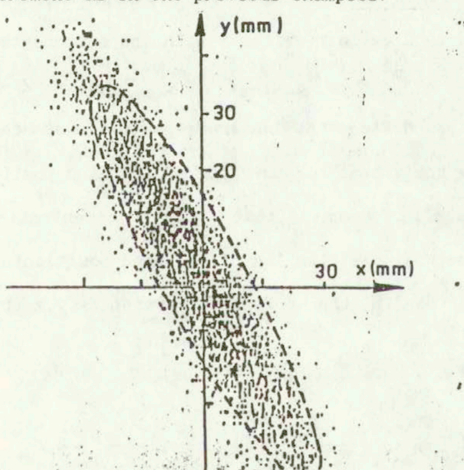

Fig. 5

3. Ellipsoids of positioning accuracy concentration

Now we shall analyse the positioning accuracy for more general cases of manipulators operating in three dimensions. In a Cartesian co-ordinate system X.Y,Z any position of a chosen reference point of the hand is defined by its three co-ordinates. Each co-ordinate is a certain function of joint positioning parameters $q_i = q_i^0 + \Delta q_i$

$$\begin{aligned} X &= X\,(q_1,\ q_2, \ldots\ldots, q_n). \\ Y &= Y\,(q_1,\ q_2, \ldots\ldots, q_n). \\ Z &= Z\,(q_1,\ q_2, \ldots\ldots, q_n) \end{aligned} \tag{10}$$

To analyse hand positioning errors we shall introduce a local co-ordinate system x, y, z, with the axes parallel to the respective axes of the basic system X, Y, Z and the origin at the desired position of the hand. Any positioning error will be represented by a certain vector v with the components

$$x = X - X^0, \qquad y = Y - Y^0, \qquad z = Z - Z^0, \tag{11}$$

where X^0, Y^0, Z^0 define the desired position of the hand and X, Y, Z are the actual co-ordinates of the hand position.

We shall assume that the concentration of the three-dimensional distribution of the hand positioning errors may be represented in the reference system x,y.z by a certain ellipsoid

$$\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 + \left(\frac{z}{\sigma_z}\right)^2 - 2\,\rho_{xy}\,\frac{x}{\sigma_x}\,\frac{y}{\sigma_y} - 2\,\rho_{yz}\,\frac{y}{\sigma_y}\,\frac{z}{\sigma_z} - 2\,\rho_{zx}\,\frac{z}{\sigma_z}\,\frac{x}{\sigma_x} = \text{const}\,, \tag{12}$$

where σ_x, σ_y, σ_z are standard deviations and ρ_{xy}, ρ_{yz}, ρ_{zx} are correlation coefficients.

Similarly as in the two-dimensional problem we shall at first assume that the distribution of joint positioning errors is uniform and that it is limited by the extreme values $\pm\sigma_i\sqrt{3}$.

It has been demonstrated in the previous paper [6] that when the joint positioning errors vary within certain limits (joint positioning tolerances), then the end points of all vectors of the error displacement from the desired position of the hand will lie inside a certain polyhedron bounded by a family of pairs of parallel planes. The equations of these planes have the following form

$$\begin{vmatrix} \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} \\ \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} \end{vmatrix} x + \begin{vmatrix} \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} \\ \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} \end{vmatrix} y + \begin{vmatrix} \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} \\ \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} \end{vmatrix} z =$$

$$= \sum_{i=1}^{n} \begin{vmatrix} \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} & \frac{\partial X}{\partial q_i} \\ \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} & \frac{\partial Y}{\partial q_i} \\ \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} & \frac{\partial Z}{\partial q_i} \end{vmatrix} \Delta q_i . \qquad (13)$$

The end point of the vector of hand positioning error moves along one of such planes when two joint positioning errors Δq_r and Δq_s change, while all remaining joint positioning errors are kept constant.

We shall obtain the two extreme positions of these planes by taking appropriately the extreme values of the joint positioning errors $\Delta q_i = +\sigma_i\sqrt{3}$ or $\Delta q_i = -\sigma_i\sqrt{3}$. Taking consecutively all the possible combinations of pairs of joint positioning errors Δq_r and Δq_s as changing parameters we obtain equations of various families of parallel planes and then their extreme positions forming the faces of the polyhedron of the positioning accuracy.

Now following the known procedure (cf.[4]) we can find the orientation and dimensions of the ellipsoid (12), which should have the same principal axes and second order moments as the polyhedron.

Having found the polyhedron in the space of positioning errors we can calculate its second order moments (volume inertia moments) with respect to the reference axes x, y, z. As the next step we can find the orientation of the principal axes 1, 2, 3 of the polyhedron and also its second order principal moments J_1, J_2, J_3.

The principal radii a, b. c of the ellipsoid can be calculated by solving the system of equations

$$\frac{4}{15}\Pi a^3 b c = J_1, \qquad \frac{4}{15}\Pi a b^3 c = J_2, \qquad \frac{4}{15}\Pi a b c^3 = J_3, \tag{14}$$

where J_1, J_2 ,J_3 are the second order principal moments calculated for the polyhedron.

3.1. Examples

As the working examples we shall determine the positioning accuracy ellipsoids for the hand of a simple 4-R manipulator with four revolute kinematic pairs. The manipulator is shown schematically in Fig. 6. Joint positions are determined by three positioning angles Θ_1, Θ_2, Θ_3. Position of the joint with the axis 4-4 has no influence on the position of the hand.

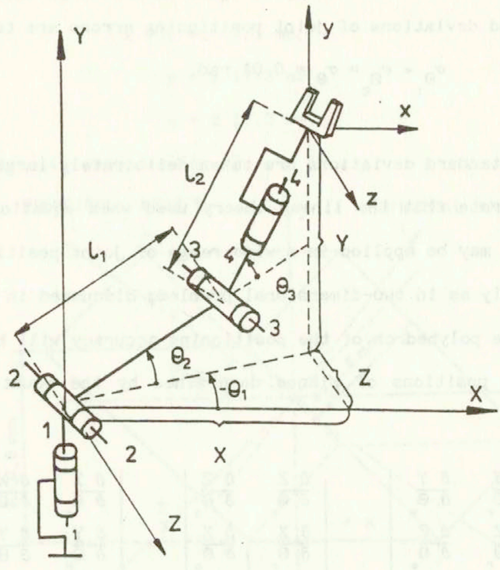

Fig.6

The position of the hand in the basic reference system X,Y,Z is determined by the co-ordinates

$$
\begin{aligned}
X &= [\, l_1 \cos\Theta_2 + l_2 \cos(\Theta_2 + \Theta_3)] \cos\Theta_1 , \\
Y &= l_1 \sin\Theta_2 + l_2 \sin(\Theta_2 + \Theta_3), \qquad (15) \\
Z &= [\, l_1 \cos\Theta_2 + l_2 \cos(\Theta_2 + \Theta_3)] \sin\Theta_1 .
\end{aligned}
$$

These co-ordinates are functions of three random independent variables Θ_1, Θ_2 Θ_3. Linear dimensions l_1 and l_2 do not change their values. Thus for the problem in question general relations (10) are written in the particular form (15).

Numerical examples for the manipulator shown in Fig. 6 will be calculated for the following data

$$l_1 = l_2 = 1000 \text{ mm} . \tag{16}$$

Standard deviations of joint positioning errors are taken to be

$$\sigma_{\Theta_1} = \sigma_{\Theta_2} = \sigma_{\Theta_3} = 0.01 \text{ rad.}$$

These standard deviations are taken deliberately large in order to demonstrate that the linear theory used when equations (13) were derived may be applied in a wide range of joint positioning errors similarly as in two-dimensional problems discussed in Section 2.

The polyhedron of the positioning accuracy will be bounded by extreme positions of planes determined by the equations [cf.eqn (13)]

$$\begin{vmatrix} \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} \\ \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} \end{vmatrix} x + \begin{vmatrix} \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} \\ \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} \end{vmatrix} y + \begin{vmatrix} \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} \\ \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} \end{vmatrix} z =$$

$$= \sum_{i=1}^{3} \begin{vmatrix} \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} & \frac{\partial X}{\partial \Theta_i} \\ \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} & \frac{\partial Y}{\partial \Theta_i} \\ \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} & \frac{\partial Z}{\partial \Theta_i} \end{vmatrix} \Delta\Theta_i , \tag{17}$$

where $\Delta \Theta_i = \pm 0{,}01 \sqrt{3}$ rad.

Example 1 .

Desired position of the hand is determined by the following nominal values of joint positioning angles

$$\Theta_1 = \Theta_2 = 0, \qquad \Theta_3 = \frac{1}{2}\Pi. \tag{18}$$

Making use of relations (15) and (17) we obtain that the polyhedron of the positioning accuracy is bounded by three pairs of planes. The equations of these planes are

$$x + y = \pm 17.3 \text{ mm},$$
$$y = \pm 17.3 \text{ mm}, \quad (19)$$
$$z = \pm 17.3 \text{ mm}.$$

The polyhedron is shown in Fig. 7.

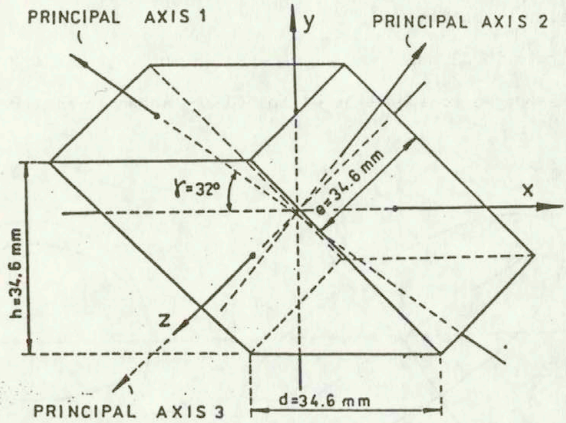

Fig. 7

The second order moments of the polyhedron with respect to the reference planes are

$$J_{xx} = \frac{1}{6} d^3 h\, e = 8.26 \cdot 10^6 \text{ mm}^5,$$
$$J_{yy} = \frac{1}{12} d\, h^3 e = 4.13 \cdot 10^6 \text{ mm}^5,$$
$$J_{zz} = \frac{1}{12} d\, h\, e^3 = 4.13 \cdot 10^6 \text{ mm}^5$$

and the mixed second order moment with respect to the planes x, z and y , z is

$$J_{x-y} = - \frac{1}{12} d^2 h^2 e = - 4.13 \cdot 10^6 \text{ mm}^5.$$

Now principal second order moments of the polyhedron can be found by constructing Mohr circles. These principal second order moments are

$J_1 = 10.85 \cdot 10^6\ mm^5$, $J_2 = 1.58 \cdot 10^6\ mm^5$, $J_3 = 4.13 \cdot 10^6\ mm^5$

The principal axis 1 makes the angle $\gamma = 31°30'$ with the direction of the x-axis as shown in the figure.

The principal radii of the ellipsoid of probability concentration can now be calculated by solving equations (14). Finally we obtain

a = 35.32 mm, b = 13.48 mm, c = 21.79 mm.

Three projections of this ellipsoid are shown in Fig. 8.

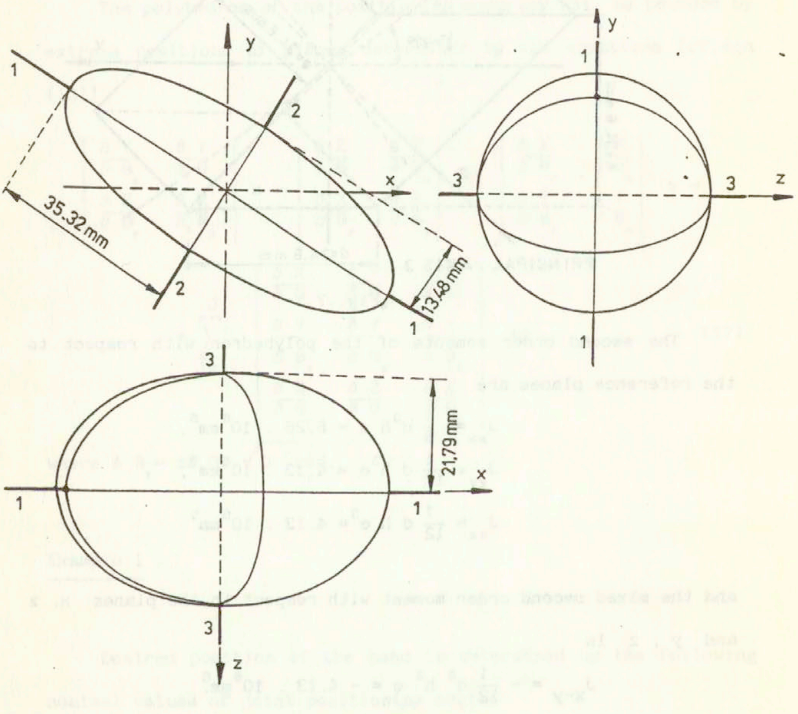

Fig.8

The theoretical ellipsoid has been compared with the results of a numerical experiment. Random small Gaussian deviations from the desired joint positions (18) have been numerically generated by a program for a personal computer calculating the displacement of the hand from its nominal position. Calculated displaced positions are shown in Fig. 9 as projections of the corresponding points. Altogether five thousand repeated cycles of the movement have been numerically simulated with randomly generated joint positioning errors. Theoretical ellipsoid from Fig. 8 is also shown in Fig.8. It can be seen that the ellipsoid obtained theoretically coincides well with the assembly of points obtained during the numerical experiment.

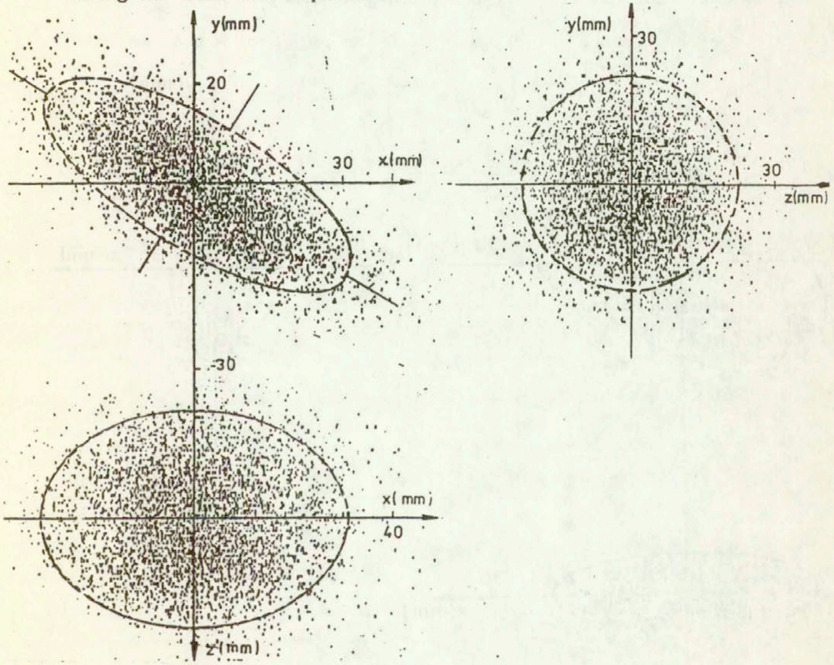

Fig.9

Example 2.

In this example the same manipulator shown in Fig.6 is considered. However, now the desired position of the hand is different than that analysed in the previous example. It is now determined by the following nominal values of joint positioning angles

$$\Theta_1 = \Theta_2 = 0 , \qquad \Theta_3 = \frac{3}{4}\pi . \qquad (20)$$

Repeating the procedure described in the Example 1 we obtain the following values of the principal radii of the ellipsoid of probability concentration of positioning accuracy

a = 23.7 mm , b = 14.0 mm , c = 6.4 mm.

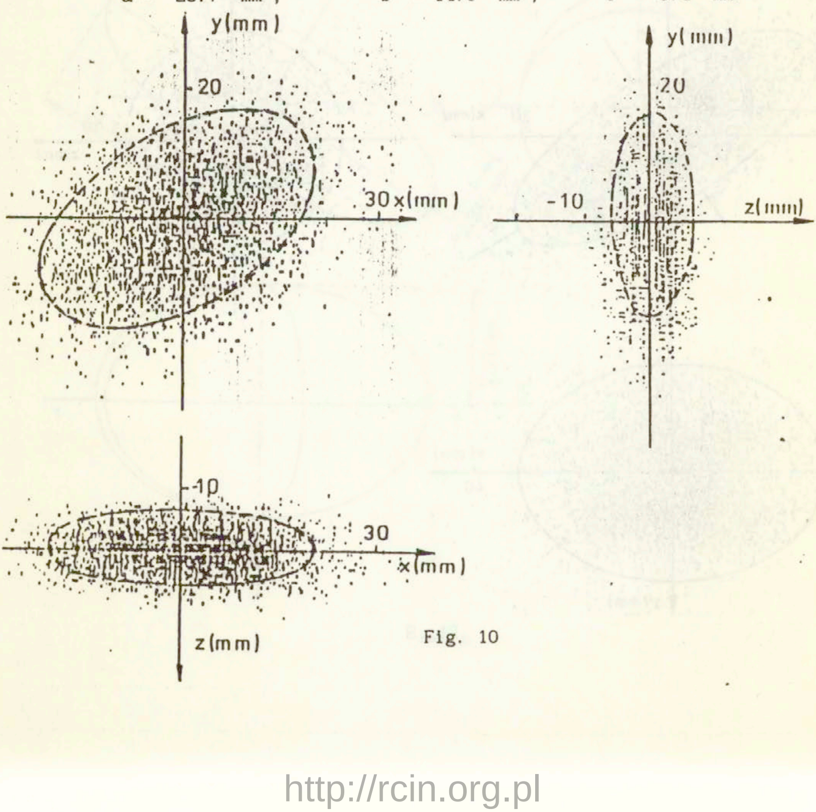

Fig. 10

The principal axis 1 makes the angle $\gamma = 28^{\circ}30'$ with the direction of the x-axis.

As in the Example 1 theoretical ellipsoid has been compared with the results of a numerical experiment analogous to that described above. Random positioning errors of the hand calculated by the computer are shown in Fig. 10 together with the projections of the theoretical ellipsoid.

4. Concluding remarks

All the examples presented above demonstrate that the theoretical ellipses and ellipsoids of the probability concentration calculated with the use of the proposed simple approximate method coincide well with the results of numerical simulation. Thus this approximate method seems to be of practical significance for the analysis of the positioning accuracy of manipulators. The examples show moreover how strongly the positioning accuracy depends on the particular position in which the manipulator is operating.

Acknowledgement: *The authors gratefully acknowlege the support of the Polish Commitee of Scientific Research under grant No.706749101.*

References

1. R.P. Paul. Robot Manipulators, Mathematics, Programming and Control, MIT Press. Cambridge, Mass., 1982

2. A. Kumar and K.J. Waldron, Numerical plotting of surfaces of positioning accuracy of manipulators, Mech. Mach. Theory, vol. 16, No. 4. 361-368, 1981

3. A. Antshev. B. Bekiarov and L. Litov, Accuracy characteristics of mechanical systems with tree-like structure. Advances in Mechanics, (in Russian) , vol 11. No. 1/2, 11-41, 1988

4. G Cramer, Mathematical Methods of Statistics, Princeton University Press, 1946

5. W. Szczepiński, On a certain method of determininig tolerance fields, (in Polish), Archiwum Budowy Maszyn, vol. 2 ,No 3, 275-284, 1955

6. W. Szczepiński, Theory of polyhedrons of positioning accuracy of manipulators, Mech. Mach. Theory, vol. 26, No. 7, 697-709, 1991