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Krzysztof Podolak

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A SURVEY

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Krzysztof Podolak
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SYSTEM IDENTIFICATION IN STRUCTURAL MECHANICS - A SURVEY

Introduction

Structural engineering basically has as its primary purpose the design of structure to a predictable level of performance. To achieve this goal some serviceable tools are developed in a framework of structural mechanics. Among them a pertinent analysis of the structure, starting with a preliminary design and expected loads plays the fundamental role. In structural analysis it is necessary to make use of abstract and mathematical models. In reality an "a priori" mathematical representation remains to be idealized and only approximates the behavioural characteristics of extremly complex structures in the real world.

The mathematical models of the structure response to mechanical action are usually formulated in terms of differential equations /ordinary or partial ones/. An information in a form of boundary and/or initial conditions is required to ensure a unique solution of mentioned differential equations. Functions satisfying differential equations together with the proper boundary and/or initial conditions result from a solution of the direct initial-boundary value problem in mathematical physics. The solution constitutes the basis for the further structural analysis. Some questions concerning the relations which connect structural mechanics with mathematical physics are discussed in [51].

The accuracy of the results of engineering analysis in prediction of the behaviour of the actual structure depends on two fundamental factors: the mathematical model of the structure and the computational algorithm for the solution of the initial-boundary value problem of structural mechanics.

The complex geometry, loading conditions and material properties characterizing the majority of the modern structures cause the analytical methods to be insufficient for solving mentioned problems.

In this situation the numerical methods are widely applied. Recently an intensive development of these methods takes place. They now permit to solve most linear and many nonlinear problems of structural mechanics with accuracy acceptable for practical purposes.

On the other hand the constitutive theory of material behaviour is much advanced. It gives the possibility of the description of mechanical properties for the broad class of materials used in engineering practice. But the fundamental question is that the validity of a material model can only be verified on the basis of suitable experiments.

Unfortunately, owing to limited capability of doing measurement the majority of contemporary experimental techniques is designed to the specimen configurations in which the stress state is statically determinate. In addition the specimens commonly utilized like slender rods and thin-walled tubes are unsuitable in examining the mechanical behaviour of materials which exhibit nonhomogeneous and anisotropic properties /for instance composities, fabricates/.

In the case of rock-like materials, often it is impossible to construct a suitable laboratory specimens in order to obtain the information needed for the prediction of rock-mass behaviour.

The substancial difficulties arise when nonlinear material properties are to be described. Then a large number of complex, multiaxial deformation states should be produced and observed in the laboratory to determine the constitutive function /model/ at least over the part of its domain of technical interest. Unfortunately, it is practically impossible to realize an experimental program which furnishes the information for the complete description of the nonlinear material properties.

The above remarks show that in many technologically important situations the results of the laboratory experiments are insufficient for the construction of the material models. In this situation, the main reason why discrepancies between theoretically predicted and actual structure behaviour often arise is unsuitable choice of a model rather than approximate method of the solution of the initial-boundary value problem of structural mechanics. Then the pressing need arises for material characterization techniques which are compatible in accuracy with the currently employed analytical methods.

One of the possible ways to achieve this goal consists in utilizing a measured response of the actual structure to a given excitation. These data are used either to produce the best functional representation of the considered structure, if its model is totally unknown or to determine values of some parameters, if a preliminary model of the structure is assumed. The determination of structural models in both mentioned situations needs suitable procedures which are developed in system identification framework. The results arising from system identification can be applied equally as in the design of structures so in the inspection of structures.

To facilitate a better understanding of listed above remarks a study from [42] as an illustrative example is outlined below. It presents an application of system identification to structural mechanics purposes. The aim of the study is to develope a material characterization technique useful for such specimens /systems/ in which statically indeterminate stress field arises. Some knowledge of a material model is required to utilize this technique. The study is restricted to linear elastic, anisotropic material in a state of plane stress. In an "a priori" model of the system only values of parameters are unknown. The model is used to formulate a boundary value problem corresponding to the experiment. The solution of this problem based on a finite element method requires an initial set of parameter values to be chosen. Experimental data are compared with those following from the numerical solution of the boundary value problem. A least squares type criterion is applied to make a comparison. An application of the criterion yields a new set of parameter values. It can be used to formulate a next boundary value problem. In such a way a trial and error process can be expressed as a closed-loop sequence of computational steps. This process continues until the numerical and experimental results agree to some predetrmined accuracy. In the described procedure both strain and displacement measurements can be utilized. The application of the procedure requires its convergence to be studied.

The notion of system identification originated from electric circuit theory was then widely developed in the framework of control theory and in the so-called "system science". Until now a large numer of papers have been written on the subject. A series of survey papers [1, 3, 12, 30] and several books [28, 31, 41, 48-50, 59, 80] have been published. At the same time many conferences were performed to present and discuss applied and fundamental papers on system identification /cf [55,58] /. The questions of the teaching of identification at academic high schools were also discussed [63] . The first papers devoted to system identification problems in structural mechanics appeared at the end of sixties /for instance [44, 56] /. The objective of these papers was to determine the characteristic parameters for linear elastic vibrating systems with viscous damping.

Thereafter the system identification was applied to various engineering problems of structural mechanics.

Some of them are listed below.

As a first example the determination of the model for structural systems undergoing seismic effects can be mentioned. Both, linear [35] and nonlinear [22] properties of structural systems are considered. The resulting models can then be used for various purposes, both in prediction of the behaviour of existing structures and aseismic design of the structures.

The paper [51] discusses in a general manner the applicability of system identification to material characterization purposes. In [42] material constants for an anisotropic, linear elastic body in a plane stress state are determined. A model describing a nonlinear elastic behaviour of rubber-like materials is considered in [37] . The paper [69] treats the designing of metal working machines. A useful procedure for testing and inspection of actual, concrete dam is developed in [10] . A brief review of some methods for model structure testing is presented in [60] . Some questions related to biomechanics, such as formulation of phenomenological models of processes occuring in living systems are studied in [54] .

A term of structural identification was introduced in a review of literature [57] for the problems referred to the application of methods in system identification to structural engineering.

This paper presents a review of the work done in this field during the last twelve years. The reference given here are, it is hoped, representative. However, the list of papers is by no means complete and is only a personal search of accessible literature.

The arrangement of the paper is the following. In Section 2 the notion of the system identification is defined and some ways of the classification of the problems referred to this subject are mentioned, In Section 3 the papers concerning the identification of quasistatic and static systems are reviewed. Section 4 is devoted to the discussion of the publications referred to the identification of dynamic systems. In Section 5 three mathematical methods are treated, which are applied in system identification. Needs for future research are discussed in Section 6. The final Section 7 presents some conclusions arising from the reviewed material.

2. Formulation and classification of system identification problems

A commonly accepted definition of the system identification in control theory as well as in structural mechanics /cf [1, 53, 66] / was proposed in [71] and it has the following form: "Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent".

In relation to structural identification "system" in the above definition means a model of the structure which reflects main physical and geometrical features of this structure. The system accepts inputs/excitation, loading/ and produces outputs /responses/. Meaning of "a class of systems" is twofold and it depends on a type of identification process, i.e., without any knowledge of a model or with an assumed model. In a first case a certain class of functionals and in a second one an "a priori" model with some unknown parameters is considered. Then, an experiment with the actual system should be carried out and the system response /output/ to suitable loading /input/ measured.

An important item is the notion of equivalence linking theory and experiment in every system identification procedure [80] . It is usually formulated in terms of a criterion and it takes the form of a functional with respect to the actual system response and the model response.

Below the classification of the system identification problems in structural mechanics is represented from the "class of models" standpoint. The models can be characterized using nonparametric representation or in parametric form /see, for example [19] /.

In the case of nonparametric system identification no assumptions are made about the structure of the model. It is often called the input-output "black-box" model of a system. The ultimate objective of the identification process is to predict the response of the considered system. As a result an overall description of the complete structure in terms of certain functions like impulse response, transfer functions etc. is obtained. For instance, in [47,66] the response of structural systems to strong earthquake ground shaking is considered on the basis of nonparametric system identification. Wiener theory of nonlinear system characterization is applied. Both, linear and nonlinear behaviour of the structures are taken into account.

Often some "a priori" information on the system permit formulation of its mathematical model in which only some parameters are unknown. The system response is assumed to depend on these parameters. In this situation a parametric system identification problem /PSIP/ can be formulated.

In view of the scope of this paper PSIP are presented below in a more detailed form. Likewise the general formulation, the PSIP, in accordance with [71], is characterized by the following three basic elements: a class of models, a class of inputs and a criterion /see also [53,66]/.

The formulation of the model structure as well as its parametrization is greatly intuitive and usually based on experience with the behaviour of actual system. The choice of the model remarkably affects the computational effort required by identification procedure, the possibility to get unique solution etc. An analysis of the theoretical "a priori" model is helpful in planning an experiment for identification purposes /cf [35] /.

As for as the class of inputs are concerned, it should be emphasized that in order to be able to identify particular effects, one must apply inputs that involve these effects in the system to identify. For instance, if the plastic model of the system is identified, the applied loading conditions must cause plastic flow in this system. Further comments on the role of a class of inputs in system identification one can find, for instance, in [1,53] .

As it was mentioned above the criterion function is closely related to the notion of the system equivalence. It should be constructed such that a good fit of the model response to the actual system response is obtained when the function is minimized as a function of the unknown model parameters. Typically, the criterion function has the form:

/2.1/ $$e(t;\mathbf{p}) = \left\| \mathbf{y}(t) - \mathbf{y}_m(t;\mathbf{p}) \right\|$$

where $\mathbf{y}(t)$, $\mathbf{y}_m(t;\mathbf{p})$ are the actual /measured/ system response and the model response, respectively, and $\mathbf{p}$ is a vector of unknown model parameters. A norm $\|\cdot\|$ must be chosen. Then the criterion function is minimized as a function of model parameters. By some minimization procedure a vector $\mathbf{p}_0$ is selected, which satisfies the condition

/2.2/ $$e(t;\mathbf{p}_0) = \min_{\mathbf{p}} [e(t;\mathbf{p})] .$$

Mostly, the criterion function is expressed in a form

/2.3/ $$e(t;\mathbf{p}) = \int_0^t [\mathbf{y}(\theta) - \mathbf{y}_m(\theta;\mathbf{p})]^T \mathbf{W} [\mathbf{y}(\theta) - \mathbf{y}_m(\theta;\mathbf{p})] d\theta$$

where $\mathbf{W}$ is a positive, weighting matrix to be specified.

The minimization of the expression /2.3/ can be interpreted as a least square criterion for error e.

Relations /2.2/, /2.3/ define an optimization problem which can be solved applying algorithms described for instance in [3,28] .

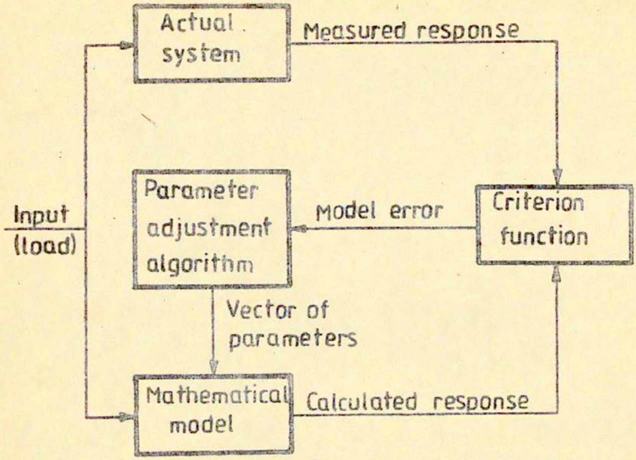

Fig.1. The diagram of the parametric system identification problem

The idea of parametric system identification problem can be represented in the simple flow chart form shown in Fig.1. As one can see there, the actual system and its mathematical model are subjected to the same input. The system output is measured and the model output is calculated as a function of the values of model parameters. The model error determined in the sense of the criretion function is then used in a parameter-adjusting algorithm in order to select the parameter vector that minimizes the criterion function.

System identification problems may be subdivided in several ways and some of these will be mentioned below.

They can be considered in both probabilistic or deterministic framework. In general, these two approaches utilize different methods in identification procedure cf [1] . Then, another kind of classification of system identification problems can be introduced, which divides them into: stochastic system identification problems and deterministic system identification problems.

Following classifications usually applied in mechanics one can consider separately identification problems of dynamical systems and identification problems of quasi-static systems.

For the farther classification of system identification problems the reader is referred to the book [50] related to shock and vibration systems.

3. Identification of quasi-static and static systems

The material characterization is the main objective of the papers concerned with quasi-static and static problems listed in this Section.

A suitable theoretical basis for material characterization in the system identification framework is developed in [51 - 53].

Paper [51] discusses the general question of formulating an initial-boundary value problem of mathematical physics as a model of a structural system. Some mathematical modelling techniques useful in the structural design process are reviewed. An application of PSIP to the material characterization is discussed and its direct relation to numerical solution algorithms of the problems of mathematical physics is emphasized.

In [52] a question of constitutive modelling based on the notion of an isolated material element is discussed as a parametric system identification problem. Some methods for selection of model parameters characterizing isolated material elements are reviewed. It is shown that the constitutive modelling leads to an inverse initial-boundary value problem. A number of significant questions related to the formulation and solution of the inverse problem are discussed. Some examples from the literature, related to both material characterization and structural identification, are presented in order to illustrate the methodology developed in the paper.

The fundamental questions relevant to the material characterization are considered in [53] . The problem of the choice of a constitutive theory of material behaviour is compactly discussed. Material characterization is presented in a form of the PSIP. The paper contains some remarks concerning the effect of changes in model parameters on the value of criterion function as well as the model response. Also the question of compatibility of models with the currently applied computational methods for solution of field problems is briefly discussed.

Apart from the general type papers mentioned above, one can find in the literature a number of publications concerning the particular problems of material characterization in which the system identification methods are utilized.

3.1. Elastic systems

In [43] some computational procedures are proposed for material characterization of elastic solids in a plane stress state. They are derived from the finite element solution of a suitable boundary value problem applying a least square type criterion function. Statically indeterminate test specimens are used to acquire the experimental data. The application of the procedure for determination of material constants characterizing a particular nonlinear elastic material is presented.

The material characterization problem for anisotropic, linear-elastic solids in plane stress state is considered in [42] . Two iterative procedures for material parameters computation based on the least-square error criterion are presented. In the characterization process strains or displacements can be utilized separately as measured quantities. The possibility of incorporating both strain and displacement measurements in the same characterization procedure is mentioned. A comparison of the suggested procedures is presented from the computational efficiency standpoint. The paper also contains a study of the effect of various kinds of errors on accuracy and convergence properties of the characterization procedures.

An approach to the material characterization problem extending the application of finite element concepts to discretization of the constitutive stress function over its domain is presented in [36, 37]. The method introduced there is used to identify the parametrized stress response function /model/ for isotropic, incompressible, nonlinear elastic solid exposed to generalized plane stress state. In the identification procedure a least squares type criterion function is applied. In order to solve a systen of algebraic equations describing identification problem the Newton-Raphson iterative method is employed.

From the solution the unknown material parameters are obtained. Some questions arising from "data corruption" as well as location of measurement points are examined. Data corruption provides a certain kind of data which are used in testing the stability and accuracy of some numerical procedures related to PSIP. They are called "corrupted data" and are produced by introducing an additive noise to data following from a known solution of a suitable initial-boundary value problem. This solution is regarded as a result of a numerical experiment. The additive noise introduced to data simulates possible errors which may appear in an identification procedure.

In [10] an approach to control of actual behaviour of concrete dams is presented. It assumes the elastic behaviour of dam-bedrock system, for prediction of the system response to various loading conditions. Some thermal effects are also included into considerations. The finite element method is applied to calculate the response of the dam. The estimates of structural parameters are derived by using a suitable procedure based on least squares method. The control system built on the basis of this approach was successfully applied to the control "in real time" of actual dams.

The applicability of the system identification to some problems in geotechnical engineering such as rock excavations and construction of embankments is discussed in [40].

In order to predict the overall responses of these geotechnical systems, the elastic model is considered.

Two procedures are suggested for the calculation of the model parameters /elastic moduli/. These are: an iterative least square procedure and a simplex search strategy.

3.2. Inelastic systems

A method for material characterization based on a similar concept to that utilized by system identification is proposed in [27] . The aim of this paper is to characterize physically nonlinear viscoelastic solid undergoing quasi-static deformations. It is emphasized that the material characterization and numerical solution of the suitable boundary value problem should be considered simultaneously. In order to solve the problem an iterative method is proposed. A case of an axially deformed, statically indeterminate nonlinear viscoelastic rod is considered to illustrate the method. An iterative procedure resulting from the quasilinearization is applied in [77] to identify material parameters of nonlinear, viscoelastic solids.

Material characterization of linear viscoelastic solids formulated as parametric system identification problem was first presented in [14] . In order to solve this identification problem a quasilinearization method was used. An application of this method to the identification problem in the theory of thermorheologically simple materials is presented in [21] . The characteristic functions describing material behaviour are determined from experimental data using differential approximation and quasilinearization. The study can be considered as an example of the more general problem of modelling nonlinear constitutive equations. In order to illustrate the feasibility and accuracy of the method a numerical example is presented.

System identification problems for nonlinear viscoelastic materials governed by Volterra integral equations are considered in [15, 25, 26] . In [15] a numerical procedure for material characterization is developed using general input-output measurements. The questions of stability and accuracy of the proposed procedure are tested by numerical experimentation using simulated, corrupted and uncorrupted data. The identification problem in a case with inputs restricted to the form given in terms of one-dimensional step functions, is considered in [25] .

The extended Gauss-Newton iterative procedure is employed to solve the optimization problem formulated on the basis of weighted least squares type of criterion. Experimental data obtained from relaxation tests are utilized in examining the predictive ability of the considered model. A quasilinearization method is proposed in [26] to solve the minimization problem arising from the material characterization procedure. Unlike [25] the generalized inputs are used for identification purposes.

In [40] a quadratic programming approach is suggested to identify the elastic perfectly plastic model representing the behaviour of some geotechnical systems. A piecewise linear approximation of the yield locus in the stress space is used in the identification procedure. The outlined study has only a preliminary character.

Applications of so-called hybrid identification are described in [74] . Some algorithms are suggested in the paper to identify viscoelastic and plastic properties of structures. The basic constitutive relation is taken in a form of regression. The use of the finite element method as a basis for describing the entire identification problem is examined. One can consider the study to be an extension of the results derived in [51] . The paper also presents a variety of possible applications of developed techniques to structural identification.

4. Identification of dynamic systems

The construction of models for the description of the overall structure behaviour constitutes the main purpose of the system identification problems presented in papers reported below. A common feature of the models considered there is that mass is lumped at nodal points which are interconnected by deformable elements. Applying this approach one can represent the actual structure by a model with a finite number of degrees of freedom. The mechanical properties of the deformable elements affect the structural model response.

Mathematical models determined by system identification can be used in engineering analysis of structures, for evaluation of structural safety under various loading conditions or to active control of complex structures in civil engineering.

The book [50] /edited 1972/ presents the current state of system identification with a special emphasis given to structural dynamics. This work collects, compares and analyses various contemporary methods used in system identification, and included details of selected recent investigations as examples. Several parts of the book are concerned with the current state in specific areas of application, such as the large structures, aerospace structures, high speed ground transportation systems, automotive structures and machine tool systems.

A comprehensive review on system identification in structural dynamics up to about 1973 is presented in [57] . The papers described in that survey are not mentioned below.

A further development of identification methods in structural dynamics is reviewed by the authors of [32, 62, 79] Some techniques devoted to identification of the dynamic characteristics of complex structures are presented in [8, 9, 38, 73, 75, 78, 82] .

In view of the form of mathematical models considered in the papers reviewed here the following presentation is divided into two parts.

4.1. Linear models

The modal analysis methods are widely applied in identification problems for linear systems. If the damping matrix satisfies certain conditions, then in the case of systems with a finite number of degrees of freedom the normal modes can be used to uncouple the equations of motion. These uncoupled equations represent the individual modal responses. For each mode the system identification problem can be formulated idepedently. By solving these problems one can identify eigenfrequencies, modal damping, effective masses, stiffness properties and mode shapes.

An application of modal analysis to identification of dynamic properties of one - dimensional chain is presented in [87] ; In [33, 34] several procedures for identification of the eigenparameters are developed utilizing a modal analysis approach. Identification of these parameters proceeds through the minimization of the least square type criterion function. In order to solve the minimization problem, gradient techniques are applied. It is shown that the application of the proposed minimization algorithms leads to a global minimum of criterion function for which the values of model parameters are calculated. Some significant questions related to a number and location of measurement points are discussed. The application od the developed procedures to the identification of actual structures like multistory buildings are studied.

An identification method is developed in [46] to determine natural frequencies, mode shapes and generalized mass, stiffness and damping matrices describing a structural system. The method uses frequency response data at a finite number of discrete frequencies. The response data can be obtained from sinusoidal vibration tests with either base excitation, or single or multipoint excitation.

A generalization of the inverse of a matrix, so-called "pseudoinverse" is presented in [35] . The pseudo-inverse is used to calculate weighted least-mean-square and weighted minimum-norm solutions to ill-conditioned linear systems. It is convenient tool in the identification of vibrating systems, where the number of measurements and forcing points are often different and the confidence in different measurements and models varies. Several mathematical methods based on the pseudo-inverse notion are presented. These are useful in the planning of vibration tests /i.e. in placing vibrators and accelerometers/, analyzing data to identify eigenparamateres, adjusting mass and stiffness matrices and identifying unknown forces acting on a known structure.

The three-storey actual structure is studied to illustrate the utilization of the proposed mathematical methods.

In [11] a numerical method is suggested for identifying the damping coefficients of a linear, multidimensional, vibrating system from its frequency response, which is known over some frequency range. The least square technique is used to construct the objective function, which depends on the difference between the actual system response and the model response. The system identification is then reduced to an optimization problem. The solution to this problem is performed in an iterative manner by processing one frequency point at a time. To illustrate the use of the proposed method some numerical examples involving viscous damping are presented.

A discreet model of a nine story steel frame is identified in [5] . The elements of mass, stiffness and damping matrics are determined from the available information about mode shapes.

The choice of the best objective function, the minimization of the objective function, and the design of the optimum experiments are the problems discussed in [2] . Objective functions in the form of ordinary least squares, maximum likelihood and maximum "a posteriori" /or Bayesian/ estimations yielding from the estimation theory are characterized. The minimization of the objective function by using the linearization is described. A main part of the paper is devoted to a formulation of criteria for the design of optimum experiments. Parallels between system identification and modern design are underlined. The examples of the heat conduction in solids are used to illustrate ideas presented in the paper. Some problems related to model identification of vibrating structures are studied in [81, 83, 85, 86] . Suitable identification procedures are also suggested in these papers.

When the relationship connecting the observed quantities and the unknown model parameters is not linear, then nonlinear least squares can be applied [4] to formulate the identification problem. The considerations presented in [4] are concentrated on the structural dynamics problems, for which the method of nonlinear least squares is discussed from application in time domain analysis, frequency analysis and modal analysis standpoint. An iterative procedure is proposed to compute the corrections to the initial guess of model parameters. The nonlinear least squares estimates obtained at the i-th iteration are compared with those calculated by using the method of ordinary least squares, maximum likelihood and Bayesian estimation.

An attempt is made to investigate the uniqueness of the system identification problem in [65] . A linear two-degree of freedom, damped, vibrating system as a model of a two-storey structure is considered. The stiffness and damping parameters are determined from a knowledge of the base acceleration and the corresponding roof or first storey response records obtained during strong ground shaking. It is assumed that there is no noise in the measurements and there is no soil-structure interactions. The theorems formulated in the paper show that proper locations of the measuring instruments lead to unique solution of the identification problem. The results obtained for the two-storey structure can be extended to an n-story system.

The problem of optimal sensor location for identification using time domain data is studies in [61] . The behaviour of the considered system is described by a set of linear ordinary differential equations. The solution of the problem results from a suitable probabilistic analysis of observation errors. In [68] the uniqueness of the results in identification of dynamic properties of soil and structural systems is considered, N degrees of freedom, lumped system consisting of masses, springs and dampers is used to model an N-story structure or an N-layered soil medium. Only a linear range of the system response is investigated. It is assumed that the mass distribution in both the structural and the soil system is known. Some concepts and the nature of nonuniqueness of identification are illustrated through a numerical example.

There are a few examples of approaches which do not use normal modes but directly identify the abstract parameters in the equations of motion.

Such a nonmodal approach to identification of vibrating structures using response time history was presented in a survey paper [70]. Error-free computer experiments on a simulated spring-mass chain were conducted to calculate improved values of the model parameters.

In [67] a structure is modelled in a form of the continuous shear beam using forced linear differential equation. A computational procedure is developed, based on a penalty function method, to identify a space-dependent stiffness coefficients. The success of the identification process is strongly dependent upon the closeness of the initial guesses to the exact values of the identified quantity, and upon the nature of the ground motions used as the input data.

4.2. Nonlinear models

Under various loading conditions the actual structures exhibit a nonlinear behaviour. Consequently, new mathematical models and different from those mentioned in 4.1 methods yielding the solutions to the system identification problems are required.

The book [76] presents some current methods applied in the identification of the nonlinear vibrating systems. It also contains a comprehensive description of techniques used for linear systems. The identification of nonlinear systems by use of models with one degree of freedom is discussed in [29, 72] . A quasilinearization method is employed in [72] to determine the model parameters.

In the paper [64] the damping forces are determined from known resonance characteristics of the nonlinear system under study. Some practical aspects related to identifiability of nonlinear elastic systems are discussed in [84].

Using a quasilinearization and nonlinear filtering approach the procedures are developed in [18] for the determination of the characteristics of the nonlinear spring. To represent the force-displacement law of the spring a nonlinear integral Volterra equation was used.

Some examples of the application of methods in system identification to construction of models describing behaviour of the structures undergoing seismic excitations are presented in [22-24] . Papers [23, 24] comprise a compact form of the detailed study presented in [22] .

In [23] three different, efficient methods are proposed for the determination of model parameters. These are the following: the direct search, filtering approach and Gauss-Newton method. An extensive experimentation using simulated data is performed. One-degree of freedom, nonlinear model of the viscous type is used to discuss the accuracy and stability of the proposed methods. Only results related to the experiments with impuls of the seismic type are reported in the paper.

It follows from the considerations that the direct search algorithm is computationally very efficient. Direct search does not require an initial estimate for the model parameters, but it requires the unknown parameters to appear linearly in the mathematical model and the measurements to be very accurate. This procedure may be used to provide estimates which can be utilized as the best initial estimates for the Gauss-Newton or filtering procedures. These latter two are exempt from the limitations of the direct method.

In [24] the filtering method is applied to determine the unknown parameters for two different models. Only one-degree of fredom vibrating systems are considered as the structural models. One of them is characterized by a piecewise function of displacements, which represents the restoring forces. This model enables the description of the elasto-plastic behaviour of the structure. Numerical experimentation using simulated data is conducted to demonstrate feasibility and accuracy of the parameter identification algorithm for the above model.

The second model of the viscous type is used to illustrate the suitability of the filtering method in the problem utilizing real data. The parameters determined by the identification procedure are employed to predict the response of the real structure.

An identification problem is considered in [16] to determine a nonlinear behaviour of the structure undergoing the dynamic loading conditions. Some preliminary results on that problem, later developed in [20], are presented. Two nonlinear filtering methods are employed in [20] to determine the unknown parameters of three-degree of freedom mathematical models, which simulate the behaviour of an actual structure undergoing seismic effects. The restoring forces in the models are assumed to be of viscous differential type. To perform the identification procedure, the data measured on a three-storey steel frame tested on a seismic table are used. Numerical experimentation using simulated data is carried out in order to demonstrate the stability of the suggested procedure. Some remarks concerning the identifiability of the considered systems are placed in the paper.

An extensive study is carried out to test the predictive ability of the identified models.

5. Mathematical methods

The diversity of mathematical methods appropriate to the solution of system identification problems is indicated by the quantity of literature devoted to the subject over the past several years. It is not the intent of this paper to review or compare all these methods. To the contrary, only three approaches are chosen to present in this Section. First, the nonlinear filtering method is described. Next, the gradient technique is presented. Finally, the quasilinearization approach is discussed.

5.1. Nonlinear filtering method

The notion of the filter means the system used for the separation of a signal which has been perturbed by the addition of noise or a random process. The problem of determining of the state of such a system from noisy measurements is called filtering.

The filtering methods arise from the estimation theory [13, 17, 41] . The general linear nonstationary filtering problem is essentially completely solved /see [39]/ , and the nonlinear filtering methods are considerably advanced.

When some criterion - evaluating a filter - is involved in a filtering problem, then this latter yields an optimum filter. One possible way of the determination of the optimal nonlinear filter is presented below.

Consider a system described by the set of nonlinear differential equations:

$$\dot{\mathbf{u}} = \mathbf{g}(\mathbf{u}), \tag{5.1}$$

where $\mathbf{u}(t)$ is an augmented state vector which contains the state variables and the model parameters as its components / $\dot{\mathbf{u}}$ denotes time derivative/.

The behaviour of the system is observed in the interval $[0,T]$. A linear form of the observation equation is assumed

$$(5.2)\qquad w = \Gamma u + \eta ,$$

where w is a vector representing the observed evolution of the system, Γ is a rectangular matrix, and η denotes the observation error vector.

The goal is to determine an optimal estimate of the state of the system at time T on the basis of observations carried out on the interval $(0,T)$. The considered procedure requires the minimization of the quadratic error functional $f(u(T),T)$ given by

$$(5.3)\qquad f(u(T),T) = \int_0^T (w-\Gamma u, w-\Gamma u)dt + (u(0)-b, \Lambda(u(0)-b))$$

where b is the best "a priori" estimate of $u(0)$, and Λ is a nonsingular symmetric matrix estabilishing the degree of confidence in such an estimate. The expression (α,β) in /5.3/ denotes the usual inner product $\sum_i \alpha_i \beta_i$ of the vectors α and β.

By taking derivatives in /5.3/ one can construct a partial differential equation for $f(u(T),T)$ in a following form

$$(5.4)\qquad f_T(c,T) = (w-\Gamma c, w-\Gamma c) - (f_c(c,T), g(c)) ,$$

where for simplicity of the notation $u(T) = c$ is introduced. In /5.4/ f_c stands for the gradient of f, i.e.

$$(5.5)\qquad f_c(c,T) = \left\{ \frac{\partial f}{\partial c_1}\ \frac{\partial f}{\partial c_2} \cdots\cdots \frac{\partial f}{\partial c_N} \right\} .$$

The initial condition for $f(c,T)$ is the following

$$(5.6)\qquad f(c,0) = (c-b, \Lambda(c-b)) .$$

If

$$(5.7) \qquad \mathbf{e}(T) = \arg\min_{\mathbf{c}} f(\mathbf{c}, T)$$

denotes the optimal filter, then

$$(5.8) \qquad f_c(\mathbf{e}, T) = 0 .$$

Taking total derivatives in /5.8/ one can obtain a differential equation for the optimal filter $\mathbf{e}(T)$ in the following form

$$(5.9) \qquad \frac{d\mathbf{e}}{dT} = - f_{cc}^{-1}(\mathbf{e}, T)\, f_{cT}(\mathbf{e}, T).$$

An expression for f_{cT} can be derived by differentiation of /5.4/ and then introduced into /5.9/. Such a transformation produces a new form of the Eq. /5.9/

$$(5.10) \qquad \frac{d\mathbf{e}}{dT} = \mathbf{g}(\mathbf{e}) + \mathbf{Q}(T)\,\Gamma^T(\mathbf{w} - \Gamma\mathbf{e}),$$

where the superscript T denotes a matrix transposition, and is the matrix given by

$$(5.11) \qquad \mathbf{Q}(T) = 2 f_{cc}(\mathbf{e}, T).$$

The Eq. / 5.10/ shows that the optimal filter $\mathbf{e}(T)$ is obtained by integrating a slightly modified form of the original system equation / 5.1/ .

As an initial condition, the following one can be used

$$(5.12) \qquad \mathbf{e}(0) = \mathbf{b} .$$

Assuming the function $\mathbf{g}(\mathbf{c})$ to be representable in the form

(5.13) $$g(c) \approx g(e) + g_e(e)(c-e)$$

in the neighbourhood of the optimal filter $e(T)$, it is possible to derive, with the use of /5.4/ - /5.6/, the differential equations for the matrix $Q(T)$

(5.14) $$\frac{dQ}{dT} = g_e(e)Q + Qg_e^T(e) - Q\Gamma^T\Gamma Q$$

with an initial condition

(5.15) $$Q(0) = \Lambda^{-1}$$

Equations /5.10/, /5.12/, /5.14/ and /5.15/, integrated simultaneously, provide the optimal filter $e(T)$ in a sequential manner.

5.2. Gradient method

It was already mentioned that the system identification problem can be formulated as an optimization problem. Once the problem including a relevant objective function is defined, an appropriate optimization technique must be chosen. Since analytical methods cannot be applied in most practical cases, owing to the complexity of the required mathematical manipulations, numerical search techniques should be utilized. These techniques are applied iteratively until no further improvement is possible. The most effective methods of search become available if the objective function derivatives are easily obtained. The resulting gradient methods may attempt to follow the function gradient closely, as in the method of steepest ascent /descent/ [45], or use it in guiding the search as in the conjugate gradient methods.

The various techniques are grouped according to the methods used to determine the two essential features of any search method, i.e., the direction and size of the subsequent step in the iterative procedure.

In the general optimization problem, no one technique can be singled out as the most efficient, and each type must be examined in relation to the problem under study.

In order to illustrate some ideas, which form the basis of the gradient methods, the steepest ascent /descent/ approach is presented below. It is well known that the greatest rate of improvement of the function will be found by moving along the gradient. This is called the direction of steepest ascent, if the maximum is sought. This direction is a local property rather than a global one. It means that a move starting in the direction of steepest ascent will not necessarily end up aligned along the new direction of steepest ascent associated with the new point. In other words, the direction of steepest ascent generally varies from point to point. The locus of infinitely small moves along the line of steepest ascent will generally be a curved line. The described above situation is indicated in Fig.2.

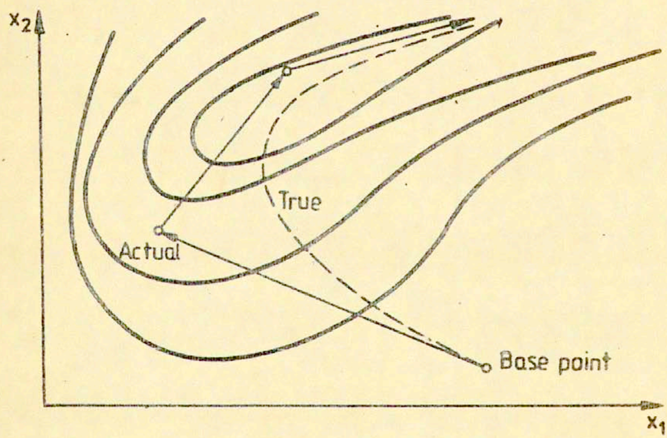

Fig.2. The sketch of the steepest ascent /descent/ sea

Let $f(\mathbf{x})$ be an differentiable objective function defined in n-dimensional space of the variables x_i, where $i = 1,2,\ldots,n$ and $\mathbf{x} = (x_1 x_2 \ldots x_n)$. Denote by S a parameter representing length of the arc of the curve drawn in this space.

The problem is to take a small step ds in such a way that the objective function increases/decreases/ as much as possible. Here

$$(5.16) \qquad (ds)^2 = \sum_{i=1}^{n} (dx_i)^2.$$

The change in $f(x)$ is given by

$$(5.17) \qquad \frac{df}{ds} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right) \frac{dx}{ds} .$$

By maximizing /minimizing/ of $\frac{df}{ds}$ subject to the condition /5.16/ the direction of steepest ascent /descent/ can be determined. To this end a modified objective function should be considered

$$(5.18) \qquad F = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \frac{dx_i}{ds} + \lambda \left[1 - \sum_{i=1}^{n} \left(\frac{dx_i}{ds}\right)^2\right],$$

where λ is a Lagranian multiplier. Differentiating with respect to dx_i/ds yields the necessary condition

$$(5.19) \qquad \frac{dx_j}{ds} = \frac{1}{2\lambda} \frac{\partial f}{\partial x_j} \qquad \text{for } j = 1,2 \ldots n.$$

This result shows that the direction of the steepest ascent /descent/ is a vector parallel to the gradient of the function $f(\mathbf{x})$.

Substituting /5.19/ into the constraint /5.16/ provides

$$(5.20) \qquad 1 = \frac{1}{4\lambda^2} \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2.$$

Solving /5.20/ for λ yields

$$2\lambda = \pm\sqrt{\sum_{j=1}^{n}\left(\frac{\partial f}{\partial x_j}\right)^2} \tag{5.21}$$

By substituting /5.19/ into /5.17/ and applying (5.21) one can obtain

$$\frac{df}{ds} = \pm\sqrt{\sum_{j=1}^{n}\left(\frac{\partial f}{\partial x_j}\right)^2} \tag{5.22}$$

The positive sign gives the direction of the greatest increase in $f(x)$, whereas the negative sign gives the direction of the greatest decrease in $f(x)$

The next problem, at the given base point $x^{(i)}$, is to decide the distance of the move. A line drawn from this point in the direction of the greatest improvement can be represented in a parametric form as

$$x_j^{(i+1)} = x_j^{(i)} \pm \left(\frac{\partial f/\partial x_j}{\sqrt{\sum_{k=1}^{n}\left(\frac{\partial f}{\partial x_k}\right)^2}} \right)^{(i)} \Delta s \,, \tag{5.23}$$

where the superscripts are used to distinguish the quantities calculated in two successive iterations. The positive sign is used for the steepest ascent and the negative one for the steepest descent.

The objective function $f(x)$ can be expressed in terms of the parameter s through /5.23/. Then the value of s the step size Δs can be determined so that f reaches its extremum. This can be adopted as the new base point, and the procedure repeated.

5.3. Quasilinearization

Quasilinearization is a mathematical method devoted to the study of nonlinear boundary value problem. In accordance with this method, the properties of the generally nonlinear function are derived from the properties of the associated family of linear functions /quasilinear representation [7] /.

The origin of quasilinearization lies in the theory of dynamic programming. In particular, an important development of this method is obtained if the considerations are restricted to the study of nonlinear operators exhibiting some kind of convexity /concavity/ because they afford a very convenient quasilinear representation in terms of the maximum (minimum) operation.

Quasilinearization yields a successive approximation procedure which is used to approach the solution of the original nonlinear boundary value problem. This procedure is quadratically convergent /cf [7] / and it requires moderate amounts of computation at each stage.

Possibly the most significant application of quasilinearization lies in its ability to solve large systems of functional equations of various kinds subject to suitable initial and boundary conditions, in a very efficient and accurate manner.

In this Section the application of the method to systems of ordinary differential equations is presented. To this end the following nonlinear system of first-order ordinary differential is considered

$$\frac{du}{dx} = g(u) , \qquad (5.24)$$

where u is the N-dimensional vector with components $u_1(x)$, $u_2(x), \ldots\ldots u_N(x)$ and $g(u)$ is the N-dimensional vector with components $g_i(u_1(x), u_2(x), \ldots, u_N(x))$ $i = 1, 2, \ldots, N$.

The boundary conditions have the form

$$\begin{aligned} u_i(0) &= \alpha_i , & i &= 1,2,\ldots,k \\ u_i(b) &= \beta_i , & i &= k+1, k+2, \ldots, N, \end{aligned} \qquad (5.25)$$

where α_i, β_i, are given scalars.

It should be mentioned here, that there is no difficulty in applying the same general technique to the case where there are nonlinear boundary conditions.

Using the basic ideas of the quasilinearization one can generate a sequence of linear equations

(5.26) $$\frac{d\mathbf{u}^{n+1}}{dx} = \mathbf{g}(\mathbf{u}^n) + \mathbf{J}(\mathbf{u}^n)(\mathbf{u}^{n+1} - \mathbf{u}^n),$$

where $\mathbf{u}^{n+1}$ is subject to boundary conditions

(5.27) $$u_i^{n+1}(0) = \alpha_i, \qquad i = 1,2,\ldots,k,$$
$$u_i^{n+1}(b) = \beta_i, \qquad i = k+1,k+2,\ldots,N,$$

with $\mathbf{u}^0$ an initial guess. The superscripts used in above relations indicate the order of the element in the approximating sequence. Here $\mathbf{J}(\mathbf{u}^n)$ is the Jacobian matrix defined by relation

$$\mathbf{J}(\mathbf{u}^n) = \frac{\partial g_i}{\partial u_j}(u_1^n, u_2^n, \ldots u_N^n) =$$

$$= \left(\frac{\partial g_i}{\partial u_j}\right)\bigg|_{\mathbf{u} = \mathbf{u}^n}, \qquad i,j = 1,2,\ldots,N.$$

The presented iterative scheme is similar to the Newton-Raphson procedure, which is used to find a sequence of approximations to the root of the scalar equation.

In general, linear differential equations /5.26/ must be integrated numerically. Since the problem is now linear, by adding solutions together properly, one can construct a solution of the original problem from the solution of a series of linear boundary value problems.

Quasilinearization offers a unified device to study the properties of the described approximating sequence and to establish appropriate intervals of convergence.

6. Needs for future research

It arises from the present survey, that most of the literature devoted to system identification in structural mechanics deals with the determination of linear systems. Most current testing and measurement techniques do not provide sufficient information to uniquely identify the nonlinear behaviour of actual structures. To avoid further damage of the structure, the forcing quantities applied to structural identification studies are usually of low magnitude. Consequently, the resulting mathematical representations of the structure are only useful in calculations involving the linear or slightly nonlinear behaviour of this structure. Therefore, more studies are needed in the subject area of nonlinear structural identification.

It is also known that structural behaviour under extreme loading conditions is nonlinear as well as inelastic. So, needs exist for a further development of identification procedures for inelastic systems.
Moreover, the possible applications of the existing methodology for system identification to determination of other structural characteristics, such as the damage state and reliability measures, should be investigated.

One of the important problems which appear, especially, when civil engineering structures are studied is to ascertain the behavioural characteristics of actual structures under the actual conditions to which they will be subjected during their lifetime. This is the main reason for the further increase of the experience in conducting of so-called full-scale tests.

System identification clearly belongs to the class of inverse problems and as such very often leads to ill-posed formulations. The way to transform these formulations into well-posed problems is by the incorporation of appropriate additional information on the functions or functionals to be determined. A proper selection of the above information from system identification standpoint, as well as studies referred to uniqueness and stability of the suitable initial-boundary value problem constitute the object of identifiability. Much work remains to be done to explore all important questions arising from this domain [6] .

The accuracy of the results of any identification procedure is degraded by a combination of measurement, modelling and numerical errors. For that reason the further development of methods of error analysis referred to system identification is required.

In view of numbers of different methods and techniques utilized in the existing literature on system identification it seems to be highly desirable to provide a solid methodological framework to this theory.

It is impossible to mention here all the needs arising from the accessible literature on structural identification. These listed above are, in the author's opinion, the most significant in future research referred to this subject. Many more suggestions are contained in the papers cited here and their references.

7. Conclusions

It is evident from the scope of the examples presented herein that the structural identification is a field in rapid development. A large number of different methods is suggested and analysed in the reported papers. Nevertheless, it is resonable to state that no one technique is superior for system identification under all situations. Currently, structural identification does not present a unified field. The reasons for this are obvious, since identification problems belong to the class of inverse problems of mathematical physics. It is well known from the existing literature that theoretical foundations of inverse problems are considerably less developed than those in the case of direct problems. It should be emphasized that only a small class of carefully chosen inverse problems is amenable to a complete analytical treatment.

In general, difficulties in solving of inverse problems arise from their intrinsic unstable nature. Then, one must expect, if any identification problem is considered, that even small errors in input data will produce arbitrarily large errors in the identified quantity. The following errors can appear in any real problem: errors in the model, measurement errors in taking data, errors associated with a method used to solve the problem for instance, discretization in space and in time . The above remarks show that the sensitivity analysis should be considered as an important component of the identification problem.

Some results following from the literature point out that the best safeguard against poor results due to errors in taking data is to have a number of redundant measurements. In other words, the dimension of the data vector should be much larger than that of the parameter vector. Owing to this situation it is possible to "filter" the noise in the data by using the least squares error method.

What is more, one can recognize the possibility that either the test data or the structural model may be random variables. Then, the original deterministic identification problem must be modified by imbedding in a probabilistic framework. Such a formulation makes it possible to exploit the tools of estimation and decision theory. Furthermore, the probabilistic representation of some identification problems seems to be more adequate than deterministic description.

As it was already emphasized, both the experiment and the analysis of the initial-boundary value problem are closely connected in any identification procedure. Because of the complexity of the problems under study, an increasing use of advanced computational methods in structural identification is observed. This involves questions of a selection of the best numerical algorithm for every identification problem. Often numerical experimentation using simulated data is utilized to show the feasibility of the suggested method.

As pointed out earlier, substantial difficulties appear when the nonlinear or inelastic model of the structure is identified. For instance, in the case of characterization of nonlinear elastic materials any extrapolation of the results to different ranges of strains from those used in the identification process may be not valid.

Similarly, essential difficulties arise, if the actual structures behaviour under extraordinary loading conditions must be studied using the data acquired from full-scale tests.

Experience to date indicates one of the most striking fact that relatively simple mathematical relations and only modest number of parameters are required to produce excellent models of rather complex structural behaviour.

The criterion which is probably most used for the structural identification purposes is the integral form of the least squared error, because it normally leads to smoother error surfaces and hence rapid convergence of computations.

References

1 K.J.ASTROM, P.EYKHOFF, "System identification - a survey", Automatica, 7, 1971, pp.123-162.

2 J.V.BECK, "Design aspects of parameter estimation", in "Basic Questions of Design Theory", W.R.SPILIERS /ed./, North Holland Publ. Comp. /American Elsevier Publ. Comp./, Amsterdam-Oxford-New York, 1974, pp.183-201.

3. G.A. BEKEY, "System identification - An introduction and a survey" Simulation, 15/4/, 1970 pp.151-166.

4 J.G.BÉLIVEAU, "The role of system identification in design", in "Basic Questions of Design Theory", W.R.SPILIERS /ed./, North-Holland Publ. Comp. /American Elsevier Publ. Comp./, Amsterdam-Oxford-New York, 1974, pp. 21-37.

5 J.G.BÉLIVEAU, "Identification of viscous damping structures from modal information", Trans. ASME, J. Appl. Mech., No 2, E43, 1976, pp. 335-339.

6 R.BELLMAN, K.J.ASTROM, "On structural identifiability", Math. Biosci., 1, 1969, pp. 329-339.

7 R.BELLMAN, R.KALABA, "Quasilinearization and Nonlinear Boundary-Value Problems", American Elsevier Publ. Comp. Inc., New York, 1965.

8 H.BERGER, J.P.CHAQUIN, "Identification of models describing a vibratory motion of complex structures" /in French/, Rech. Aerosp., No 4, 1979, pp. 283-285.

9 J.BESSROUR, J.M.SEMICOURT, J.L.TEBEC, "Identfication of multidimensional linear systems by use of an experimentally determined apparent mass" /in French/, Méc. Mater. Elec., No 367-368, 1980, pp. 228-237.

10 P.BONALDI, A.Di MONACO, M.FANELLI, G.GIUSEPPETTI, R.RICCIONI, "Concrete dam problems: an outline of the role, potential and limitations of numerical analysis", in "Criteria and Assumptions for Numerical Analysis of Dams", /Naylor,D.J., Stagg,K.G., Zienkiewicz,C.C., eds./ - Proc. Int. Symp. held in Swansea, U.K., 8-11 September, 1975.

11 P.CARAVANI, W.T.THOMSON, "Identification of damping coefficients in multidimensional linear systems", J.Appl. Mech., 41, 1974, pp. 379-382.

12 M.CUENOD, A.P.SAGE, "Comparison of some methods used for process identification", Automatica, 4, 1968, pp. 235-269,

13 R.DEUTSCH, "Estimation Theory", Prentice-Hall, Inc., Englewood Cliffs, N.I., 1965.

14 N.DISTÉFANO, "On the identification problem in linear viscoelasticity", ZAMM, 50, 1970, pp. 683-690.

15 N.DISTÉFANO, "Some numerical aspects in the identification of a class of nonlinear viscoelastic materials", ZAMM, 52, 1972, pp. 389-395.

16 N.DISTÉFANO, "Preliminary results on the identification problem in nonlinear structural dynamics", Proc. of the Third UCEER Conf. in Ann Arbor, Michigan, May 1974.

17 N.DISTÉFANO, "Nonlinear Processes in Engineering", Academic Press, New York and London, 1974.

18 N.DISTÉFANO, "On the identification of a nonlinear viscoelastic spring under dynamic conditions", Anniversary Volume dedicated to Prof. Y.N. Rabotnov, February 1975.

19 N.DISTÉFANO, D.NAGY, "Parametric and adaptive non-parametric system identification procedures in structural mechanics", Proc. 1971 Summer Comput. Simulation Conf., Boston, Massachusetts, I, 1971, pp. 688-695.

20 N.DISTÉFANO, B.PENA-PARDO, "System identification of frames under seismic loads", J. of the Engin. Mech. Div., Proc. ASCE, Vol. 102, No EM2, April 1976.

21 N.DISTÉFANO, K.PISTER, "On the identification problem for thermorheologically simple materials", Acta Mechanica, 13, 1972, pp. 179-190.

22 N.DISTÉFANO, A.RATH, "Modeling and identification in nonlinear structural dynamics - I. One degree of freedom models", Rep. No EERC 74-15, College of Engineering, University of California, Berkeley, California, December 1974.

23 N.DISTÉFANO, A.RATH, "System identification in nonlinear structural seismic dynamics", Comp. Meth. in Appl, Mech. and Engin., 5, 1975, pp. 353-372.

24 N.DISTÉFANO, A.RATH, "Sequential identification of hysteretic and viscous models in structural seismic dynamics", Comp. Meth. in Appl. Mech. and Engin., 6, 1975, pp. 219-232.

25 N.DISTÉFANO, R.TODESCHINI, "Modeling, identification and prediction of a class of nonlinear viscoelastic materials /I/", Int. J.Solids Struct., 9, 1973, pp. 805-818.

26 N.DISTÉFANO, R.TODESCHINI, "Modeling, identification and prediction of a class of nonlinear viscoelastic materials /II/", Int. J. Solids Struct., 9, 1973, pp.1431-1438.

27 R.DONG, K.PISTER, R.DUNHAM, "Mechanical characterization of nonlinear viscoelastic solids for iterative solution of boundary value problems", Acta Mechanica, 9, 1970, pp. 36-48.

28 P.EYKHOFF, "System identification - Parameter and State Estimation" John Wiley and Sons, Inc., New York, N.Y., 1974.

29 V.GIBA, J.HERGOTT, D.LACIAK, "Identification of nonlinear mechanical system with one degree of freedom" (in Slovak), Strojnicky Čas., No 5, 29, 1978, pp. 521-522.

30 R.GOODSON, M.POLIS /eds./, "Identification of Parameters in Distributed Systems", Proc. of 1974 Joint Automatic Control Conf., Austin, Texas, Amer. Soc. of Mech., Engrs, New York 1974.

31 D.GRAUPE, "Identification of Systems", Robert E. Krieger Publ. Comp., Huntington, New York, 1976.

32 G.S.HART, J.T.P.YAO, "System identification in structural dynamics", J.Engng. Mech. Div., Proc. ASCE, No EM6, Vol. 103, Dec. 1977, pp. 1089-1104.

33 P.IBAÑEZ, "Identification of dynamic structural models from experimental data", Engin. Rep. UCLA-ENG-7225, School of Engineering and Applied Science, University of California, Los Angeles, March 1972.

34 P.IBAÑEZ, "Identification of dynamic parameters of linear and nonlinear structural models from experimental data", Nucl. Engin. and Design, 25, 1973, pp. 30-41.

35 P.IBAÑEZ, "Methods for the identification of dynamic parameters of mathematical structural models from experimental data", Nucl. Engin. and Design, 27, 1974, pp. 209-219.

36 R.H.IDING, "Identification of nonlinear materials by finite element methods", Ph. D. Dissertation, Graduate Division, Univ. of California, Berkeley 1973.

37 R.IDING, K.PISTER, R.TAYLOR, "Identification of nonlinear elastic solids by a finite element method", Comp. Meth. in Appl. Mech. and Engin., 4, 1974, pp. 121-142.

38 K.D.JAHN, "Experimental identification of vibrating mechanical subsystems for the simulation of the vibra-

tion characteristics of a complex system" /in German/, Feinwerk-techn. Messtechn., No 2, Vol. 86, 1978, pp. 101-105.

39 A.H. JAZWIŃSKI, "Stochastic Processes and Filtering Theory", Academic Press, New York and London, 1970.

40 L. JURINA, G. MAIER, K. PODOLAK, "On model identification problems in rock mechanics", presented at the Third National Congress of the AIMETA, Cagliari, 13-16 October, 1976.

41 H.H. KAGIWADA, "System Identification. Methods and Applications", Addison-Wesley Publ. Comp., Inc., London-Amsterdam-...-Tokyo, 1974.

42 K.T. KAVANAGH, "Experiment versus analysis; computational techniques for the description of static material response", Int. J. for Num. Meth. in Engin., 5, 1973, pp. 503-515.

43 K. KAVANAGH, R. CLOUGH, "Finite element applications in the characterization of elastic solids", Int. J. Solids Struct., 7, 1971, pp. 11-23.

44 F. KOZIN, C.H. KOZIN, "A moment technique for system parameter identification", Shock and Vibration Bulletin, No 38, 1968, pp. 119-131.

45 G. LEITMANN /ed./, "Optimization Techniques", Academic Press., Inc., New York 1962.

46 M. LINK, A. VOLLAN, "Identification of structural system parameters from dynamic response data", Z. Flugwiss. und Weltraumforsch., No 3, Vol. 2, 1978, pp. 165-174.

47 P.Z. MARMARELIS, F.E. UDWADIA, "The identification of building structural systems. II. The nonlinear case", Bull. of the Seism. Soc. of Amer., 66, No 1, 1976, pp. 153-171.

48 R.K. MEHRA, D.G. LAINIOTIS /eds./, "System Identification. Advances and Case Studies", Academic Press, New York, San Francisco, London, 1976.

49 G.A. PHILIPSON, "Identification of Distributed Systems", American Elsevier Publ., Comp. Inc., 1971.

50 W.D. PILKEY, R. COHEN /eds./, "System Identification of Vibrating Structures: Mathematical Models from Test Data", 1972 Winter Annual Meeting of the ASME, New York, November 1972.

51 K.PISTER, "Mathematical modeling for structural analysis and design", Nucl. Engin. and Design, 18, 1972, pp. 353-375.

52 K.PISTER, "Constitutive modeling and numerical solution on field problems", Nucl. Engin. and Design., 28, 1974, pp. 137-146.

53 K.PISTER, "Some thoughts on the material identification problem", /to be pusblished/.

54 K.PISTER, N.DISTÉFANO, "On some modeling and identification problems in biomechanics", J. of Biomedical Systems, Vol. 1, No 2, 1970, pp. 32-47.

55 N.S.RAJBMAN, /ed./, "Identification and system parameter estimation", Part: 1, 2, 3, Proc. of the Fourth IFAC Symposium, Tbilisi, September 21-27, North-Holland Publ. Comp., Amsterdam-New York-Oxford, 1978.

56 J.P.RANEY, "Identification of complex structures using near resonance testing", Shock and Vibration Bulletin, No 38, 1968, pp. 23-32.

57 R.RODEMAN, J.T.P.YAO, "Structural identification - Literature review", Techn. Rep. No CE-STR-73-3, School of Civil Engineering, Purdue University, West Lafayette, Indiana, December 1973.

58 A.RUBERTI, /ed./, "Distributed parameter systems: Modelling and identification", Proc. of the IFIP Working Conf., Rome, Italy, June 21-24, 1976, Berlin, Springer-Verlag 1978.

59 A.SAGE, J.MELSA, "System Identification", Academic Press, 1971.

60 T.SODERSTROM, "On model structure testing in system identification", Int. J. of Control, No 1, Vol. 28, July 1977, pp. 1-18.

61 P.C.SHAH, F.E.UDWADIA, "A methodology for optimal sensor locations for identification of dynamic systems", Trans. ASME, J. Appl. Mech., No 1, Vol. 45, March 1978, pp. 188-196.

62 SHICH-CHI LIU, J.T.P.YAO, "Structural identification concept", J.Struct. Div., Proc. ASCE, No ST12, Vol. 104, Dec. 1978, pp. 1845-1858.

63 S.THOMPSON, "An introduction to identification", Proc. 2nd Brit. Conf. Teach. Vibr. and Noise, Sheffield 1977, pp. 233-244.

64 A.TONDL, "Some properties of nonlinear system characteristics and their application to damping identification", Acta Technica ČSAV, No 2, Vol.18, 1973, pp.166--179.

65 F.E.UDWADIA, "On some unicity problems in building systems identification from strong motion records", Proc. Fifth European Conf. on Earthquake Engineering, Istanbul 1975.

66 F.E.UDWADIA, P.MARMARELIS, "The identification of building structural systems. I. The linear case", Bull. of the Seism. Soc. of Amer., 66, No 1, 1976, pp. 125-151.

67 F.E.UDWADIA, P.C.SHAH, "Identification of structures through records obtained during strong ground motion", presented at the ASME Design Engineering Technical Conference, held in Washington, 17-19 September, 1975.

68 F.E.UDWADIA, D.K.SHARMA, P.C.SHAH, "Uniqueness of damping and stiffness distributions in the identification of soil and structural systems", Trans. ASME, J.Appl. Mech., No 1, Vol. 45, March 1978, pp. 181-187.

69 J.WICHER, "Identification problems of technical systems, especially related to mechanical systems", IFTR Reports, No 67, 1975 /in Polish/.

70 J.YOUNG, F.ON, "Mathematical modeling via direct use of vibration data", SAE Paper 690615, Los Angeles, California, October 1969.

71 L.A.ZADECH, "From circuit theory to system theory", Proc. IRE, 50, 1962, pp. 856-865.

72 G.A.ZUPP, Jr., S.B.CHILDS, "Applications of quasilinearization theory to system identification", NASA Technical Note, D-5300, July 1969.

73 Л.Ю.АКИНФИЕВА, "Об одной задаче идентификации динамических моделей механических колебательных систем", Пробл. мех. управ. движ.,Пермь, 1980, 3-10.

74 Я.ВАЛЕНТА, Н.МАТОУШЕК, И.ПЛУНДРОВА, М.ШАТРА, "Использование метода конечных элементов при идентификации упруго-пластического ответа тел", Успехи механики, вып. 2, т.2, 1979, 101-144.

75 И.Н.КАРАБАН, В.Н.КАРАБАН, Э.Г.ЧАЙКА, "Идентификация многомассовых механических систем", Теория механ. и машин. Респ. межвед. темат. науч.-техн.Сб.,вып. 20, 1976, 150-154.

76 В.О.КОНОНЕНКО, Н.П.ПЛАХТИЕНКО, "Методы идентификации механических нелинейных колебательных систем", Наук. думка, Киев, 1976.

77 Г.Х.МУРЗАХАНОВ, "Идентификация реологических характеристик нелинейных, нестабильных вязкоупругих сред", Тр. моск. энерг. ин-та, вып. 280, 1976, 72-76.

78 В.Д.ПЕТРОВ, "Идентификация сложных систем на основе анализа подсистем", в сб. "Динамика и прочность упруг. и гидроупруг. систем", Наука, Москва 1975, 12-17.

79 Л.Пуст, "Идентификация динамических систем", Успехи механики, вып. 1/2, т.1, 1978, 183-205.

80 Н.С.РАЙБМАН, "Что такое идентификация", Наука, Москва, 1970.

81 С.Ф.РЕДЬКО, "Идентификация жесткостей механических систем по собственным частотам", Колебания, прочность и устойчивость слож. мех. систем, Киев 1979, 46-49.

82 С.Ф.РЕДЬКО, В.П.ЯКОВЛЕВ, "Способ декомпозиции задачи идентификации механических систем", Колебания, прочность и устойчивость слож. мех. систем, Киев 1979, 49-57.

83 С.Ф.РЕДЬКО, Ю.В.КРЕМЕНТУЛО, В.Ф.УШКАЛОВ, В.П.ШАБЕЛЬСКИЙ, В.П.ЯКОВЛЕВ, "Определение некоторых параметров механических систем с использованием функций чувствительности", Сложн. системы упр., Киев, 1978, 82-89.

84 Е.С.СТАНЧЕВ, "Идентификация параметров нелинейно-упругих механических систем" /болг./, Теор. и прилож. мех.: 3-й Нац. Конгр.-Варна 1977, София 1977, 133-138.

85 Д.И.ТЕЛИЗЕ, "Параметрическая идентификация линейных динамических систем", Сообщ. АН Груз. ССР, вып. 1, т. 92, 1978, 157-160.

86 В.А.ТРЕГУБОВ, "Экспериментально-расчетный метод динамической идентификации объекта при колебательных движениях", Материалы науч. техн. конф. (С.Горн. Машин). Кривой Рог 1979, 12-22.

87 В.Ф.УШКАЛОВ, "Идентификация параметров многомассовой модели одномерной системы", в Сб. "Нагруженность, колебания и прочность слож. мех. систем", Наук. думка, Киев, 1977, 37-43.

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