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Some concepts of distributive justice in bargaining problems

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Abstract

In this paper we study the problems associated with distributive justice in an abstract framework originally conceived for the analysis of social choice and bargaining problems. Induced social choice correspondences are derived by considering alternatives which are invariant under permutations of the status-quo point. We study in particular the fairness correspondence and a generalized Walrasian bargainig solution and establish links between the two concepts. The analysis in this paper can proceed far beyond where our paper ends.

Key words

Distributive Justice, Bargaining Problems, Fairness, Solution, Social Choice Correspondence.

1. Introduction

The objective of this paper is to study some problems and concepts of distributive justice in a framework of analysis oryginally conceived for the theory of social choice and bargaining. Distributive justice has traditionally been studied in the context of an exchange economy with or without production. The preferences of the agents have usually been defined on their consumptions bundles. In this paper we attempt to release this theory from it narrows confines, and explore possibilities in an abstract setting. However, to make meaningful statements some restrictions have to be imposed. Thus we assume that the state space is identically decomposable and the feasible set is symmetric (concepts to be explained later).

What is the benefit that arises out of such an analysis? To answer this question, we must first accept that the theory of distributive justice is concerned with equity criteria. Thus, our analysis helps us to study problems of equity and efficiency in abstract choice theoretic settings. The problem of social choice is more pervasive than the problem of allocating consumables amongst a finite number of agents. So, by our approach we are able to study equity and efficiency problems in more fundamental situations.

The point of view adopted in this paper is that the rule by which society actually chooses one or more alternative is a bargaining problem, determined by institutional characteristics. Whether such solutions are "just" or "not" is answered by inspecting induced social choice rules and their distributional implications. This paper owes its origin to a study by Thomson (1983), of similar problems in the context of a pure exchange economy.

2. Framework

In this section we develop a general framework for the analysis of bargaining problems. The framework we propose is a synthesis of the one developed by Nash (1950) and the one developed by Yaari (1981), the latter being a context in which problems of distributive justice can be analyzed. The analytical framework of this paper, also draws heavily on Lahiri (1991).

To capture the essentials of a bargaining problems, we start with a nonempty set X and a list of n real-valued functions $u_1, u_2, \ldots, u_n$, defined on X i.e. $u_i : X \mapsto \mathbf{R}$ for all $i \in \{1, \ldots, n\}$. The set X is the universe of discourse and its elements are entities among which society could conceivably be called upon to exercise a choice. Elements of X will be called alternatives and the set X of all conceivable alternatives will be referred to as the choice space. The functious $u_1, u_2, \ldots, u_n$ are utility functions, which represent the preferences over the choice space of the individuals who make up the society under consideration. In other words, "society" consists of n individuals who are identified by the utility functions $u_1, u_2, \ldots, u_n$ all of which are functions assigning real numbers to elements of the choice space.

The choice space X consists of all conceivable alternatives that might arise as options in any hypothetical situation. However, every specific choice situation is characterized by a subset of X, consisting of those alternatives that are feasible in that particular choice situation, taking into account all the extraneous factors. This gives rise to the notion of the set of feasible alternatives (or feasible set, for short) for which the symbol F will be used. We shall agree to call an n-tuple of utility functions $(u_i)_{i=1}^n$, an utility profile for the society, where for each $i \in \{1, \ldots, n\}$, $u_i : X \mapsto \mathbf{R}$ is a real valued function defined on X.

A social choice problem is an ordered pair $S = ((u_i)_{i=1}^n, F)$ where $(u_i)_{i=1}^n$ is an utility profile and $F \subseteq X$ is a feasible set of alternatives.

Let $x_0 \in X$ be an alternative which we shall refer to as the status-quo point. The underlying interpretation of a status-quo point is that if the agents agree on an alternative in the feasible set then they select this alternative; in the absence of an agreement they remain at x_0.

A bargaining problem is an ordered pair (S, x_0) where $S = ((u_i)_{i=1}^n, F)$ is a social choice problem at $x_0 \in X$.

A significant assumption that pervades a major portion of bargaining theory is that given a bargaining problem (S, x_0) where $S = ((u_i)_{i=1}^n, F)$, $x_0 \in F$. The meaning of this assumption is that the status-quo point is itself a feasible alternative.

Let Σ denote a class of bargaining problems. A solution (or a choice correspondence (CC)) defined on Σ is a function that associates with every problem in Σ, a nonempty subset of feasible alternatives for that problem; these alternatives are interpreted as the set of possible compromises reached by the agents (or recommended to them, in the event of an impartial arbitrator being in charge of deciding on the outcomes). The set of values taken by the solution, when applied to a particular problem, is the solution outcome of the problem. Thus, formally, a solution on Σ is a nonempty valued correspondence $G : \Sigma \mapsto X$ such that for all $(S, x_0) \in \Sigma$, $G[(S, x_0)] \subseteq F$.

The intuitive interpretation of the solution G is that of a model of some first stage of negotiations in which a subset of the feasible set is identified, from amongst which the final outcome will eventually be selected, through a process left unspecified.

A solution $G : \Sigma \mapsto X$ is said to be Pareto efficient if $x \in G[(S, x_0)]$ implies, there does not exist any $x' \in F$ such that $u_i(x') \geq u_i(x)$ $\forall i = 1, \ldots, n$ and $u_j(x') > u_j(x)$ for some $j \in \{1, \ldots, n\}$ where $S = ((u_i)_{i=1}^n, F)$. The requirement of Pareto efficiency for a solution concept is standard in welfare economics. This principle enjoys wide acceptance, the main reason for which is that a failure to do so would lead to the untenable position that a solution may have to be enforced, against the will of all individuals.

The main solutions to bargaining problems that have been investigated are the Nash (1950) solution, the Kalai–Smorodinsky (1975) solution, the Egalitarian solution of Kalai (1977), to mention a few. Our purpose in this paper is to analyse the goals of distributive justice in the context of bargaining problems. Distributive justice implies possible redistribution of resources among the agents if necessary. Nothing that has been said about the framework of bargaining problems discussed above, allows for the possibility of an analytical representation of redistribution. Our objective is to get as much mileage out of the above set up. Unless we make specific assumptions about the choice space X, we shall not be able to proceed with our analysis.

Let us say that the choice space X is decomposable if $\exists$ sets $Y_1, \ldots, Y_n$ such that $X = \prod_{i=1}^{n} Y_i$ i.e. X is the Cartesian product of n–spaces. Each Y_i is interpreted to contain all the conceivable personalized outcomes relevant to the i^{th} agent. We say that the choice space X is identically decomposable if $\exists$ a set Y such that $X = Y^n$ i.e. X is the n–fold Cartesian product of Y. Identical decomposability, allows for comparability of the personalized outcomes of the different agents. However, we still have not been able to adequately capture the notion of redistributions required to guarantee distributive justice. How do we ensure that a redistribution of a feasible outcome is itself feasible?

Let Π^n be the class of all permutations of order n. A feasible set $F \subseteq X$ is said to be symmetric if $\forall \pi \in \Pi^n$, $\pi(F) = F$; i.e. a rearrangement of the personalized outcomes of a feasible alternative is once again a feasible alternative.

We shall assume unless otherwise mentioned that, X is identically decomposable and $\forall (S, x_0) \in \Sigma$ where $S = ((u_i)_{i=1}^{n}, F)$, F is a symmetric feasible set in X.

Let $P : \Sigma \mapsto X$ be the Pareto correspondence i.e. $x \in P[(S, x_0)] \Leftrightarrow$ there does not exist any $x' \in F$ such that $u_i(x') \geq u_i(x)$ $\forall i = 1, \ldots, n$ and $u_j(x') > u_j(x)$ for some $j \in \{1, \ldots, n\}$, where $S = ((u_i)_{i=1}^{n}, F)$.

Let (S, x_0) be a bargaining problem with $S = ((u_i)_{i=1}^{n}, F)$. Let $x \in F$. Agent i is said to regret $\pi \in \Pi^n$ at x if $u_i(\pi(x)) > u_i(x)$. The concept of regret generalizes (in many ways) the concept of envy in exchange economies, due to Foley (1967) and Varian (1974). First, we allow for externalities. Second we allow for arbitrary permutations, not merely a switch between two agents.

Let $Q : \Sigma \mapsto X$ be the regret–free correspondence i.e. $x \in Q[(S, x_0)] \Leftrightarrow \forall i \in \{1, \ldots, n\}$, $\forall \pi \in \Pi^n$, $u_i(x) \geq u_i(\pi(x))$.

Denote $P[(S, x_0)] \cap Q[(S, x_0)] = PQ[(S, x_0)]$ $\forall (S, x_0) \in \Sigma$. $PQ : \Sigma \mapsto X$ will be called the fairness correspondence.

Our first preliminary result is as follows :

Proposition 1 *If $x \in P[(S, x_0)]$, then $\forall \pi \in \Pi^n$, $\exists$ an agent i (possibly depending on x and π) such that i does not regret π at x.*

Proof. Suppose towards a contradiction that there exist no such agent for π at x. Hence $u_i(\pi(x)) > u_i(x)$ $\forall i \in \{1, \ldots, n\}$. Since F is symmetric, $\pi(x) \in F$ and thus contradicts that $x \in P[(S, x_0)]$.

Q.E.D.

We can further strengthen Proposition 1 to obtain the appropriate generalization of a well–known result if we allow for selfish preference. Assume for the sake of the next two propositions that $\forall i \in \{1, \ldots, n\}$, $\forall x_i \in Y$, $u_i(x_i, x_{-i}) = u_i(x_i, y_{-i})$, where $x_{-i} \in Y^{n-1}$, $y_{-i} \in Y^{n-1}$,

$$x_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n),$$
$$y_{-i} = (y_1, y_2, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n).$$

Proposition 2 (Varian (1974), Thomson and Varian (1985)) *If $x \in P[(S, x_0)]$ and preferences are selfish (as defined above), then there is some agent who does not regret any permutation at x.*

Proof. Suppose towards a contradiction (and without loss of generality) that the agent 1 regrets π at x. Clearly there exists some other agent (say agent 2) whose personalized outcome gives, agent 1 higher utility that his own. Now suppose agent 2 regrets π' at x. Thus, by the same argument, there exists some other agent (say agent 3) whose personalized outcome gives agent 2 higher utility that his own. Since the total number of agents is finite at some stage say stage k there must form a cycle. Consider the permutation $\overline{\pi}$

$$\overline{\pi}(x_1, \ldots, x_n) = (x_2, x_3, \ldots, x_k, x_1, x_{k+1}, x_{k+2}, \ldots, x_n).$$

$$\text{Clearly,} \quad u_i(\overline{\pi}(x)) > u_i(x),\ i = 1, \ldots, k$$
$$u_i(\overline{\pi}(x)) = u_i(x),\ i = k+1, \ldots, n.$$

This contradicts that $x \in P[(S, x_0)]$.

Q.E.D.

A related proposition under the same assumptions has a similar proof to the above.

Proposition 3 (Varian (1974), Thomson and Varian (1985)) *If $x \in P[(S, x_0)]$ and preferences are selfish, then there exists some agent*

$$i \in \{1, \ldots, n\} \quad \pi_k(x) = x_i \Rightarrow u_k(x) \geq u_k(\pi(x))\ \forall k \in \{1, \ldots, n\} \quad \textit{where}$$
$$\pi(x) = (\pi_1(x), \ldots, \pi_k(x), \ldots, \pi_n(x)).$$

Thus at any Pareto efficient allocation there is a natural way to say which agents are "best–off" (see Proposition 2) and which agents are "worst–off" (see Proposition 3). Before we end this section, let us make the following observation : A bargaining solution $G : \Sigma \mapsto X$, is called a social choice correspondence if $G[(S, x_0)]$ is independent of x_0 $\forall (S, x_0) \in \Sigma$. Under such circumstances we can denote $G[(S, x_0)]$ by $\overline{G}[S]$. It is instructive to note that $\overline{P}$, $\overline{Q}$ and $\overline{PQ}$ are social choice correspondences.

3. Acceptable Outcomes and Permutation Invariance

Distributive justice is ultimately a problem in social choice. It may in the process of its implementation, entail redistributions of the initial endowments of the agents. This would imply a permutation of the status–quo point — a definite change from what prevailed before the redistribution. In our framework, and in most (non–authoritarian) economic situations, solutions are defined for bargaining problems. We naturally have to organize a marriage of distributive justice and solutions to bargaining problems, consistent with our framework, if we desire to proceed with our analysis. First we propose a few basic acceptability definitions.

Definition 1 *Given a social choice problem $S = ((u_i)_{i=1}^n, F)$, and a solution $G : \Sigma \mapsto X$ we say that a pair $(x, x_0) \in F \times F$ is a weakly G–acceptable configuration for S if*

(a) $x \in G[(S, x_0)]$

(b) $\forall \pi \in \Pi^n, \forall i,\ \exists x' \in G[(S, \pi(x_0))]$ *s.t.* $u_i(x) \geq u_i(x')$.

Definition 2 *Given a social choice problem $S = ((u_i)_{i=1}^n, F)$, and a solution $G : \Sigma \mapsto X$ we say that a pair $(x, x_0) \in F \times F$ is a semi-strictly G–acceptable configuration for S if*

(a) $x \in G[(S, x_0)]$

(b) $\forall \pi \in \Pi^n, \exists x' \in G[(S, \pi(x_0))]$ *s.t.* $u_i(x) \geq u_i(x') \forall i$.

It is instructive to note that if G is Pareto efficient and (x, x_0) is a semi–strictly G acceptable configuration then $(x, \pi(x_0))$ is also a semi–strictly G acceptable configuration $\forall \pi \in \Pi^n$.

Definition 3 *Given a social choice problem* $S = ((u_i)_{i=1}^n, F)$*, and a solution* $G : \Sigma \mapsto X$ *we say that a pair* (x, x_0) *is a strictly G–acceptable configuration for* S *if*

(a) $x \in G[(S, x_0)]$

(b) $\forall \pi \in \Pi^n, \forall x' \in G[(S, \pi(x_0))]$, $u_i(x) \geq u_i(x') \; \forall i$.

From these three definitions we can derive in a natural way three criteria of acceptability of outcomes.

Definition 4 *Given a social choice problem* $S = ((u_i)_{i=1}^n, F)$*, and a solution* $G : \Sigma \mapsto X$*, we say that*

(a) *x is a weakly G–acceptable outcome for S if $\exists x_0 \in F$ such that (x, x_0) is a weakly G–acceptable configuration for S.*

(b) *x is a semi–strictly G–acceptable outcome for S, if $\exists x_0 \in F$ such that (x, x_0) is a semi–strictly G–acceptable configuration for S.*

(c) *x is a strictly G–acceptable outcome for S, if $\exists x_0 \in F$ such that (x, x_0) is a strictly G–acceptable configuration for S.*

Definition 4 alongwith a Pareto efficiency criteria imposed on G, paves the way for the concept of permutation invariant outcomes. These are outcomes which belong to the solution inspite of the permutations in the status–quo outcomes. Before we show how this occurs let us formalize the above concept in the following definition.

Definition 5 *Given a social choice problem S and a solution $G : \Sigma \mapsto X$, we say that :*

(a) *x is a permutation invariant outcome with respect to G for S if there exist $x_0 \in F$ such that $\forall \pi \in \Pi^n$, $x \in G[(S, \pi(x_0))]$*

(b) *x is a strictly permutation invariant outcome with respect to G for S, if $\exists x_0 \in F$ such that $\forall \pi \in \Pi^n$, $G[(S, \pi(x_0))] = \{x' \in F \mid \forall i, \; u_i(x') = u_i(x)\}$.*

We are now in a position to assert the following propositions.

Proposition 4 *Let S be a social choice problem, $G : \Sigma \mapsto X$ be a solution such that $G(.) \subseteq P(.)$, and the following property (P) is satisfied by G :*
Property (P). For every social choice problem $S = ((u_i)_{i=1}^n, F)$, $\forall x, x' \in F$, $x \in G[(S, x_0)]$ for some $(S, x_0) \in \Sigma$ and $u_i(x) = u_i(x') \; \forall i$ implies $x' \in G\left[(S, x_0^0)\right]$.

(a) *If x is a semi–strictly G–acceptable outcome for S then x is a permutation invariant outcome with respect to G for S*

(b) *If x is a strictly G–acceptable outcome for S, then x is a strictly permutation invariant outcome with respect to G for S.*

Proof. (a) Suppose X is a semi–strictly G–acceptable outcome for $S = ((u_i)_{i=1}^n, F)$. Therefore there exists $x_0 \in F$, such that $(S, x_0) \in \Sigma$, $x \in G[(S, x_0)]$ and $\forall \pi \in \Pi^n$, $\exists x' \in G[(S, \pi(x_0))]$ $u_i(x) \geq u_i(x') \forall i$.
Since $G(.) \subseteq P(.)$ by hypothesis, we have $u_1(x) = u_1(x'), \forall i$. By property (P) applied to $(S, \pi(x_0))$, we get $x \in G[(S, \pi(x_0))]$.

Hence x is a permutation invariant outcome with respect to G for S.

(b) Suppose x is a strictly G–acceptable outcome for $S = ((u_i)_{i=1}^n, F)$. Therefore there exists $x_0 \in F$, such that $(S, x_0) \in \Sigma$, $x \in G[(S, x_0)]$ and $\forall \pi \in \Pi^n$, $\forall x' \in G[(S, \pi(x_0))]$, $u_i(x) \geq u_i(x')\ \forall i$.

Since $G(.) \subseteq P(.)$ by hypothesis, we have $u_i(x) = u_i(x')\ \forall i$.

- $x' \in G[(S, \pi(x_0))] \Rightarrow u_i(x') = u_i(x)\ \forall i$
  - $G[(S, \pi(x_0))] \subseteq \{x' \in F \mid u_i(x') = u_i(x)\ \forall i\}$.

Conversely, suppose $y \in \{x' \in F \mid u_i(x') = u_i(x)\ \forall i\}$. By property (P), $y \in G[(S, \pi(x_0))]$.
Let us now show that $x \in G[(S, \pi(x_0))]\ \forall \pi \in \Pi^n$.

Let $x' \in G[(S, \pi(x_0))]$. Interchanging the role of x and x' in the statement of property (P) and applying strict G–acceptability of x for G at S we get since $G(.) \subseteq P(.)$,

1. $u_i(x) = u_i(x')\ \forall i$
2. $x \in G[(S, \pi(x_0))]$

- $G[(S, \pi(x_0))]\ \forall \pi \in \Pi^n$.

Now applying property (P) to y and x once again at $(S, \pi(x_0))$, we get since $u_i(y) = u_i(x)\ \forall i$, that $y \in G[(S, \pi(x_0))]$

- $\{x' \in F \mid u_i(x) = u_i(x)\ \forall i\} \subseteq G[(S, \pi(x_0))]$
- $\{x' \in F \mid u_i(x') = u_i(x)\ \forall i\} = G[(S, \pi(x_0))]$
- x is strictly permutation invariant with respect to G at S.

Q.E.D.

An interesting discussion of the above concepts in relation to a pure exchange economy is available in Thomson (1983).

A result of general interest is the following :

Lemma 1 *If $G : \Sigma \mapsto X$ and $G' : \Sigma \mapsto X$ are two solutions such that $G[(S, x_0)] \subseteq G'[(S, x_0]$, $\in \Sigma$ and if $x \in G[(S, x)]$ for some $(S, x) \in \Sigma$ implies that $\forall \pi \in \Pi^n$, $x \in G[(S, \pi(x))]$, then $x \in G'[(S, x)]$.*

Proof.

$$\begin{aligned} x \in g[(S, x)] &\Leftrightarrow \forall \pi \in \Pi^n, x \in G[(S, \pi(x))] \\ &\Rightarrow \forall \pi \in \Pi^n, x \in G'[(S, \pi(x))] \\ &\Rightarrow x \in G'[(S, x)]. \end{aligned}$$

Q.E.D.

It should be noted that under the conditions of the above lemma we have the stronger result that $x \in G'[(S, \pi(x))]\ \forall \pi \in \Pi^n$.

Let $I : \Sigma \mapsto X$ defined by $I[(S, x_0)] \equiv \{x \in F \mid u_i(x) \geq u_i(x_0)\ \forall i\}$ denote the <u>individually rational bargaining solution</u>, where $S \equiv ((u_i)_{i=1}^n, F)$.

Proposition 5 *Given a social choice problem $S \equiv ((u_i)_{i=1}^n, F)$, the set $\{x \in F \mid x \in I[(S, x)] \Leftrightarrow x \in I[(S, \pi(x))]\ \forall \pi \in \Pi^n\} \equiv \overline{Q}[S]$. (Refer to the definition of a social choice correspondence given earlier).*

Proof. Let $y \in \{x \in F \mid x \in I[(S,x)] \Leftrightarrow x \in I[(S,\pi(x))]\ \forall \pi \in \Pi^n\}$

- $y \in I[(S,\pi(y))]\, \forall \pi \in \Pi^n$
  - $u_i(y) \geq u_i(\pi(y))\, \forall \pi \in \Pi^n$
    - $y \in \overline{Q}[S]$.

Conversely suppose $y \in \overline{Q}[S]$

- $u_i(y) \geq u_i(\pi(y))\, \forall \pi \in \Pi^n$
- $y \in I[(S,\pi(y))]\, \forall \pi \in \Pi^n$.

Q.E.D.

Proposition 5 can be strengthened further. Let $IP : \Sigma \mapsto X$ defined by $I[(S,x_0)] \cap P[(S,x_0)]$ $\forall (S,x_0) \in \Sigma$ denote the individually rational Pareto–efficient bargaining solution.

Proposition 6 *Given a social choice problem* $S \equiv ((u_i)_{i=1}^n, F)$, *the set* $\{x \in F \mid x \in IP[(S,x_0)] \Leftrightarrow x \in IP[(S,x_0)]\, \forall \pi \in \Pi^n\} \equiv \overline{PQ}[S]$.

Proof. Similar to the proof of Proposition 5.

Q.E.D.

Let Γ be the class of all social choice problems $S = ((u_i)_{i=1}^n, F)$. For the purpose of our subsequent analysis we will assume that as before, X is identically decomposable.

A social choice correspondence can also be defined as a nonempty valued correspondence $H : \Gamma \mapsto X$ such that for all $S \equiv ((u_i)_{i=1}^n, F)$, $H(S) \subseteq F$. Our above analysis, reveals that by using the concepts of acceptability and permutation invariance we can obtain from a bargaining solution, an induced social choice correspondence which displays desirable properties. One of these desirable properties i.e. a status–quo alternative should itself be a candidate for a solution helped us in characterizing the fair social choice correspondence.

4. A Generalization of the Walrasian bargaining Solution and an Induced Social Choice Correspondence

The Walrasian correspondence of general competitive analysis can be suitably generalized. We provide one such generalization, consistent with our framework. We assume as before, that X is identically decomposable i.e. $X = Y^n$ where Y is some set.

Definition 6 *Let* $W : \Sigma \mapsto X$ *be defined as follows :* $\forall (S,x_0) \in \Sigma$,
$W[(S,x_0)] = \{x \in F \mid \exists u : Y \mapsto \mathbf{R}$ *such that for all i.* $u(x_i) \leq u(x_{0i})$ *and* $u(x_i') \leq u(x_{0i})$ *implies* $u_i(x_i, x_{-i}) \geq u_i(x_i', x_{-i})\}$.
Then W is called the generalized Walrasian bargaining solution. Associated with each $x \in W[(S,x_0)]$, *is a function* $u : Y \mapsto \mathbf{R}$, *as can be observed from Definition 5. The u associated with* $[x,(S,x_0)]$ *will be called an indicator for* $[x,(S,x_0)]$.

It should be noted that, unlike general competitive analysis, there is nothing in our definition of a generalized Walrasian bargaining solution, which can easily be adjusted to imply $W(.) \subseteq P(.)$. However, we may associate with W the following induced social choice correspondence $\overline{W}$:

$\overline{W} : \Gamma \mapsto X$ is defined as follows : $\overline{W}(S) = \{x \in F \mid \exists x_0 \in F$ and $u : Y \mapsto \mathbf{R}$ such that $x \in W[(S,x_0)]$ $u(x_{0i}) - u(x_{0j})\ \forall i,j$ where u is the indicator function for $[x,(S,x_0)]\}$.
$\overline{W}$ is the appropriate generalization of the Walrasian correspondence from equal incomes.

The following proposition establishes a desirable link :

Proposition 7 *Suppose preferences are selfish. Then* $\overline{W} \subseteq \overline{Q}$.

Proof. Let $x \in \overline{W}[S]$, $u : Y \mapsto \mathbf{R}$ be the associated indicator function, and x_0 the associated status–quo point,

- $u(x_{0i}) = u(x_{0j})\ \forall i, j$.

Let $\pi \in \Pi^n$, and suppose $\pi_i(x) = x_j$

- $u(x_j) \leq u(x_{0j})$
- $u(x_j) \leq u(x_{0j}) = u(x_{0i})$

But $x \in \overline{W}(S) \Rightarrow u_i(x_i) \geq u_i(x_j)$

- $u_i(x) \geq u_i(\pi(x))$
- $x \in \overline{Q}[S]$.

Q.E.D.

Let $\overline{WP} = \overline{W} \cap \overline{P}$ i.e. Pareto efficient allocations belonging to $\overline{W}$. Then we can prove the following proposition, which strengthens Proposition 7.

Proposition 8 *Suppose preferences are selfish. Then* $\overline{WP} \subseteq \overline{QP}$.

Proof. Analogous to the above proof.

Proposition 8 comes very close to a standard result in the theory of justice, which says that Walrasian allocations from equal incomes are fair. For reasons mentioned earlier we need to impose Pareto efficiency on the generalized Walrasian social choice correspondence from equal indicator values to arrive at the desired result.

As a final result we obtain an interesting characterization of $\overline{W}$, which parallels an equivalent result of Thomson (1983), for pure exchange economies.

Proposition 9 *Given any social choice problem* $S \in \Gamma$, $\overline{W}[S] = \{x \in F \mid x \in W[(S, x)] \Leftrightarrow x \in W[(S, \pi(x))]\ \forall \pi \in \Pi^n\}$.

Proof. Let $x \in W[(S, x)] \Leftrightarrow x \in W[(S, \pi(x))]\ \forall \pi \in \Pi^n$.

Let π be such that $\pi_i(x) = x_j$. Let u be the indicator associated with $[x, (S, \pi(x))]$.

- $u(x_i) \leq u(\pi_i(x)) = u(x_j)$

Now choosing π' to be such that $\pi'_j(x) = x_i$, we get,

$u(x_j) \leq u(x_i)$

- $u(x_i) = u(x_j)\ \forall i, j$
- $x \in \overline{W}[S]$.

Conversely, let $x \in \overline{W}[S]$, x_0 be the status–quo point and u be the associated indicator function,

- $u(x_{0i}) = u(\pi_i(x_0))\ \forall \pi \in \Pi^n$

Also, $u_i(x') \leq u_i(x)\ \forall x' = (x'_i, x_{-i})$ such that $u(x'_i) \leq u(x_{0i})$

$\Leftrightarrow u_i(x') \leq u_i(x)\ \forall x' = (x'_i, x_{-i})$ such that $u(x'_i) \leq u(\pi_i(x_0))$

- $x \in W[(S, x)] \Leftrightarrow x \in W[(S, \pi(x))]\ \forall \pi \in \Pi^n$.

This proves the proposition.

Q.E.D.

5. Conclusion

This paper attempts at a generalization to an abstract setting, the results obtained by Thomson (1983). The various concepts introduced in this paper, which were earlier studied in the context of a pure exchange economy, are shown to be valid under very general conditions. This is one significant contribution of this paper.

Bargaining theory and social choice theory can in their right be developed in an abstract framework. This paper shows that a considerable portion of the theory of distributive justice can also be analyzed in a similar abstract framework. In fact the analysis, based on identically decomposable choice spaces and symmetric feasible sets can go far beyond where our paper ends.

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