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THE METHOD OF COMPARATIVE FACTOR ON UNCERTAIN MULTIPLE ATTRIBUTE(CRITERIA) DECISION MAKING

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Abstract: This paper will deal with the problem of making decisions under conditions of uncertainty. Often, however,we must make a choice and are naturally concerned about whether it is a best or optimal choice. In the paper, the concept of multiple attribute(criteria) decision making will be extended under uncertain conditions. The comparative and subcomparative factor will have been defined. Some algebraic properties of the comparative and subcomparatve factor as well as corresponding theorems will have been obtained. The method of comparative factor will have been proposed on the basis of the properties and theorms of the comparative and subcomparative factor.

Keywords: the method of comparative factor, comparative factor, subcomparative factor, the optimal programme, the optimal ordered number, MCDM

1. Introduction

Much of life, of course,involves making choices under uncertainty, that is, choosing from some set of alternative courses of action in situations where we are uncertain about the actual consequences that will occur for each course of action being considered. Thus, all of decision problems have certain and uncertain general characteristics. These characteristics constitute the formal description of the problems and provide the structure for solutions. The decision problem under study may be represented by a model in terms of the following definition.

Definition 1. 1 Let F_k $(k=1, 2, \cdots, n)$ be the set of the programme of the attribute k of decision making, ordering

$$V = \{\alpha(\square) \mid \alpha(\square) = [\square(X_1),\square(X_2),\cdots,\square(X_n)], \square(X_k) = [X_k(a),X_k(b)] \in F_k, X_k(a),X_k(b)\in R, k = 1,2,\cdots,n\}$$

so call $V\{\square\}$ programme space of the uncertain multiple attribute(criteria) decision making. Denoted the programme in $V\{\square\}$ by $\alpha(\square)$, $\alpha(\square)$ is the interval vector. Then, the programme space is composed of all programmes under the decisional rule.

Definition 1. 2 Let $\alpha(\square)$, $\beta(\square)\in V\{\square\}$, $\alpha(\square)=[\square(X_1), \square(X_2), \cdots, \square(X_n)]$, $\square(X_k)=[X_k(a), X_k(b)]$, $k=1, 2, \cdots, n$; $\beta(\square)=[\square(Y_1), \square(Y_2), \cdots, \square(Y_n)]$, $\square(Y_k)=[Y_k(a), Y_k(b)]$, $k=1, 2, \cdots, n$; then, (1) $\alpha(\square)=\beta(\square)$, if and only if $X_k(a)=Y_k(a)$, $Y_k(b)=X_k(b)$, $k=1, 2, \cdots, n$; (2)$\alpha(\square)<\beta(\square)$, if and only if $X_k(b)<Y_k(a)$, $k=1, 2, \cdots, n$, (3) $\alpha(\square)\prec\beta(\square)$, if and only if $X_k(b)\leqq Y_k(a)$, $k=1, 2, \cdots, n$, but $\alpha(\square)\neq\beta(\square)$; (4) $\alpha(\square)\leqq\beta(\square)$, if and only if $X_k(b)\leqq Y_k(a)$, $k=1, 2, \cdots, n$.

Definition 1. 3 Let $\alpha(\square)_0\in V\{\square\}$, then, (1) $\alpha(\square)_0$ is the inferior programme in $V\{\square\}$, if and only if at least exists one programme $\alpha(\square)_1\in V\{\square\}$, subject to $\alpha(\square)_0\prec\alpha(\square)_1$; (2)$\alpha(\square)_0$ is the noninferior programme in $V\{\square\}$, if and only if there is no programme $\alpha(\square)$ in $V\{\square$

}, subject to $\alpha(\boxtimes)_0 \prec \alpha(\boxtimes)$; (3) $\alpha(\boxtimes)_0$ is the weak noninferior pogramme in $V\{\boxtimes\}$, if and only if there is no prgramme $\alpha(\boxtimes)$ in $V\{\boxtimes\}$, subject to $\alpha(\boxtimes)_0 < \alpha(\boxtimes)$; (4)$\alpha(\boxtimes)_0$ is the optimal programme in $V\{\boxtimes\}$, if and only if to any programme $\alpha(\boxtimes)(\neq \alpha(\boxtimes)_0)$ in $V\{\boxtimes\}$, there be $\alpha(\boxtimes) \prec \alpha(\boxtimes)_0$; (5) $\alpha(\boxtimes)_0$ is the strong optimal programme in $V\{\boxtimes\}$, if and only if to any programme $\alpha(\boxtimes)$ $(\neq \alpha(\boxtimes)_0)$, there be $\alpha(\boxtimes) < \alpha(\boxtimes)_0$.

2. The Method of Comparative Factor Under Untertain Conditions

Definition 2. 1 Let the interval factor $\alpha(\boxtimes) \in V\{\boxtimes\}$, $\alpha(\boxtimes) = [\boxtimes(X_1), \boxtimes(X_2), \cdots, \boxtimes(X_n)]$, $\boxtimes(X_k) = [X_k(a), X_k(b)]$, $k = 1,2,\cdots, n$

$$\varphi_k[\alpha(\boxtimes)] = \begin{cases} 0 & when X_k(a) > 0 \\ 1/2 & when X_k(a)X_k(b) \leqq 0, k = 1,2,\cdots,n; \\ 1 & when X_k(b) < 0 \end{cases}$$

$$\varphi'_k[\alpha(\boxtimes)] = \begin{cases} 0 & when X_k(a) > 0 \ \ or \ \ X_k(a)X_k(b) \leqq 0, k = 1,2,\cdots,n; \\ 1 & when X_k(b) < 0 \end{cases}$$

then, $\Phi[\alpha(\boxtimes)] = \sum_{k=1}^{n} \varphi_k[\alpha(\boxtimes)]$ and $\Phi'[\alpha(\boxtimes)] = \sum_{k=1}^{n} \varphi'_k[\alpha(\boxtimes)]$

are called the comparative and subcomparative factor of the interval vector $\alpha(\boxtimes)$.

Definition 2. 2 Let $\alpha(\boxtimes)$, $\beta(\boxtimes)$ in $V\{\boxtimes\}$. (1) The sum of $\alpha(\boxtimes)$ and $\beta(\boxtimes)$ is defined by

$$\alpha(\boxtimes) + \beta(\boxtimes) = [\boxtimes(X_1) + \boxtimes(Y_1), \boxtimes(X_2) + \boxtimes(Y_2), \cdots, \boxtimes(X_n) + \boxtimes(Y_n)]$$

(2) The product of scalar $\lambda \in R$ and interval vector $\alpha(\boxtimes)$ is defined by

$$\lambda\alpha(\boxtimes) = [\lambda\boxtimes(X_1), \lambda\boxtimes(X_2), \cdots, \lambda\boxtimes(X_n)]$$

Definition 2. 3 Let $\boxtimes(X_k) = [X_k(a), X_k(b)]$, $\boxtimes(Y_k) = [Y_k(a), Y_k(b)]$, $\lambda \in R$, there are the following operation: (1) addition operation, $\boxtimes(X_k) + \boxtimes(Y_k) = [X_k(a) + Y_k(a), X_k(b) + Y_k(b)]$; (2) subtration operation, $\boxtimes(Y_k) - \boxtimes(X_k) = [\min(Y_k(a) - X_k(a), Y_k(b) - X_k(b)), \max(Y_k(a) - X_k(a), Y_k(b) - X_k(b))]$; (3) mutiplication operation, $\lambda\boxtimes(X_k) = [\min(\lambda X_k(a), \lambda X_k(b)), \max(\lambda X_k(a), \lambda Y_k(b))]$.

Evidently there are the comparative and subcomparative factor for everyone interval vector. In other words, everyone programme has the comparative and subcomparative factor in the uncertain multiple attribute(criteria) decision marking. According to Definition 2. 1, we got the following theorem 2. 1.

Theorem 2. 1 Let $\alpha(\boxtimes)$, $\beta(\boxtimes)$ in $V\{\boxtimes\}$, then, (1) $0 \leqq \Phi[\alpha(\boxtimes)] \leqq n$, $0 \leqq \Phi'[\alpha(\boxtimes)] \leqq n$; (2) $\Phi[\alpha(\boxtimes) - \alpha(\boxtimes)] = n/2$, $\Phi'[\alpha(\boxtimes) - \alpha(\boxtimes)] = 0$; (3) $\Phi[\alpha(\boxtimes) + \beta(\boxtimes)] \leqq \Phi[\alpha(\boxtimes)] + \Phi[\beta(\boxtimes)]$; $\Phi'[\alpha(\boxtimes) + \beta(\boxtimes)] \leqq \Phi'[\alpha(\boxtimes)] + \Phi'[\beta(\boxtimes)]$;

(4) $\Phi[\lambda\alpha(\boxtimes)] = \begin{cases} \Phi[\alpha(\boxtimes)], & when \ \lambda \ in \ R^+ \\ \Phi[-\alpha(\boxtimes)], & when \ \lambda \ in \ R^- \end{cases}$, $\Phi'[\lambda\alpha(\boxtimes)] = \begin{cases} \Phi'[\alpha(\boxtimes)], & when \ \lambda \ in \ R^+ \\ \Phi'[-\alpha(\boxtimes)], & when \ \lambda \ in \ R^- \end{cases}$.

Theorem 2. 2 Let $\alpha(\boxtimes)$, $\beta(\boxtimes)$ in $V\{\boxtimes\}$, then (1) $\alpha(\boxtimes) < \beta(\boxtimes)$, if and only if $\Phi[\beta(\boxtimes) - \alpha(\boxtimes)] = 0$; (2) If $\alpha(\boxtimes) \prec \beta(\boxtimes)$, then $\Phi[\beta(\boxtimes) - \alpha(\boxtimes)] \leqq (n-1)/2$ and $\Phi'[\beta(\boxtimes) - \alpha(\boxtimes)] = 0$; (3) If $\alpha(\boxtimes)$

$\leq\beta(\square)$, then $\Phi'[\beta(\square)-\alpha(\square)]=0$.

Proof Suppose $\alpha(\square)=[\square(X_1), \square(X_2), \cdots, \square(X_n)]$, $\square(X_k)=[X_k(a), X_k(b)]$

$\beta(\square)=[\square(Y_1), \square(Y_2), \cdots, \square(Y_n)]$, $\square(Y_k)=[Y_k(a), Y_k(b)]$,

$\gamma(\square)=\beta(\square)-\alpha(\square)=[\square(Y_1)-\square(X_1), \square(Y_2)-\square(X_2), \cdots, \square(Y_n)-\square(X_n)]$

$=[\square(Z_1), \square(X_2), \cdots, \square(Z_n)]$, $\square(Z_k)=[Z_k(a), Z_k(b)]$, $k=1, 2, \cdots, n$

From Definition 1.2, 2.2, 2.3, there are (1) $\alpha(\square)<\beta(\square)$, if and only if $X_k(b)<Y_k(a)$, $k=1, 2, \cdots, n$ if and only if $Z_k(a)>0$, where $\gamma(\square)=\beta(\square)-\alpha(\square)$, then $\Phi[\beta(\square)-\alpha(\square)]=0$.

(2) $\alpha(\square)\prec\beta(\square)$, if and only if $X_k(b)\leq Y_k(a)$, $k=1, 2, \cdots, n$ and $\alpha(\square)\neq\beta(\square)$, if and only if $Z_k(a)\geq 0$ and at least one $Z_{k0}(a)>0$, $k=1, 2, \cdots n$, $k\neq k_0$, where $\gamma(\square)=\beta(\square)-\alpha(\square)$, then $\Phi[\beta(\square)-\alpha(\square)]\leq(n-1)/2$ and $\Phi'[\beta(\square)-\alpha(\square)]=0$.

(3) $\alpha(\square)\leq\beta(\square)$, if and only if $X_k(b)\leq Y_k(a)$, $k=1, 2, \cdots, n$, if and only if $Z_k(a)\geq 0$, $k=1, 2, \cdots, n$, where $\gamma(\square)=\beta(\square)-\alpha(\square)$, then $\Phi'[\beta(\square)-\alpha(\square)]=0$.

Theorem 2. 3 Let $\alpha(\square)$, $\beta(\square)$ in $V\{\square\}$, then, (1) If $\alpha(\square)<\beta(\square)$, therefore $\Phi[\gamma(\square)-\alpha(\square)]\leq\Phi[\gamma(\square)-\beta(\square)]$ and $\Phi'[\gamma(\square)-\alpha(\square)]\leq\Phi'[\gamma(\square)-\beta(\square)]$, for everyone $\gamma(\square)\in V\{\square\}$; (2) If $\alpha(\square)\prec\beta(\square)$, therefore $\Phi[\gamma(\square)-\alpha(\square)]\leq\Phi[\gamma(\square)-\beta(\square)]+(n-1)/2$ and $\Phi'[\gamma(\square)-\alpha(\square)]\leq\Phi'[\gamma(\square)-\beta(\square)]$, for everyone $\gamma(\square)\in V\{\square\}$; (3) If $\alpha(\square)\leq\beta(\square)$, therefore $\Phi'[\gamma(\square)-\alpha(\square)]\leq\Phi'[\gamma(\square)-\beta(\square)]$, for everyone $\gamma(\square)\in V\{\square\}$.

Proof (1) From Theorem 2. 2(1) and $\alpha(\square)<\beta(\square)$, know $\Phi[\beta(\square)-\alpha(\square)]=0$. So using Theorem 2. 1(3), there is

$$\begin{aligned}\Phi[\gamma(\square)-\alpha(\square)]&\leq\Phi[\gamma(\square)-\beta(\square)]+\Phi[\beta(\square)-\alpha(\square)]\\&\leq\Phi[\gamma(\square)-\beta(\square)]\end{aligned}$$

Likewise $\Phi'[\gamma(\square)-\alpha(\square)]\leq\Phi'[\gamma(\square)-\beta(\square)]$ for everyone $\gamma(\square)\in V\{\square\}$

(2) From Theorem 2. 2(2) and $\alpha(\square)\prec\beta(\square)$, know $\Phi[\beta(\square)-\alpha(\square)]\leq(n-1)/2$ and $\Phi'[\beta(\square)-\alpha(\square)]=0$. So using Theorem 2. 1(3), there are

$$\begin{aligned}\Phi[\gamma(\square)-\alpha(\square)]&\leq\Phi[\gamma(\square)-\beta(\square)]+\Phi[\beta(\square)-\alpha(\square)]\\&\leq\Phi[\gamma(\square)-\beta(\square)]+(n-1)/2\\\Phi'[\gamma(\square)-\alpha(\square)]&\leq\Phi'[\gamma(\square)-\beta(\square)]+\Phi'[\beta(\square)-\alpha(\square)]\\&\leq\Phi'[\gamma(\square)-\beta(\square)]\quad\text{for everyone }\gamma(\square)\in V\{\square\}\end{aligned}$$

(3) The demostration is so similar to the proof of Theorem 2. 3(2) that we omit it.

Definition 2. 4 Let $\alpha(\square)$, $\beta(\square)\in V\{\square\}$

$$H[\alpha(\square)]=\sum_{\beta(\square)\in V\{\square\}}\Phi[\beta(\square)-\alpha(\square)]$$

so call $H[\alpha(\square)]$ the optimal ordered number of the programme $\alpha(\square)$ in $V\{\square\}$.

Obviously, since Definition 2. 4, we got the desired following result.

Theorem 2. 4 If $\alpha(\square)_0$ is the (strong) optimal programme in $V\{\square\}$, then $H[\alpha(\square)]=\max\{H[\alpha(\square)_0]\mid \text{any }\alpha(\square)\in V\{\square\}\}$.

Theorem 2. 5 If $H[\alpha(\square)_0]=\max\{H[\alpha(\square)]\mid \text{any }\alpha(\square)\in V\{\square\}\}$, then $\alpha(\square)_0$ is the noninferior programme in $V\{\square\}$.

Corollary 1 Suppose there are m programmes in $V\{\square\}$, then, (1) If $\alpha(\square)_0$ is the strong optimal progrmme in $V\{\square\}$, if and only if $H[\alpha(\square)_0]=(m-1)n+n/2$; (2) If $\alpha(\square)_0$ is the optimal programme in $V\{\Omega\}$, if and only if $n(m+1)/2 \leq H[\alpha(\Omega)_0] \leq n(m-1)$.

Corollary 2 Suppose the (strong) optimal programme exists in $V\{\Omega\}$ and $H[\alpha(\Omega)_0] = \max\{H[\alpha(\Omega)] \mid \text{any } \alpha(\Omega) \in V\{\Omega\}\}$, then $\alpha(\Omega)_0$ is the (strong) optimal programme.

Theorem 2. 6 If $\alpha(\Omega)_0, \alpha(\Omega)_1 \in V\{\Omega\}$, $H[\alpha(\Omega)_0] = \max\{H[\alpha(\Omega)] \mid \text{any } \alpha(\Omega) \in V\{\Omega\}\}$ and $H[\alpha(\Omega)_1] = \max\{H[\alpha(\Omega)] \mid \text{any } \alpha(\Omega) \in V\{\Omega\} / \{\alpha(\Omega)_0\}\}$, then, (1) $\alpha(\Omega)_1$ is the noninferior programme in $V\{\Omega\} / \{\alpha(\Omega)_0\}$; (2) If $\alpha(\Omega)_0 \not\succ \alpha(\Omega)_1$, therefore $\alpha(\Omega)_1$ is also the noninferior programme in $V\{\Omega\}$; (3) If $\alpha(\Omega)_0 \not\succ \alpha(\Omega)_1$, therefore $\alpha(\Omega)_1$ is also the weak noninferior programme in $V\{\Omega\}$.

Theorem 2. 7 If $\alpha(\Omega)_0$ is not the noninferior programme in $V\{\Omega\}$, so there must exist the noninferior programme $\alpha(\Omega)_e$ in $V\{\Omega\}$, subject to $H[\alpha(\Omega)_e] > H[\alpha(\Omega)_0]$.

The process of the proof of the above Theorems and Corollarys is too long to have to omit it because of limited space.

Pay attention to: Theorem 2.6 (3) provides the possibility and the way of seeking which may (weak) noninferior programme. Theorem 2. 7 actually is a method of arranging in order according to the big and small of the optimal ordered number, so , from the proof conclusion above, the noinferior programme generally is put in the front, the inferior programme generally is put in after.

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