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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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FLEXIBLE THREATS IN BARGAINING

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Abstract: We present a model of bargaining, in which one party issues threats characterized by a number of parameters. In real life negotiation threats are often "flexible", i.e. they are not all or nothing affairs. Our model emphasizes such a nature of threat strategies, and explores possible implications of extending the notion of a threat along that line.

Keywords: Multi-person decision making, Game theory, Bargaining, Threats.

1. Introduction

Let us imagine the following situation. In a firm, a trade union wants to change the conditions of employment. In negotiating for that the union may issue threats of strikes. In real life it often happens that the threatened strikes vary in their intensities (this can be done, for instance, by stopping work only in certain parts of the enterprise or by withdrawing labour for only a couple of hours a day). A choice of that intensity is an important union's decision. Our model takes this into account. Moreover, the union chooses not only the strike intensity but also decides the possible length of the threatened strike. It chooses, also, the points of time during the possible strike, at which it will be willing to negotiate with the firm's management.

If an agreement is reached during the strike, the action is called off and employees go back to work under the newly agreed upon terms of employment. If, however, an agreement is not achieved until the end of strike, labor returns to work under the old terms.

In this paper we are interested in situations analogical to the outlined above. We model the entire process as a multi-stage game. The game is solved through the technique of solving backwards, the solution concept used at each step of this backward procedure being the well known Nash (1950) solution (which was quite often used in describing labor-management bargaining, e.g. see Svejnar (1980), Grout (1984), McDonald and Solow (1981)). Threats of strikes have been also introduced in several earlier models (e.g. DeBrock and Roth (1981) and Barrett and Pattanaik (1989)), but the strikes there were announced complete or not at all. We believe that allowing variable intensities of threats as well as introducing other parameters, takes into account important factors of real-life situations and adds an interesting analytical feature.

2. The Model

Having in mind our example of union-firm's management negotiations we shall denote the two parties in our game by U and F. Both players take into account their utilities over a time horizon $t = 0, 1, \ldots, T$. Since U is not satisfied with the status quo situation, he wants to bargain over a change of a vector v of decisions. Since F has the power to change v, then U can issue a threat (of strike action) in order to force F to make concessions. In general, various such actions are possible, which are harmful to F to different extents. The normalized *intensity* of the threatened action will be denoted by $s \in [0,1]$. Then, U has the possibility of choosing $s \in [0,1]$, while F controls the decision vector v. We assume that other characteristics are constant over time, and that the parties' utilities are additive over time. Thus, we can denote the utilities attained during a single time period t by $x_U(v,s)$ and $x_F(v,s)$.

The point of departure in our model is characterized by a certain vector v^0 (and, of course, $s = 0$). U would like to attain an agreement which would change v^0 to a certain v^A such that $x_U(v^A,0) > x_U(v^0,0)$. Typically, however, such an agreement would be disadvantageous to F, i.e. $x_F(v^A,0) < x_F(v^0,0)$, and that is why the use of a threat may be necessary. Usually an implementation of a threat is costly to F as well as to U.

To solve the bargaining problems which arise in our model we shall use Nash's scheme applied, in case the parties fail to agree, at successive stages of the game. The negotiations, however, need not take place in each time period. We assume that U can choose periods in which talks could take place. The set of such time periods will be denoted by B and called the *bargaining schedule.* The threatener, U, announces also the possible *length of action* $\tau \in \{0,1,\ldots,T\}$. It is natural that $B \subseteq \{0,1,\ldots,\tau\}$. Moreover, we assume that the announced s is constant over the period $0 - \tau$.

Then, we have introduced three parameters that characterize a threat, namely, the intensity of the threatened action s, its possible length τ, and the bargaining schedule B. The question we shall concentrate on is the role and the optimal choice of the triple (s,τ,B).

Let us denote the set of all *feasible agreement outcomes at t* by Z_t:

$$Z_t = \{(X_U, X_F) : X_U = t x_U(v^0, s) + (T\text{-}t)\, x_U(v,0), \quad X_F = t x_F(v^0, s) + (T\text{-}t)\, x_F(v,0),\ v \in V\}, \quad t \in \{0,1,\ldots,\tau\} \qquad (1)$$

The Nash bargaining solution at consecutive stages are computed through a backward analysis. Let t_L denote the last period at which the talks can take place, i.e. $t_L = \max\{t: t \in B\}$. Thus, if the parties do not agree at t_L then the final disagreement outcome will be $X^d(s,\tau) = (X^d_U(.),X^d_F(.))$, where

$$X_i^d(s,\tau) = \tau x_i(v^0, s) + (T\text{-}t)\, x_i(v^0,0), \quad i \in \{U,F\}\,. \qquad (2)$$

Thus, at t_L we have the bargaining problem (Z_{t_L}, X^d) which can be solved according to Nash's scheme. This solution, which we denote by $N(Z_{t_L}, X^d)$, is used as a disagreement point (in the sense of Nash (1953)) when solving the bargaining problem for the prior-to-t_L time period t_{L-1} at which negotiations can take place (i.e. from B). And this process is repeated, yielding, finally, a bargaining solution for $t_1 = \min\{t: t \in B\}$ (a similar sequential scheme has been discussed in Binmore (1987) and Livne (1987)).

Note that the bargaining solutions depend on s, τ, and B. Denoting the solution reached at t by X(s,τ,B,t), the process can be written as follows:

$$
\begin{aligned}
X(s,\tau,B,t_L) &= N(Z_{t_L}, X^d(s,\tau)) ,\\
X(s,\tau,B,t_{L-1}) &= N(Z_{t_{L-1}}, X(s,\tau,B,t_L)) ,\\
&\ \vdots \\
X(s,\tau,B,t_1) &= N(Z_{t_1}, X(s,\tau,B,t_2)) ,
\end{aligned}
\tag{3}
$$

where N(G) denotes Nash's solution of a bargaining game G.

The sequence of events in our game is the following. At the very beginning U announces the triple (s,τ,B). Then, in case $t_1 > 0$ the threatened action is immediately on up to t_1-1. At the beginning of the first period of negotiations t_1, the parties must decide whether to accept X(s,τ,B,t_1). In case of acceptance the strategic part ends - a decision vector corresponding to the agreement v^A is computed and implemented up to T. If X(s,τ,B,t_1) is not accepted, then the threat action continues up to t_2-1. At t_2 there is a chance of the agreement X(s,τ,B,t_2), etc. Eventually, if the parties failed to agree even at t_L, then the outcome would be given by (2).

3. Optimal Threats

We shall concentrate on the problem U faces, of how to determine the optimal *flexible threat* described by a triple (s,τ,B), where $s \in [0,1]$, $\tau \in \{0,1,...,T\}$, and $B \subseteq \{0,1,..., \tau\}$. Because of the lack of space only the main results will be presented. For details as well as the proofs of theorems presented see (Pattanaik,Stefanski ,1991).

Consider first two bargaining schedules B and $\bar{B}$ such that $\bar{B} = B \cup \{\bar{t}\}$, where $\bar{t} < \min\{t: t \in B\} = t_1$. As it follows from (3) it can be proved that $X(s,\tau,\bar{B},\bar{t}) \geq X(s,\tau,B,t)$. From this inequality, in turn, it follows that the optimal bargaining schedule B should contain the time period t = 0, i.e. the optimal B should have the form $B = \{0\} \cup B'$, where $B' \subseteq \{1,2,...,\tau\}$.

Moreover, from (3) we have $X(s,\tau,B,t') \geq X(s,\tau,B,t'')$ if $t' < t''$. Then, for a given s,τ, and for B of the above mentioned form, X(s,τ,B,0) is the dominating solution, and thus the parties will accept it, i.e. the agreement will be reached at the time $t_A = 0$. This is, in fact, due to our implicit assumption about complete information in the game.

This means that the U must consider the following problem when looking for an optimal threat:

$$
(s^*,\tau^*,B^*) = \arg\max_{(s,\tau,B)\in\Omega} X_U(s,\tau,B,0) , \tag{4}
$$

where $\Omega = \{(s,\tau,B):\ s \in [0,1];\ \tau \in \{0,1,...T\};\ B = \{0\} \cup B',\ B' \subseteq \{1,2,...,\tau\}\}$.

Problem (4) can be solved in three steps. First we shall consider the choice of the best bargaining schedule B for given s and τ, i.e. we shall be looking for a function $\hat{B}$ such that

$$
\hat{B} = \arg\max_{\bar{B}\in \mathbf{B}} X_U(s,\tau,\bar{B}(s,\tau),0) , \tag{5}
$$

where $\bar{B}:[0,1]\times\{0,1,...,T\}\rightarrow 2^{\{0,1,...,\tau\}}$, and **B** is the set of all such functions. In the second step, we consider the choice of the length of the threatened action as a function of its intensity, s. Lastly, we derive the optimal threat intensity. Given the optimal choice of s, and given the length of action as a function of s, we get the optimal length of the threatened action. Further, the optimal choice of the bargaining schedule is determined once we have the optimal intensity and length of the threatened action.

It appears that the optimal bargaining schedule $\hat{B}(s,\tau)$ depends on the relative positions of the points $X^d(s,\tau)$ and $X^d(s,T)$ (see (2)) within the set Z_0 (determined according to (1)).Consider the possible positions of the point $X^d(s,\tau)$. From (2) and the assumption that a threatened action is costly to F as well as to U (like a strike) it follows that the set of possible positions of $X^d(s,\tau)$ can be determined as follows:

$$D_S=\{(X_U,X_F)\in Z_0:\ X_i>Tx_i(v^0,s),\ i\in\{U,F\}\quad\text{or}\quad X_i=Tx_i(v^0,s),\ i\in\{U,F\}\}\tag{6}$$

Now, let us construct a line segment joining $X^d(s,T)$ with a given point $X\in Z_0$. This line will be treated as a set and denoted by L(X). We shall consider a family of such lines: $\mathcal{L}=\{L(X): X\in P(Z_0)\}$, where $P(Z_0)$ denotes a Pareto frontier of Z_0. A function describing $P(Z_0)$ will be denoted by f_0. From the family $\mathcal{L}$ we shall distinguish a particular line L(X), $X=(X_U,X_F)\in P(Z_0)$, such that its slope M is equal to the negative slope of the Pareto frontier of Z_0 at X, i.e.

$$M=\frac{f_0(X_U)-X_F^d(s,T)}{X_U-X_U^d(s,T)}=-\frac{df_0(X_U)}{dX_U}\ .\tag{7}$$

This line will be used to divide D_s into three sets A_s, C_s, and E_s as indicated in Fig.1.

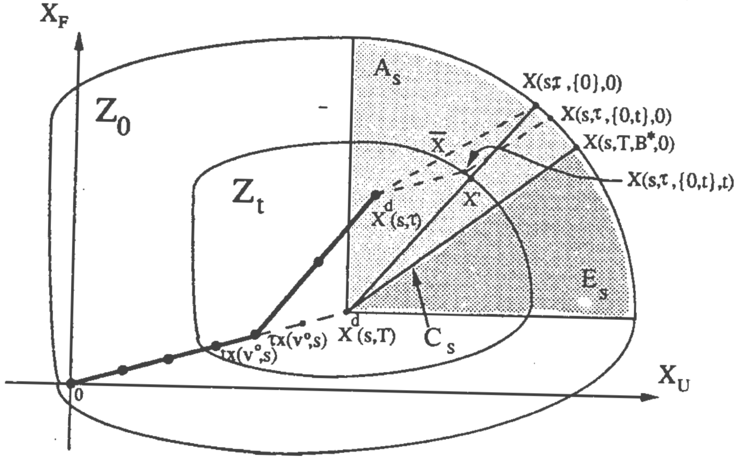

Figure 1

Now we are in a position to formulate a proposition which determines the optimal choice of B, given s and τ (see (Pattanaik,Stefanski,1991) for the proof).

Proposition 1. For a given threatened action intensity $s \in [0,1]$ and length of action $\tau \in \{0,1,...,T\}$ the optimal (for U) choice of a bargaining schedule B is $B = \hat{B}(s,\tau)$, where the mapping $\hat{B}$ is determined as follows:

$$\hat{B}(s,\tau) = \begin{cases} \{0,1,...,\tau\} & \text{if } X^d(s,\tau) \in A_s \\ \{0\} \cup B' & \text{if } X^d(s,\tau) \in C_s \\ \{0\} & \text{if } X^d(s,\tau) \in E_s \end{cases} \tag{8}$$

where B' is any subset of $\{1,2,...,\tau\}$.

As it follows from Proposition 1, in certain situations, i.e. for certain s and τ, it is optimal to declare that negotiations can go on at each time period during the strike action, while in others it is optimal to announce that the talks could take place only once, at the very beginning. It can also happen that it does not matter whether B includes later negotiation stages or not. Since the threat with $B = \{0\}$ seems to be the strongest (since it excludes the possibility of a later compromise) it is interesting that sometimes it pays to declare weaker threats.

The next step in solving (4) is to find the optimal length τ of an action, for given action intensity s and with the optimal schedule chosen according to (8). In other words we are looking for a mapping $\hat{\tau}$ such that:

$$\hat{\tau} = \arg\max_{\tilde{\tau} \in \Theta} X_U(s, \tilde{\tau}(s), \hat{B}(s, \hat{\tau}(s)), 0) \quad , \tag{9}$$

where $\tilde{\tau}: [0,1] \to \{0,1,...,T\}$, and Θ is the set of all such functions. The solution to (9) is given by the following proposition.

Proposition 2. The mapping $\hat{\tau}$ which describes the optimal choice of the length of action as a function of action intensity s is the following:

$$\hat{\tau}(s) = \begin{cases} T & \text{if } X^d(0,0) \in A_s \\ \text{any } \tau \in \{0,1,...,T\} & \text{if } X^d(0,0) \in C_s \\ 0 & \text{if } X^d(0,0) \in E_s \end{cases} \tag{10}$$

The third and last step in finding the optimal threat is to find the best action intensity in such a way that

$$s^* = \arg\max_{s \in [0,1]} X_U(s, \hat{\tau}(s), \hat{B}(s, \hat{\tau}(s)), 0) \tag{11}$$

Let us denote $Z = \{X: X = x(v,0),\ v \in V\}$ and recall that $N_U(G)$ denotes the U's component of the Nash solution of a bargaining game G. The following proposition allows to simplify the way of solving (11):

Proposition 3. Problem (11) is equivalent to the one-stage problem,

$$s* \; = \arg\max_{s\in[0,1]} X_U(s, \hat{\tau}(s), \hat{B}(s, \hat{\tau}(s)), 0) = \arg\max_{s\in[0,1]} N_U(Z, x(v^0, s)) \; . \tag{12}$$

The one stage problem (12) is usually relatively easy to solve.

Thus, we can conclude by saying that the optimal flexible threat, as a solution to (4), is characterized by the following action intensity, length of action, and bargaining schedule:

$$\begin{aligned} &s^* \, , \\ &\tau^* = \hat{\tau}(s^*) \, , \\ &B^* = \hat{B}(s^*, \tau^*) \, , \end{aligned}$$

where s^* is computed according to (12), and the mappings $\hat{\tau}$ and $\hat{B}$ are given, respectively, by (10) and (8) .

4. Concluding Remark

In the paper, the notion of multi-dimentional, flexible threats been introduced. Threats of this kind are facts of economic life, e.g. they are often used in negotiation processes such as labor-management bargaining or in politics.

In general, our finding is that the three parameters characterizing the optimal threat are interconnected, what means that they should be considered jointly. Although the propositions presented suggest a way of deriving optimal threat parameters, the results of the analysis should be treated as methodological rather then substantive.

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LOGICAL NEURAL NETWORKS IN PROBLEMS OF CONCEPT ELICITATION AND VALIDATION

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Abstract: We derive a distributed, neural network-oriented architecture of concept elicitation. The proposed structure is directly implied by logical constructs of fuzzy sets and reflects a logical nature of the problem. The concepts are formally described by means of disjunctive forms of several conjunctive terms defined in a space of features of the objects. The use of positive and negative instances will give rise to two different distributed structures. They separately contribute to decisions about the described concept and its counterconcept and allow to characterize a range of concept descriptors provided by the network.

Keywords: concept elicitation, knowledge acquisition, distributed information processing, logic processing, fuzzy sets, decision making.

1. Introduction

The problem of elicitation of concepts from a given collection of objects (such as e.g., patterns or decision situations) becomes an essential issue in many areas of applications, see Gilmore (1986), Michalski (1983), Valiant (1985). The background requirement is that a concept should represent its available objects (positive instances) to the highest extent and exclude all objects viewed as its negative instances.

The approach developed in this paper looks at this problem by describing a concept as a sort of multivalued disjunctive form of conjunctive terms. In this way we can directly cope with an underlying logical framework of the concept elicitation. We will introduce a distributed neural network-based structure realizing this expression. Afterwards learning is worked out by changing parameters of the conjunctive and disjunctive terms.

In comparison to a standard way in which one attempts to handle the concept itself we will propose an additional neural network. Its role is to form a model of a counterconcept using negative instances. Combining the two neural networks (for the concept and the counterconcept) one derives an interesting and useful property of consistency of the concept across the objects of the domain.

In the remainder of the paper we will treat all objects as elements of a multidimensional unit hypercube, say $\mathbf{x} \in [0,1]^n$. Furthermore logical operations will be realized by means of triangular norms (t- and s-norms). We will start with basic functional components (Section 2). Logical processings\ in neural networks leading to conjunctive and disjunctive concept descriptions is studied in Section 3. The issue of concept elicitation completed with the aid of two dedicated neural networks is considered in Section 4. Finally numerical experiments are reported.

2. AND and OR Logical Neurons

The AND logical neuron with n inputs $x_1, x_2, ..., x_n$ realizes a function

$$y = (x_1 \text{ OR } w_1) \text{ AND } (x_2 \text{ OR } w_2) \text{ AND} \dots \text{AND } (x_n \text{ OR } w_n) \quad (1)$$

An influence of each variable (x_i) is first modulated by a weight factor w_i. The higher the value of w_i the less evident impact of x_i. In limit cases on gets: if $w_i = 0$ x_i OR $w_i = x_i$, $w_i = 1$ x_i OR w_i = 1 (the result does not depend on x_i). The global aggregation is carried out by ANDing successive partial results. The neuron as described by (1) conveys only excitatory characteristics i.e., higher values of input signals generate higher values of output y. On the other hand, an inhibitory performance of the neuron is achieved by incorporating complements of x_i, $\bar{x}_i = 1 - x_i$. This extends the generic equation into the form

$$y = (x_1 \text{ OR } w_1) \text{ AND } (x_2 \text{ OR } w_2) \text{ AND} \dots \text{AND } (x_n \text{ OR } w_n) \text{ AND}$$
$$\text{AND } (\bar{x}_1 \text{ or } w_{n+1}) \text{ AND } (\bar{x}_2 \text{ OR } w_{2+n}) \text{ AND} \dots \text{AND } (\bar{x}_n \text{ Or } w_{2n})$$

The equation is rewritten in terms of triangular norms

$$y = \underset{i=1}{\overset{n}{T}} (x_i \text{ s } w_i) \text{ t } \underset{i=1}{\overset{n}{T}} (\bar{x}_i \text{ s } w_{i+n})$$

Setting n = 1, the AND neuron has two inputs

$$y = (x_1 \text{ s } w_1) \text{ t } (\bar{x}_1 \text{ s } w_2)$$

Let us illustrate its performance in this simple case. To accomplish it we first have to specify triangular norms. Note that t- and s-norms implementing the neuron could be selected independently (their duality is not necessary). Let us discuss s-norm as a probabilistic sum and treat the t-norm as a product,

$$y = \min((x_1 + w_1 - x_1 w_1), (\bar{x}_1 + w_2 - \bar{x}_1 w_2)) =$$
$$= \min (w_1 + (1 - w_1)x_1)$$

The output of the neuron for $w_1 = 0.2$ and $w_2 = 0.7$ is visualized in Fig. 1

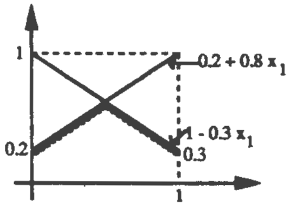

Fig. 1 AND neuron with $w_1 = 0.2$ and $w_2 = 0.7$

Note that the neuron favors a certain region of the input signal. The selectivity of the neuron for a given region of input values can be achieved by setting up appropriate connections w_1 and w_2.

The OR logical neuron is composed of input signals by carrying out OR operations. Its equation reads as

$$y = (x_1 \text{ AND } w_1) \text{ OR } (x_2 \text{ AND } w_2) \text{ OR } \ldots \text{ OR } (x_n \text{ AND } w_n) \qquad (2)$$

or

$$y = \underset{i=1}{\overset{n}{S}} (x_i \, t \, w_i)$$

3. Logical Processing in Neural Network Structures

3.1 Conjunctive and disjunctive classes of concept description

The logical formulas expressed by (1) - (2) can be directly mapped onto a series of logical elements realizing weighted AND and OR operations. They are structured as a three layer neural network in which all nodes in the hidden layer are of the AND type while the output layer has a single OR neuron. Its topology clearly reveals that the network is fully connected. Furthermore, in comparison to standard neural networks, the discussed network is composed of different processing units (neurons). This type of heterogeneous topology enhances representation capabilities of the structure.

From a functional point of view each layer completes a distinct function:

- each neuron of the hidden layer forms a region of the feature space by AND-ing (and weighting) x_i's and these complements. The output z_j, $j = 1,2,\ldots,h$, describes then a single conjunction of the features; refer also to the functions of the AND neuron described in Section 2. Denote the regions formed by the hidden layer by $\Omega_1, \Omega_2, \ldots, \Omega_h$, respectively.
- the output layer describes the concept in disjunctive form (OR) of all the regions provided by the hidden layer. Again their individual contribution is modified by connections v_j, $j = 1,2,\ldots,h$.

The network given above defines the concept in a disjunctive form of several conjunctive expression in the feature space.

A dual structure describing the concept can be viewed as a product (conjunction) of disjunctive forms. The resulting topology is characterized by a hidden layer consisting of OR neurons and followed by a single AND processing unit at the output.

3.2 Learning in the network

Learning in the network can be accomplished with the aid of standard optimization techniques. The idea introduced here follows basic concepts of the BP algorithm. The process of learning, as it is obvious from the statement of the problem, if fully supervised. Given is a collection of positive and negative instances. The performance index is defined as a sum of squared errors.

$$Q = \sum_{k=1}^{N} (t_k - N(x_k, \mathbf{connections}))^2$$

Where $N(\bullet)$ denotes the output of the neural network for instance $\mathbf{x}_k$, while t_k takes on two binary values

$$t_k = \begin{cases} 1, & \text{for positive instances} \\ 0, & \text{for negative instances} \end{cases}$$

The connections are successively updated following the adjustment rule which is stated symbolically as

$$\mathbf{adjustment\text{-}of\text{-}connections} \sim -\frac{\partial Q}{\mathbf{connections}}$$

Detailed considerations, final numerical schemes as well as some learning suggestions in cases of nondifferentiable triangular norms are well documented and can be found elsewhere, see Pedrycz (1991, 1992).

4. Concept Elicitation Through Two Logical Neural Networks

The description of the concept is performed in a supervised mode applying positive and negative instances (examples). These instances are used to guide learning in the neural network. In comparison to a common approach we take advantage of positive and negative instances constructing two separate networks.

(i) The first network is trained to build a descriptor of the concept. Its output denoted by $y_p(= N_p(\mathbf{x}))$ characterizes a degree to which $\mathbf{x}$ can be viewed as compatible with this concept. The values of y_p close to 1 represent a high level of compatibility.

(ii) The second network is constructed to represent counterconcept. In its training the negative instances are renamed and viewed as "positive" instances of the counterconcept (in other words the membership values of the instances are flipped, $0 \leftrightarrow 1$). After training the output of the network $y_n(= N_n(\mathbf{x}))$ specifies a degree to which $\mathbf{x}$ is prototypical in the sense of the counterconcept.

Note that the constructions of the two networks ($N_p(\bullet)$, $N_n(\bullet)$) are carried out separately. Each object $\mathbf{x}$ can be categorized as equivalent to the concept on the following rule of assignment,

if $N_p(\mathbf{x}) > N_n(\mathbf{x})$ then $\mathbf{x}$ is concept-consistent

From a structural point of view the two neural networks $N_p(\bullet)$ and $N_n(\bullet)$ are put in parallel and followed by a simple discriminator performing maximum over y_p and y_n.

Obviously, the way in which the networks have been designed does not impose any relationship between y_p and y_n for a given object in the feature space. Generally speaking, the equality $y_p + y_n = 1$ could not be satisfied, however for the zero performance index Q it holds for each element of the training set. The general property is that

$$y_p + y_n < 1$$

which illustrates a lack of evidence when the concept is verfied with respect to all objects in the feature space. This interesting observation has far reaching consequences on defining confidence associated with the constructed concept. Formally we put it down accordingly:

the concept Ω in $[0,1]^n$ elicited on the basis of the training set $\mathfrak{F}$ is γ-consistent, $\gamma \in [0,1]$, if it consists of objects $\mathbf{x}$ satisfying the following consistency condition

$$N_p(\mathbf{x}) + N_n(\mathbf{x}) > \gamma \quad \text{and} \quad N_p(\mathbf{x}) > N_n(\mathbf{x}) \tag{3}$$

Let

$$B(\gamma) = \{\mathbf{x} \in [0,1]^n | (3) \text{ is satisfied}\}$$

denote all objects satisfying the condition of γ-consistency. An obvious relationship holds:

$$B(\gamma_1) \subset B(\gamma_2) \quad \text{if } \gamma_1 \geq \gamma_2$$

5. Illustrative Studies

An example below will be used to illustrate the performance of the method and comment on derived results of concept building. The training set consists of few positive and negative examples distributed in the corners of the unit square, see Fig. 2

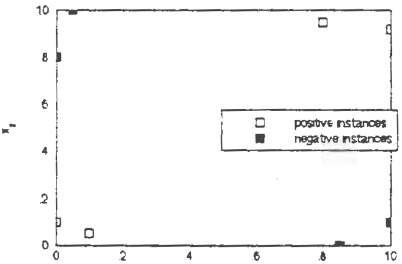

Fig. 2 Training set used in the experiment

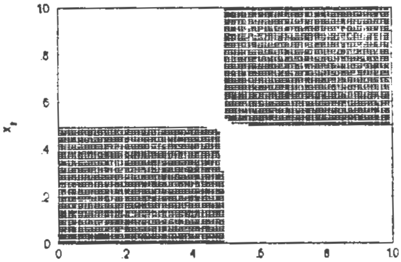

Fig. 3 Objects consistent with the concept at level 0.0, B(0)

The triangular norms used in the experiment were chosen as a probabilistic sum and a product.

The learning with the two nodes in the hidden layer produced good mapping results. The formulas resulting from the networks are given below

$$y_p = (0.983 \text{ AND } z_1) \text{ OR } z_2$$
$$z_1 = (0.988 \text{ OR } x_1) \text{ AND } (0.992 \text{ OR } x_2) \text{ AND } \bar{x}_1 \text{ AND } \bar{x}_2$$
$$z_2 = (0.011 \text{ OR } x_1) \text{ AND } (0.004 \text{ OR } x_2)$$
$$y_n = (0.996 \text{ AND } z_1) \text{ OR } (0.99 \text{ AND } z_2)$$
$$z_1 = (0.008 \text{ OR } x_1) \text{ AND } (0.997 \text{ OR } \bar{x}_1) \text{ AND } \bar{x}_2$$
$$z_2 = (0.011 \text{ OR } x_2) \text{ AND } \bar{x}_1$$

One can recognize that the training examples can be structured into a concept of (multivalued) equivalency, namely $x_1 = x_2$.

The overall distribution of the values of the sum $(y_p + y_n)$ for the uniformly distributed elements of $[0,1]^2$ is given in Fig. 3. Subsequently Fig. 4 and 5 illustrate regions consistent with the concept at the level $\gamma = 0.95$ and residual areas (the objects of which are neither accepted nor rejected).

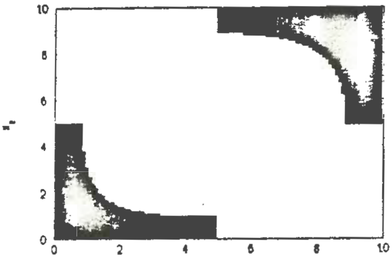

Fig. 4 Objects concept-consistent at $\gamma = 0.95$, B(0.95)

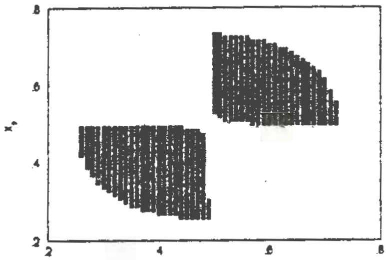

Fig. 5 $y_p + y_n < 0.90$, $y_p > y_n$

6. Conclusions

We have proposed logical neural networks to structure examples into concepts. The two separate networks are designed to handle descriptors of the concept and its associated counterconcept. This information put together makes it possible to express consistency of any object with respect to the concept as well as to form subsets of objects being essentially consistent with the concept.

Acknowledgment

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7. References

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