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Editors:

Roman Kulikowski
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Andrzej Straszak

Systems Research Institute
Polish Academy of Sciences
Warsaw, Poland

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AN APPROACH TO GROUP DECISION MAKING UNDER UNCERTAINTY WITH APPLICATION TO PROJECT SELECTION

G. R. Sotirov, E.B. Krasteva

Systems Optimization Laboratory, Faculty of Automatics
Technical University
Sofia 1156, Bulgaria

Abstract In this paper we present an interactive procedure for group decision making under imprecision of judgments applied to problems containing explicitly given resource constraints, such as project selection. Alternatives (projects) are evaluated in a quantitative scale. Based on these estimates fuzzy group preferences are determined and projects are divided into domination levels. Necessary quantities of resources are given in the form of intervals. Projects from successive levels are selected until maximum resource utilization is achieved.

Keywords strength of preference, fuzzy preference, aggregation, domination level, project selection.

INTRODUCTION AND PROBLEM STATEMENT

Decision making problems solved now usually require knowledge and experience of a group of experts. This is caused by necessity to consider possible influence of a lot of nonmeasurable factors which cannot be judged by a single person. Expert estimates are often imprecise due to insufficiency of information and verbal way of expression. That is why ordinal judgments are usually preferred or fuzzy set theory is applied to handle uncertainty.

Group decision making problem is stated as follows. Let A be a finite non-empty set of alternatives. They are evaluated by each member of a group of experts for a set of criteria. Based on these estimates an alternative is chosen or all alternatives are ranked. We assume that an expert may evaluate alternatives for all criteria or only for some of them.

A good example of decision making problem whose solution is usually based on expert judgments is project selection: out of a set of projects (alternatives) a subset must be chosen in such a way that best results for a set of criteria to be achieved and projects to be realized with available resources, i.e. explicitly given constraints are included in the model. It can be solved through mathematical programming or specific multiattribute decision making procedures. Project selection is usually modelled as 0-1 linear programming problem, either deterministic (Ringuest and Graves, 1989; Stewart, 1991), or fuzzy (Dias, 1988), which can be solved using available techniques and software. But, on the other hand, in case of project interactions the model is nonlinear and it is difficult to specify its parameters (Fox, Baker and Bryant, 1984). That is why multiattribute decision making methods handling explicitly given constraints are advantageous because of simple and psychologically appealing data they require.

GROUP DECISION MAKING METHODS AND THEIR APPLICATION TO PROJECT SELECTION

Group decision making methods differ according to the kind of expert estimates and presence of uncertainty. The latter usually stems from insufficiency of available information and verbal way of stating judgments. Estimates can be ordinal or cardinal. Ordinal ones include ranking alternatives and pairwise comparisons. Individual estimates are aggregated to form a group one through social choice functions (Hwang and Lin, 1987).

Since an alternative is usually preferred to another not absolutely but to a certain degree, it is better to model estimates with fuzzy preferences and to apply fuzzy social choice functions (Montero, 1987; Tanino, 1990).

When estimates are cardinal, utility functions can be used. A problem associated with this type of evaluation is that it is difficult to specify exact numerical estimates, especially when criteria are qualitative. An expert can usually give only verbal judgments. In this case fuzzy multiattribute utility functions (Seo and Sakawa, 1985), or ranking fuzzy numbers (Buckley, 1985) are used.

To solve project selection problems group decision making methods must be modified to handle explicitly given resource constraints. For cardinal estimates usually utility functions are applied (Ahmed and Gupta, 1987). Methods based on ordinal data are less investigated. Such methods are proposed by Cook and Seiford (1982), Brans, Vincke and Mareschal (1986).

PROPOSED APPROACH

The proposed approach is based on converting quantitative level estimates of projects into fuzzy group preferences. Resource requirements are presented in interval form.

The following notation is used: m - number of projects (alternatives); p - number of criteria; n_k - number of criteria for which expert k evaluates projects; t - number of resources; n_s - number of experts evaluating requirements for resource s; l_{iqk} - level assigned by expert k to project i for criterion q; w_{qk} - level assigned to criterion q by expert k; $s_{qk}(i,j)$ - expert k's strength of preference of project i to project j for criterion q; $r_{qk}(i,j)$ - expert k's fuzzy preference of project i to project j for criterion q; $r_k(i,j)$ - expert k's fuzzy preference of project i to project j; r(i,j) - fuzzy group preference of project i to project j; G_s - total amount of resource s; $[g_{iks}{}^L, g_{iks}{}^U]$ - lower and upper bound of amount of resource s necessary for realization of project i according to expert k; $[g_{is}{}^L, g_{is}{}^U]$ - lower and upper bound of amount of resource s necessary for realization of project i according to the group. In all notations $i,j=1,...,m$, $k=1,...,n$, $q=1,...,n_k$, $s=1,...,n_k$.

A fuzzy preference on a set of alternatives A is determined by a fuzzy set on the product set A x A, i.e. by a membership function μ_R: A x A --> [0,1] over the set A x A. We consider a finite set A. In this case fuzzy preference can be represented by a mxm matrix R with elements $r(i,j) \in [0,1]$ defined from the membership function μ_R:

$$r(i,j) = \mu_R(i,j), \; i,j=1,...,m \qquad (1)$$

Element r(i,j) represents the degree of preference of alternative i to alternative j; r(i,j)=0.5 means indifference between the two alternatives, r(i,j)=0 means definite preference of alternative j to alternative i, and r(i,j)=1 means definite preference of alternative i to alternative j. It is assumed that

$$r(i,j) + r(j,i) = 1, \; i,j=1,...,m \qquad (2)$$

Fuzzy preferences model real opinions better than crisp ones but it is not easy for an expert to define exact numeric values for them. That is why we propose to obtain fuzzy preferences from evaluations in a quantitative scale which is widely used.

A scale with a finite number of levels L is introduced. Levels are numbered from the best to the worst, L-1 corresponding to the best and 0 corresponding to worst. Each expert assigns a level to each alternative for each criterion. Strength of preference is determined for each pair of alternatives using the idea described by Cook and Kress (1985):

$$s_{qk}(i,j) = l_{iqk} - l_{jqk} \qquad (3)$$

Positive values of $s_{qk}(i,j)$ show that expert k prefers alternative i to alternative j and vice versa.

Krasteva, Narula and Sotirov (1992) propose to determine fuzzy preference of expert k for each pair of alternatives (i,j) with respect to criterion q as follows :

$$r_{qk}(i,j) = 0.5\left(1 + \frac{s_{qk}(i,j)}{s_{qk}^*}\right) \text{ if } s_{qk}^* \neq 0 \qquad (4)$$

$$r_{qk}(i,j) = 0.5 \text{ if } s_{qk}^* = 0 \qquad (5)$$

$$\text{where } s_{qk}^* = \max_{i,j} s_{qk}(i,j) \qquad (6)$$

Aggregation by criteria is done using weighting technique:

$$r_k(i,j) = \sum_{q=1}^{n_k} \frac{w_{qk}}{\sum_{q=1}^{n_k} w_{qk}} r_{qk}(i,j) \qquad (7)$$

where w_{qk} – weight of criterion q assigned by expert k. Criterion weights are defined as levels in the same scale.

Aggregation by experts is done using the function proposed by Tanino (1984):

$$r(i,j) = \frac{\sum_{k=1}^{n} \max (r_k(i,j) - 0.5, 0)}{\sum_{k=1}^{n} |r_k(i,j) - 0.5|}, \quad i \neq j \qquad (8)$$

$$r(i,j)=0.5, \qquad i=j \qquad (9)$$

Estimates r(i,j) are not defined if $r_k(i,j)=0.5$, i,j=1,...,m, for all k=1,...,n, i.e. if both alternatives are equally preferred by all experts. In this a case r(i,j) is assigned a value of 0.5.

It follows from (8),(9) that $r(i,j) \in [0,1]$, r(i,j)=1, i,j=1,...,m, iff $r_k(i,j) \geq 0.5$ for each k, k=1,...,n and r(i,j)=0, i,j=1,...,m, iff $r_k(i,j)<0.5$ for each k, k=1,...,n.

If r(i,j)=1, then alternative j is dominated by alternative i. We propose a way to divide alternatives into domination levels applying domination degrees $0.5<\alpha \leq 1$ chosen by decision maker (DM).

Definition: Alternative j belongs to domination level α, $0.5<\alpha \leq 1$, iff:

$$\text{(i)} \ \alpha \geq r(i,j), \ i \in \{1,2,...,n\}, \text{ and} \qquad (10)$$

$$\text{(ii)} \ \frac{k_+}{k_+ + k_-} \geq \alpha, \qquad (11)$$

where k_+ – number of experts for whom $r_k(i,j) > 0.5$; k_- – number of experts for whom $r_k(i,j) < 0.5$.

The second condition reflects the principle of nondictatorship stated by Arrow (1951).

Division of alternatives is done until there remains small number of alternatives in each level, e.g. no more than 3.

To solve project selection problem required amounts of resources for the alternatives (projects) must be known. We assume that an expert may evaluate necessary quantities of all resources or only some of them. Since an expert usually can judge only minimum and maximum possible amount of resource, estimates are presented in interval form. Bounds of group intervals are formed from individual as follows (Krasteva, Sotirov and Dobrev, 1992):

$$g_{is}^L = \min_k g_{iks}^L, \ k=1,...,n_s \qquad (12)$$

$$g_{is}^U = \max_k g_{iks}^U, \ k=1,...,n_s \qquad (13)$$

Total requirements of resource s for a subset of projects (domination level) are defined after (Buckley and Chanas, 1989):

$$[g_{is}^L, g_{is}^U]+[g_{js}^L, g_{is}^U]=[g_{is}^L+g_{js}^L, g_{is}^U+g_{js}^U] \qquad (14)$$

INTERACTIVE PROCEDURE

The proposed interactive procedure includes the following steps.

Step 1. DM determines the number of quantitative scale levels.

Step 2. Experts specify levels of projects and criteria, l_{iqk} and w_{qk} respectively, as well as upper and lower bounds of necessary quantities of resources $[g_{iks}^L, g_{iks}^U]$, i=1,...,m, k=1,...,n, q=1,...,n_k, s=1,...,n_k.

Step 3. Strength of preference $s_{qk}(i,j)$, q=1,...,n_k,k=1,...,n, is determined for each pair of projects i,j=1,...,m, using (3).

Step 4. Individual fuzzy preferences $r_{qk}(i,j)$, q=1,...,n_k, k=1,...,n, are obtained for each pair of projects i,j=1,...,m, using (4)-(6).

Step 5. Aggregation by criteria is performed using (7).

Step 6. Aggregation by experts is performed and projects of domination level 1 are separated applying (8).

Step 7. If each domination level contains no more than 3 projects, then go to step 9. Otherwise go to step 8.

Step 8. Degree of domination α is selected by DM and projects of domination level α are determined using (10),(11). Go to step 7.

Step 9. Lower and upper bounds of required amounts of resources for each project $[g_{is}^L, g_{is}^U]$, i=1,...,m, s=1,...,t, are determined using (12),(13),.

Step 10. Lower and upper bounds of required amounts of each resource for projects of each domination level $\underline{\alpha}$ $[g_{s\underline{\alpha}}^L, g_{s\underline{\alpha}}^U]$, $s=1,\ldots,t$, are determined using (14).

Step 11. The specified values of _ are arranged in increasing order: $\underline{\alpha}_1<\underline{\alpha}_2<\underline{\alpha}_3<\ldots<\underline{\alpha}_v<1$. Let $P_{\underline{\alpha}_z}$ be the set of projects of domination level $\underline{\alpha}_z$, $z=1,\ldots,v$, P – the set of selected projects, $[g_{sP}^L, g_{sP}^U]$ – lower and upper bound of necessary amount of resource s for selected projects set P. Resources are arranged by DM in order of priority decrease. Let s_h be resource of priority h, $h=1,\ldots,t$. Set domination level counter $N:=0$, resource counter $h:=1$, $P=P^0$, where P^0 – set of projects not belonging to any domination level.

Step 12. Resource utilization check:
- if $G_{sh} < g_{shP}^L$, then go to step 13;
- if $G_{sh} \in [g_{shP}^L, g_{shP}^U]$, then go to step 14;
- if $G_{sh} > g_{shP}^U$, then go to step 15.

Step 13. Set $N:=N+1$, $\underline{\alpha}=\underline{\alpha}_N$, $P - P \cup P_{\underline{\alpha}_N}$. Go to step 12.

Step 14. Set $h:=h+1$. If $h>t$ or if DM doesn't want to encounter the remaining resources, then stop. Otherwise go to step 12.

Step 15. Set $P=P \setminus P_{\underline{\alpha}_N}$, $N:=N-1$. Go to step 12.

CONCLUSIONS

An approach and interactive procedure for solving group decision making problems of project selection under imprecision of judgments is proposed. It is based on expert estimates expressed in a convenient form: levels in a quantitative scale for the projects and intervals for the resource requirements. Projects are divided into domination levels based on aggregated group fuzzy preference. Choice is done on the basis of resource utilization check. The procedure is computationally simple and easy to apply.

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