Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume II: Applications

Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

SRI PAS IBS PAN

Systems Research Institute
Polish Academy of Sciences

Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications

Editors

Krassimir Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 83-894-7554-5

A cohesion measure for expert preferences in group decision-making

Ana Tapia-Rosero[(1)(2)], **Guy De Tré**[(2)]

[(1)] Dept. of Electrical and Computer Engineering
ESPOL University
Campus Gustavo Galindo V., Km. 30.5 Via Perimetral, Guayaquil, Ecuador

[(2)] Dept. of Telecommunications and Information Processing
Ghent University
Sint-Pietersnieuwstraat 41, B-9000, Ghent, Belgium
Ana.Tapia@UGent.be, Guy.DeTre@UGent.be

Abstract

In this paper, we propose a cohesion measure to select a cluster of expert preferences in a group decision-making context. Through preference modeling, membership functions are used to express opinions setting the level of agreement over a specific criterion. Taking into account that it is possible to gather a large number of opinions through social media (e.g., facebook, twitter, linkedin, etc.) it is important to handle them properly. Thus, this proposal uses a shape-similarity approach to cluster similar opinions, represents each cluster by means of an interval-valued fuzzy set and provides a cohesion measure to calculate the level of togetherness among membership functions that are present in a cluster. The cohesion measure allows us to discriminate clusters that are relevant to represent expert preferences. An example that illustrates the application of the cohesion measure for expert preferences has been included.

Keywords: Cohesion measure, shape-symbolic notation, expert preference, intuitionistic fuzzy set, group decision-making.

Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications *(K.T. Atanassow, W. Homenda, O. Hryniewicz, J. Kacprzyk, M. Krawczak, Z. Nahorski, E. Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2014*

1 Introduction

Nowadays, it is common to involve a large number of participants to express their opinions by means of social media. Furthermore, several entities like governments and businesses, are increasingly interested in extracting useful information from it. Within this regard, opinions from different perspectives might be useful to make a high impact decision in a decision-making context. Here, a large group of people will be considered as experts. Furthermore, we consider that it is possible to group similar opinions in order to take a final decision among a reduced amount of them, i.e. groups of expert opinions considered as representative.

Using soft computing techniques, a person could express his/her preferences over a specific criterion through membership functions assigning his/her preferences P(x) as a matter of degree, i.e. $0 \leq \text{P(x)} \leq 1$, where 0 denotes the lowest preference level on the value x and 1 denotes the highest level of preference. On the assumption that similarly shaped membership functions reflect similar opinions, a method based on shape-similarity for clustering similar opinions is used [9].

Bearing in mind that it is possible to express a group of individual experts' preferences by using the proper representation, this proposal uses an approximate fuzzy set. Here, we will make use of an interval-valued fuzzy set and its mapping to an intuitionistic fuzzy set [1] to depict each cluster of opinions. Considering that several clusters might be obtained, it is necessary a measure that allows us to discriminate clusters that are relevant to represent the expert preferences. Within this paper, we propose a cohesion measure which takes into account the togetherness of membership functions in a cluster. The main advantage of this proposal lies in the fact that it is possible to obtain a representative cluster even in cases where some of the contained membership functions do not overlap but are close enough to be considered similar.

For the sake of illustration, let us consider the following example. A local government is wondering the proper locations of video cameras, for safety purposes, taking into account the residents' opinions. In this case, it is possible that some residents consider that a small distance between cameras is desired, since this may increase safety; while other residents might perceive small distances as an invasion of privacy. Considering that different perspectives of the selected problem deserve to be analyzed, the proposed cohesion measure allows us to discriminate representative groups of opinions. Here, the representativeness of a group is given by its trend and its cardinality. Afterwards, with representative opinions from different perspectives we could decide the suitability of locating the video cameras.

The remainder of this paper is structured as follows. Section 2 gives some preliminary concepts for clustering similar opinions. Section 3 defines our proposed cohesion measure and describes some desired features of representativeness according to the framework where these will be used. Section 4 shows an illustrative example to follow this proposal while demonstrates its applicability in a decision-making context. Section 5 concludes the paper and presents some opportunities for future work.

2 Preliminaries

This section defines preliminary concepts to properly understand the remaining sections. These include basic concepts on fuzzy sets, some definitions to cluster similarly shaped membership functions and the general idea of aggregation.

2.1 Basic Concepts on Fuzzy Sets

In a decision-making context, from the preference point of view, a membership function μ_A represents a set of more or less preferred values of a decision variable x in a fuzzy set A. Hereby, $\mu_A(x)$ represents the intensity of preference or preference level in favor of value x [2].

Without loss of generality, trapezoidal membership functions have been selected in this paper to represent the expert preferences over criteria. These might be convenient for experts due to the simplicity in selecting parameters *a, b, c*, and *d* (Equation 1) to represent their preferences [3].

$$\mu_A(x) = \begin{cases} 0 & , \quad x \leq a \\ \frac{x-a}{b-a} & , \quad a < x < b \\ 1 & , \quad b \leq x \leq c \\ \frac{d-x}{d-c} & , \quad c < x < d \\ 0 & , \quad x \geq d \end{cases} \tag{1}$$

2.2 A Shape Based Approach

To make this paper self-contained, trough this section we will recall some definitions used in [9].

Definition 1 *A **symbolic-character** is a representation of a segment in a membership function as a pair $\langle t,r \rangle$ with $t \in T$ and $r \in S$; where t represents the category of the segment and r depicts its relative length by means of a linguistic term.*

Within this paper, T = {+, −, 0, 1, L, I, H} and the linguistic term set S is depicted in Figure 1. Hereby, each segment of the membership function uses a sign {+,−} to represent its slope, a value {0, 1} to represent its preference level on the criterion and a letter {L, I, H} to denote a *low, intermediate* or *high* point (e.g., a peak in a triangular membership function corresponds to a high point annotated as *H*). Moreover, linguistic terms express the relative length of the segment on the X-axis compared to the sum of all segments' lengths (e.g., the label *ES* corresponds to an "extremely short" segment while label *EL* corresponds to an "extremely long" segment).

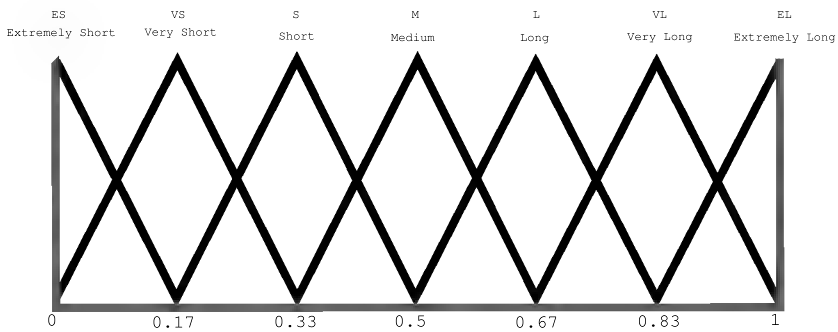

Figure 1: Linguistic terms and its semantics represented by triangular membership functions.

Figure 2 shows a trapezoidal membership function with five segments, each of them represented by a shape-symbolic character.

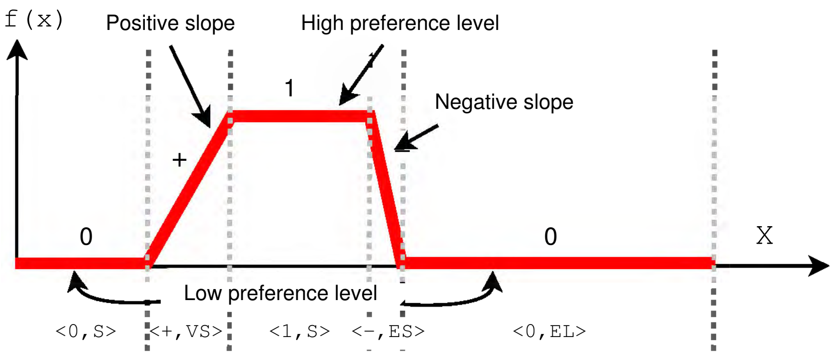

Figure 2: Segments of a trapezium and their corresponding shape-symbolic characters.

Definition 2 *A **shape-symbolic notation** depicts a membership function trough a sequence of shape-symbolic characters.*

Thus, the *shape-symbolic notation* for Figure 2 could be expressed as:

$$\langle 0,S\rangle\ \langle +,VS\rangle\ \langle 1,S\rangle\ \langle -,ES\rangle\ \langle 0,EL\rangle$$

2.3 Clustering Similar Opinions

On the assumption that similarly shaped membership functions reflect similar opinions, we use a shape similarity method proposed in [9]. The shape-similarity method receives as inputs several membership functions, each of them representing the opinion of an expert over a specific criterion, and builds clusters of similar opinions (Figure 3).

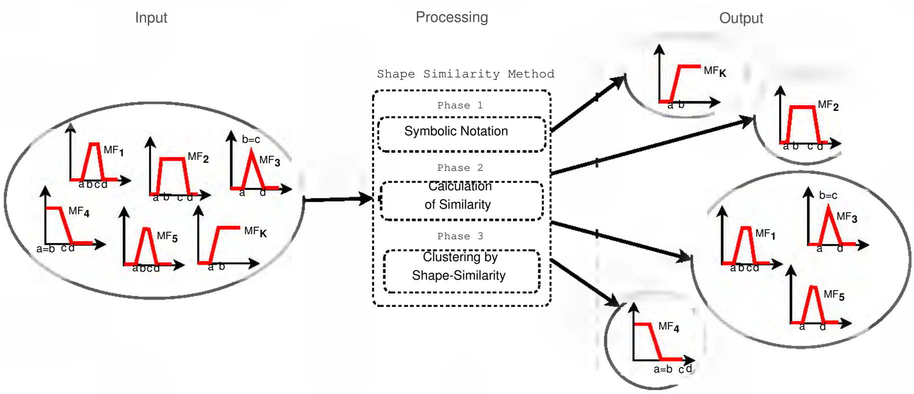

Figure 3: General architecture of the shape-similarity method.

The shape-similarity method has three phases summarized as follows:

1. A shape-symbolic notation for each normalized membership function is built.

2. A similarity measure in the unit interval among shape-symbolic notations is obtained, where 0 denotes no similarity and 1 denotes full similarity between them.

3. A clustering step is performed based on the aforementioned similarity measure between notations. The clustering stops when the highest similarity is considered too low according to a previously determined threshold.

Hereafter we will consider that different clusters containing similarly shaped membership functions were obtained and clusters that consist of a single membership function might be present. Figure 4 shows a sample of the obtained clusters, where it is feasible to identify different shapes for clusters 1, 29 and 30.

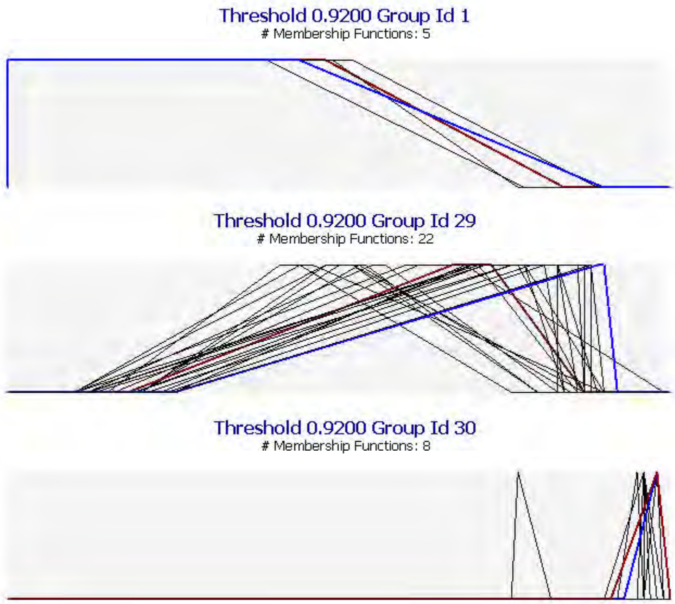

Figure 4: Sample of clusters containing similarly shaped membership functions.

According to the *a, b, c* and *d* values in Equation 1: Cluster 1 depicts the highest preference for values "above b" and the lowest preference for values "below a"; Cluster 29 depicts the highest preference for values "between b and c" and the lowest preference for values "below a" and values "above d"; and, Cluster 30 depicts the highest preference for a specific value b=c and the lowest preference for values "below a" and those "above d".

Within the scope of this proposal, the main disadvantage on summarizing expert preferences is that loss of information is possible. However, the use of shape-symbolic notations and, as we will see in the next section, the use of approximate fuzzy sets tries to avoid information loss. This is due to the mapping between membership functions and shape-symbolic notations, and the use of interval-valued fuzzy sets.

2.4 IVFS and IFS

We can notice that each cluster allows us to obtain the closest approximation to represent a group of expert opinions by means of its upper and lower bounds. Here, we represent a group of expert opinions through an interval-valued fuzzy set (IVFS) defined by Atanassov [1] as follows:

Definition 3 *An* interval valued fuzzy set *A (over a basic set E) is given by a function M_A(x) where M_A: E→INT(*$[0,1]$*), the set of all subintervals of the unit interval, i.e. for every x ∈ E, M_A(x) is an interval within* $[0,1]$.

IVFSs allows us to keep as much information as possible because they assign as membership an interval instead of a single number [8].

Furthermore an intuitionistic fuzzy set (IFS) [1] A in X is given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \tag{2}$$

where $\mu_A : X \to [0,1]$, represents the degree of membership of x in A; and $\nu_A : X \to [0,1]$, represents the degree of nonmembership of x in A, such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{3}$$

Notice that it is possible to create a mapping between the previously obtained interval-valued fuzzy set (IVFS) and its corresponding intuitionistic fuzzy set (IFS) [1] given by:

Definition 4 *(a) The map* f *assigns to every IVFS* A *an IFS* B = f(A) *given by* μ_B(x)= *inf* M_A(x), ν_B (x)= *sup* $M_A(x)$.

(b) The map g *assigns to every IFS* B *an IVFS* A = g(B) *given by* $M_A(x)$= $[\mu_B(x), 1 - \nu_B(x)]$.

Lemma 1 *(a) For every IVFS* A, g(f(A)) = A.

(b) For every IFS B, f(g(B)) = B.

For the sake of illustration, let us consider Figure 5 which corresponds to Cluster 29. Here, we could graphically observe the closest approximation to its upper and lower bounds represented by a solid and a dashed blue line respectively.

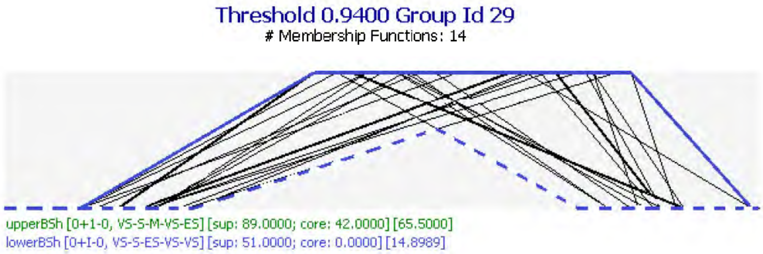

Figure 5: Cluster 29 and its corresponding upper and lower bounds.

2.5 Shape-Symbolic Notation for IVFS

Heretofore, the shape-symbolic notation for membership functions includes two components: a first component to represent the category of a segment(i.e., slope, point or level of preference), and a second component to represent its relative length through linguistic terms (i.e., from "ES" to "EL" corresponding to "extremely short" and "extremely long" respectively). However, considering that an IVFS provides a lower and an upper bound for each value, a third component to represent the *width* of each segment as shown in Figure 6 is needed.

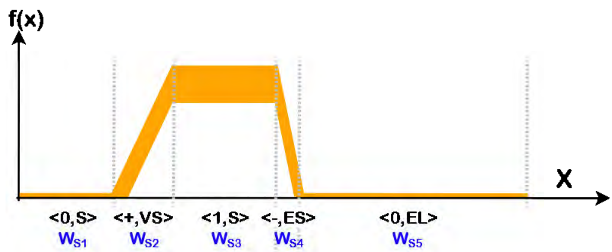

Figure 6: Shape-symbolic representation for IFS.

Although in the remainder of this paper we only consider the *width* component represented by linguistic terms, several strategies can be used (e.g., the use of interval values) and subject to further study. Linguistic terms have been selected considering that they are used in the second component and that they allow us to keep a simple notation. Considering that the width component could be associated with two consecutive linguistic terms, for the sake of simplicity we will use the linguistic term with the highest membership degree.

The linguistic term set U, representing the width component, includes the labels, linguistic terms and semantics indicated in Table 1.

Label	*Linguistic term*	*Semantic value*
EN	extremely thin	(0, 0, 0.17)
VN	very thin	(0, 0.17, 0.33)
N	thin	(0.17, 0.33, 0.5)
M	medium	(0.33, 0.5, 0.67)
K	thick	(0.5, 0.67, 0.83)
VK	very thick	(0.67, 0.83, 1)
EK	extremely thick	(0.83, 1, 1)

Table 1: Linguistic term set U and its semantics represented by triangular membership functions.

It is worth to mention that each segment might has a different *width* component in order to obtain the best approximate membership function. The proposed symbolic representation for IVFSs will facilitate the calculation of the cohesion measure for expert preferences.

2.6 Aggregation

Aggregation is the process of combining several numerical values into a single representative value [7]. Within this paper, some aggregation functions (i.e., aggregation operators in the fuzzy set context) will be used to combine numerical values that will allow us to discriminate a representative cluster among a group.

The definition of an aggregation function, taken from [7], has the domain $\mathbb{I}$ which is a nonempty real interval while the integer *n* represents the number of its variables as follows:

Definition 5 *An* aggregation function *in* $\mathbb{I}$ *is a function* $A^{(n)}(x) : \mathbb{I}^n \to \mathbb{I}$ *that*

(i) is nondecreasing (in each variable)

(ii) fulfills the boundary conditions

$$\inf_{x \in \mathbb{I}^n} A^{(n)}(x) = \inf \mathbb{I} \;\; and \;\; \sup_{x \in \mathbb{I}^n} A^{(n)}(x) = \sup \mathbb{I}$$

Based on the aforementioned definition, there are several aggregation operators that might be used including the arithmetic mean, conjunctive and disjunctive aggregators. The properties of each aggregation operator might be considered as a guide, for their proper selection, according to the context where the aggregators will be used. Although the study of different aggregators and their properties are out of the scope of this paper, within this section we will show the generalized conjunction/disjunction (GDC) function that is used to illustrate our approach.

2.6.1 Generalized Conjunction/Disjunction

GDC is a continuous logic function that integrates conjunctive and disjunctive properties in a single function [6], denoted as $y = x_1 \diamondsuit \ldots \diamondsuit x_n, x_i \in I = [0,1], i = 1, \ldots, n$, and $y \in I$. GDC includes two parameters: the *andness* and the *orness*. The *andness*, $\alpha \in I$, expresses the conjunction degree while the *orness*, $\omega \in I$, expresses the disjunction degree [4]. These parameters are complementary, i.e., $\alpha + \omega = 1$.

In [4] the location of GDC with respect to conjunction and disjunction is defined as follows:

$$\begin{aligned} x_1 \diamondsuit \ldots \diamondsuit x_n &= \omega(x_1 \vee \ldots \vee x_n) + (1-\omega)(x_1 \wedge \ldots \wedge x_n) \\ &= (1-\alpha)(x_1 \vee \ldots \vee x_n) + \alpha(x_1 \wedge \ldots \wedge x_n) \\ &= \omega(x_1 \vee \ldots \vee x_n) + \alpha(x_1 \wedge \ldots \wedge x_n) \end{aligned}$$

If $\alpha > 0.5 > \omega$, the expression $x_1 \diamondsuit \ldots \diamondsuit x_n$ is called *partial conjunction* and is denoted as $x_1 \Delta \ldots \Delta x_n$. If $\alpha < 0.5 < \omega$, the expression $x_1 \diamondsuit \ldots \diamondsuit x_n$ is called *partial disjunction* and is denoted as $x_1 \nabla \ldots \nabla x_n$. If $\alpha = \omega = 0.5$, the expression $x_1 \diamondsuit \ldots \diamondsuit x_n$ is called *neutrality function*, which is implemented as the arithmetic mean and is denoted as $x_1 \ominus \ldots \ominus x_n$.

Although the GDC can be implemented in various ways, within this paper we will only consider the multiplicative form (Equation 4). The multiplicative form is primarily applied for estimating the level of satisfaction of requirements [6] as follows:

$$x_1 \diamondsuit x_2 = (x_1 \nabla x_2)^q (x_1 \Delta x_2)^{1-q},\ 0 \leq q \leq 1. \tag{4}$$

Here q is used to adjust the level of orness and $1-q$ is used to adjust the level of andness. The aforementioned expression will allow us to obtain high levels of requirements x_1 and x_2.

3 A Cohesion Measure for Expert Preferences

The aim of this section is to provide a measure that allows us to discriminate clusters that are relevant to represent expert preferences in a group decision-making context. Thus, on the assumption that similar opinions are clustered by a shape based approach, a cohesion measure is defined as follows:

Definition 6 *The* shape-cohesion *is a togetherness measure among membership functions that are part of a cluster grouped by shape-similarity.*

A first attempt to compute the shape-cohesion measure is a straightforward geometrical approach. This geometrical approach takes into account the area contained between the boundaries (dark gray) compared to the total available area (light gray) as shown in Figure 7.

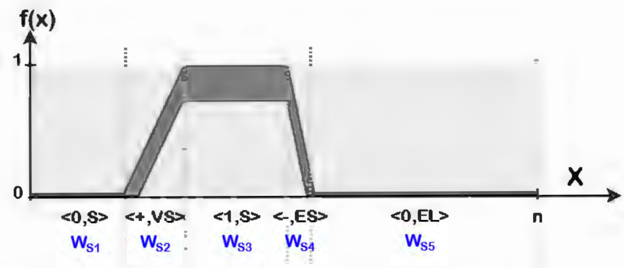

Figure 7: Present areas in a cluster grouped by shape-similarity.

The total available area is computed based on the domain of the membership function, while the area between the upper and lower boundaries might be computed using several strategies. Furthermore, different ways to compute the area for the IVFS allows us to obtain the same value. One of the strategies is based on the computation of a convex polygon area.

Equation 5 sets a general form to obtain the area between the upper and the lower boundaries on cluster c_i with threshold t given by:

$$area(c_i, t) = 1 - \frac{A^U - A^L}{A^T}. \tag{5}$$

Hereby, A^U denotes the area under the upper bound, A^L denotes the area under the lower bound and A^T corresponds to the total present area.

As an example, of this geometrical approach, let us consider Figure 8 which illustrates the cohesion measure for cluster 30 with thresholds 0.94 and 0.95.

Bearing in mind that the aim of this proposal is to discriminate representative clusters among a large group, it is useful to reduce the complexity of the aforementioned calculation through an approximation. Thus, our next attempt to obtain a cohesion measure takes into account that each segment of the membership function could be approximated to a rectangle as shown in Figure 9.

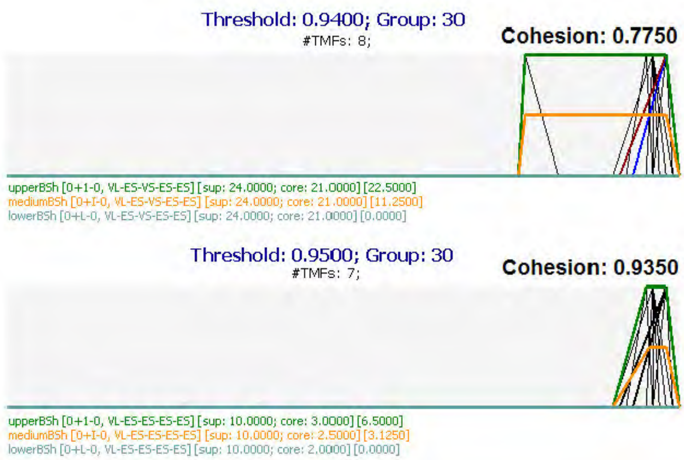

Figure 8: Cluster 30 with two different thresholds and their corresponding cohesion measures.

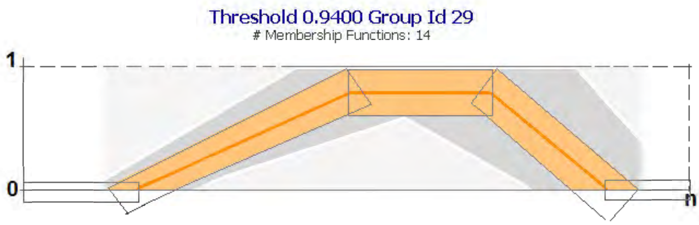

Figure 9: IVFS denoting segments by approximate rectangles.

Here, the rectangles are drawn around a solid line that represents the average between the lower and upper bounds. Notice that each rectangle is represented by its *length* and its *width*, and these values could be obtained through the symbolic-notation for the interval-valued fuzzy set explained in Subsection 2.5.

It is important to remember that the second component of the symbolic-notation represents the length of the segment on the X-axis. Thus, the *length* value could be directly used in segments expressing a specific preference level (i.e., segments without a slope). In the case that a rectangle with a slope (i.e., segments with a sign on the first component of their symbolic-notation) is present, a small computation is needed to obtain the length value properly. This additional computation, based on the Pythagoras' theorem, is feasible considering that we have the length value on the X-axis, as well as the starting and ending coordinates of the segment representing the rectangle's length. Regardless the category of the segment, the rectangle's *width* value corresponds to the third component of the shape-symbolic notation.

The aforementioned led us to a cohesion measure computed by means of a symbolic-notation, and that it is reduced to one minus the sum of all segment areas as follows:

$$cohesion(n_c) = 1 - \sum_{i=1}^{N} l(n_c(i)) \times w(n_c(i)). \qquad (6)$$

Hereby, n_c denotes the symbolic-notation of cluster c, N is the number of segments that are present in symbolic-notation n_c, $l(n_c(i))$ corresponds to the length component in notation n_c of segment i and $w(n_c(i))$ corresponds to the width component in notation n_c of segment i.

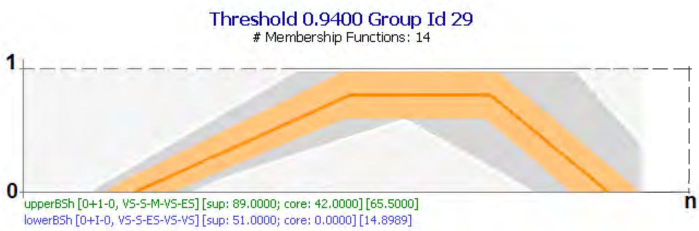

Figure 10: IVFS denoting its average and width for each segment.

In Cluster 30, we notice that the obtained cohesion value represents, to some extent, what we expect. A higher value shows more togetherness among the membership functions that are present in the cluster, while a lower value shows less togetherness. The main advantage of the proposed cohesion measure lies in that it is possible to obtain a representative cluster where some of the contained membership functions do not overlap but are close enough to be considered similar.

Notwithstanding, any cluster with a single membership function will obtain the highest cohesion value. Therefore, we consider that a representative cluster should also take into account the *relative number of membership functions* r_c. Here, r_c is given by the ratio between the number of membership functions in cluster c and the total number of membership functions.

If we compare the number of membership functions that belongs to each cluster, in a decision-making problem, we could evaluate if a specific cluster represents a majority, a minority or the same number of opinions expressed by the membership functions present in other clusters. It is achievable to have a solution, for some problems, based on this number. However, in other problems could be more important a reliable minority than a crowd.

Thus, any cluster might be categorized as *representative* based on a combination of the proposed cohesion measure and the relative number of membership functions. There are several strategies to integrate these values in the same formula. Let d_c^s be a discriminant of cluster *c* using strategy *s*. Therefore, if *s1* is a strategy using the α parameter denoting the importance of each component then Equation 7 represents the discriminant of cluster *c* in strategy *s1* as follows:

$$d_c^{s1}(\alpha) = \alpha \cdot cohesion(n_c) + (1 - \alpha) \cdot r_c. \tag{7}$$

Another strategy, *s2*, could use more specialized aggregators based on the Generalized Conjunction/Disjunction (GDC) which can be interpreted as a combination of conjunction and disjunction [6]. GDC in addition to model the relative importance of the criteria also satisfies an adjustable level of simultaneity among them.

$$d_c^{s2}(q) = (cohesion(n_c) \nabla r_c)^q \, (cohesion(n_c) \Delta r_c)^{1-q}, \; 0 \leq q \leq 1. \tag{8}$$

In this case, q is used to adjust the level of orness and $1 - q$ is used to adjust the level of andness. The aforementioned expression will allow us to obtain high levels of cohesion and the relative number of membership functions within the cluster.

The presented approach has the advantage that the result will be obtained based on the value of the selected parameters denoting the importance of each component (i.e., the cohesion measure and the relative number of membership functions). Moreover, more strategies might be considered according to the problem to be solved.

4 Illustrative Example

The cohesion measure for expert preferences presented in Section 3 is illustrated in the following example. Consider a local government which encourages the participation of all its residents. The local government has k=120 residents who are considered as “experts”’by the mayor of the city. All the experts were asked to supply their opinions over the “appropriate distance” (criterion) among video cameras for safety purposes. Our proposal suggests that each expert expresses what he/she understands to be the desired distance using parameters *a, b, c* and *d* to represent a membership function limited to the domain [0,n].

Since all opinions contribute to the final decision then different perspectives of the selected problem deserve to be analyzed. In this example, 120 opinions were randomly generated and different perspectives have been grouped by similarity as explained in Section 2.3. There are groups of opinions where residents perceive small distances between cameras as an increase of safety, groups representing the opinion of residents that might perceive small distances as an invasion of privacy, among others.

In order to select one or more representative clusters, within this example we will consider that a *representative* group of opinions is “A cluster with a ‘high’ level of togetherness and a ‘high’ number of membership functions”. Notice that this statement requires to satisfy simultaneously both components: the cohesion measure, as the togetherness level, and the relative number of membership functions.

For illustration purposes, let us consider cluster c=30 with threshold 0.95. The symbolic-notation n_{30} of this cluster is given by:

$$n_{30} = \langle 0, VL, EN\rangle\langle +, ES, EN\rangle\langle 1, VS, VN\rangle\langle -, ES, EN\rangle\langle 0, ES, EN\rangle.$$

Once we obtained different clusters of opinions, in order to categorize the clusters as representative, we should obtain the cohesion measure $cohesion(n_c)$ and the relative number of membership functions r_c for each cluster. Thus, the cohesion measure for cluster 30 using Equation 6 is:

$$\begin{aligned} cohesion(n_{30}) &= 1 - \textstyle\sum_{i=1}^{5} l(n_{30}(i)) \times w(n_{30}(i)) \\ &= 1 - (EN.VL + \sqrt{2}.ES + VN.VS + \sqrt{3}.ES + EN.ES) \\ &= 1 - [0(0.83) + \sqrt{2}(0) + 0.17(0.17) + \sqrt{3}(0) + 0(0)] \\ &= 1 - 0.0289 \\ &= 0.9711 \end{aligned}$$

It is worth to mention that segments i=2 and i=4 correspond to slopes in the membership function. Thus, the length value should be calculated considering that the length value represented in the symbolic-notation corresponds to the X-axis as mentioned in Section 3.

Then, the relative number of membership functions for cluster 30 denoted as r_{30} should be obtained. This value is obtained by the ratio between the number of membership functions in the cluster and the total number of membership functions. Considering that cluster 30 contains seven membership functions and the total number is 120, then the *relative number of membership functions* $r_{30} = 0.0583$.

After that a strategy to discriminate representative clusters must be selected. Within this paper, we proposed two strategies.

The strategy *s1* which uses the α parameter denoting the importance for the cohesion measure and the $1-\alpha$ value denoting the importance of the relative number of membership functions within the analyzed cluster based on Equation 7. Let us consider that discriminant d_{30}^{s1} with $\alpha = 0.75$ is obtained as follows:

$$\begin{aligned} d_{30}^{s1}(0.75) &= 0.75 \cdot cohesion(n_{30}) + (1-0.75) \cdot r_{30} \\ &= 0.75(0.9711) + (0.25)(0.058) \\ &= 0.7429 \end{aligned}$$

We can notice that the final values will depend directly on the selected α parameter. Here the 0.75 value was selected for illustration purposes.

The strategy *s2* uses a more specialized aggregator based on the Generalized Conjunction/Disjunction (GDC) which uses a parameter that specifies the desired level of conjunction (andness) or disjunction (orness).

Within this example a representative cluster is “A cluster with a ‘high’ level of togetherness and a ‘high’ number of membership functions”. Thus, it is necessary to satisfy the cohesion measure and the relative number of membership functions as a partial conjunction. Furthermore, considering that this aggregator must be implemented in several ways, it is possible to specify minimum and maximum values for each component. For example, consider that we would like to take into account those clusters with a level of togetherness above 0.5 and those with a minimun of five membership functions. Thus, let us tune up our example considering:

- A level of conjunction or andness as low as possible.
- A cohesion measure above 0.5
- The number of membership functions in the range of [5,120].
- The cohesion measure two times more important than the relative number of membership functions.

In this case the discriminant using strategy *s2* is obtained as follows:

$$\begin{aligned} d_{30}^{s2}(0.67) &= \max(0, \tfrac{cohesion(n_{30})-0.5}{1-0.5})^{0.67} \max[0, \min(1, \tfrac{120-r_{30}}{r_{30}-5})]^{0.33} \\ &= \max(0, \tfrac{0.9711-0.5}{0.5})^{0.67} \max[0, \min(1, \tfrac{120-7}{120-5})]^{0.33} \\ &= \max(0, 0.9422)^{0.67} \max[0, \min(1, 0.9826)]^{0.33} \\ &= (0.9422)^{0.67}(0.9826)^{0.33} \\ &= 0.95538 \end{aligned}$$

Notice that the implementation of the aggregator is out of the scope of this paper, but its use has been included for illustration purposes. More information of the use of graded logic with a detailed example can be found in [5].

After obtaining the cohesion measures and relative number of membership functions for all the clusters, a suitable strategy to discriminate representative ones should be selected. Thus, it is possible to select one or more representative groups of opinions based on the proposed cohesion measure. Afterwards, with representative opinions from different perspectives we could decide the suitability of locating the video cameras.

As expected, the proposed cohesion measure allows us to select a group with a high level of togetherness among them combined with a desired number of membership functions.

5 Conclusions

In this paper, a large number of expert preferences, expressed as membership functions, are gathered through social media. A shape-similarity approach is used to cluster similar preferences and a mapping from interval-valued fuzzy sets to intuitionistic fuzzy sets is used to represent a group of expert preferences over a specific criterion. Furthermore, an extension of the shape-symbolic notation, that includes a width component, expressing the hesitation margin of the group is proposed. Here, our main contribution is the shape-cohesion as a togetherness measure among membership functions that are part of a cluster grouped by shape-similarity. The cohesion measure combined with the relative number of membership functions allows us to discriminate clusters that are relevant to represent expert preferences in a group decision-making context.

Some crowdsource applications on different areas could be explored and they might be considered as opportunities for future work. Different strategies to select a representative cluster analyzing more cluster features, including an improvement to the proposed cohesion measure, are subject to further study.

References

[1] ATANASSOV, K. Interval valued intuitionistic fuzzy sets. *Intuitionistic Fuzzy Sets 31* (1999), 343–349.

[2] DUBOIS, D., AND PRADE, H. The three semantics of fuzzy sets. *Fuzzy Sets and Systems 90*, 2 (Sept. 1997), 141–150.

[3] DUBOIS, D., AND PRADE, H. *Fundamentals of Fuzzy Sets (THE HANDBOOKS OF FUZZY SETS Volume 7)*. Springer, 2000.

[4] DUJMOVIĆ, J. J. A comparison of andness/orness indicators. *Proceedings of the 11th Information Processing and Management of Uncertainty international conference (IPMU 2006)* (2006), 691–698.

[5] DUJMOVIĆ, J. J., DE TRÉ, G., AND VAN DE WEGHE, N. LSP suitability maps. *Soft Computing* (2010).

[6] DUJMOVIĆ, J. J., AND LARSEN, H. L. Generalized conjunction/disjunction. *International Journal of Approximate Reasoning 46*, 3 (Dec. 2007), 423–446.

[7] GRABISCH, M., MARICHAL, J.-L., MESIAR, R., AND PAP, E. *Aggregation Functions (Encyclopedia of Mathematics and its Applications)*, 1st ed. Cambridge University Press, New York, 2009.

[8] SANZ, J., FERNÁNDEZ, A., BUSTINCE, H., AND HERRERA, F. A genetic tuning to improve the performance of Fuzzy Rule-Based Classification Systems with Interval-Valued Fuzzy Sets: Degree of ignorance and lateral position. *International Journal of Approximate Reasoning 52*, 6 (Sept. 2011), 751–766.

[9] TAPIA-ROSERO, A., BRONSELAER, A., AND DE TRÉ, G. A method based on shape-similarity for detecting similar opinions in group decision-making. *Information Sciences 258* (Feb. 2014), 291 – 311.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2013

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN 838947554-5
9 788389 475541