POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

Redaktor techniczny
Iwona Dobrzyńska

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PREFACE

The present volume comprises the papers from the third Italian-Polish Conference which was held in Warszawa in May 1976, 10 to 11. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Kraków (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis applied to the economic, management, data processing and technological systems.

From the past, such meetings permit the exchange of views and experiences between the specialists from both countries what is the Institutes, applying to both sides, shows the natural path, the concentration on the systems and control theory, specially in the economic-mathematical directions, the essential development of the quantitative analysis and the particularly applicability of large computer systems.

The meaning of such meetings, however, goes far beyond that, what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — begin the friendly links between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

I. OPTIMIZATION AND CONTROL THEORY

C. Bonivento
G. Capitani
A. Tonielli
University of Bolonia

LINEAR DYNAMIC MODELLING FOR PROCESS CONTROL IN TECHNICAL AND ECONOMIC AREAS

1. INTRODUCTION

This paper will not propose completely new results but is an attempt to reconsider some methodological aspects of dynamic modelling for process control on the basis of a number of experiences carried out during the last five years at the Istituto di Automatica of the University of Bologna (Italy). Such experiences cover applications from engineering systems, such as chemical and power plants, to economic systems, such as a national monetary sector. The purpose of the mathematical modelling and the authors' cultural extraction constitute obvious limitations to the choice of the arguments and their development. Anyway further propositions about similarities and differences in modelling problems arising from technical and economic areas can be found, for example in [1], [2], [3].

Models are generally constructed for forecasting, hypothesis verification, dimensional design, control or regulator synthesis. The first two uses are classically emphasized in economic applications [4], while the latter ones are typical in engineering literature and practice [5].

But, as it is known, recently this distinction seems to become more and more slight. In fact the necessity of researching more significant tools for designing and testing management and control policies, especially at the macroeconomic level, has motivated the beginning of a common work between control theorists and economists in various countries [6], [7], [18], [19], so that the implications and the use of modern control theory now can be found also in modelling economic systems.

A model for control is quite different from a forecasting one. The latter should incorporate as many dynamic mechanisms of the reality as possible, so that it is usual to have very large and complex sets of nonlinear equations whose coefficients are very often statistically poorly determined on the basis of the available limited historical data.

Moreover the outturn of such a kind of models heavily depends on the consideration of correct future behaviours of input variables.

On the contrary, a model for control is preferably as simple as possible consistently with certain precision requirements. It is, for example linear because it must describe the behaviour of the reality only in the neighbourhood of some predetermined (desired) trajectories of the input and output variables. In fact the purpose of the regulator to be designed from the knowledge of such a model is to keep small the always present deviations from nominal paths. Moreover it is clear that only those endogenous variables which appear in the performance criterium are to be related to the exogenous variables.

Two important facts rise out from the previous considerations. Firstly, a model for control doesn't necessarily require parameters with a direct physical or economic significance in that these parameters will only be used for calculating the parameters of the regulator. Secondly the goodness of the model must be definitively decided on the basis of the success of the control policies eventually simulated on a larger theoretical model.

One may conclude that if the design of a control system is the final goal, rather simple empirical models can be obtained from i/o sequences (if available): this approach is usually adopted in technical applications. But if a better insight and understanding of the system internal properties is the aim, theoretical models (i.e. obtained from available *a-priori* knowledge of behaviour principles) are required.

In practice any modelling procedure is a blend of the two previous approaches and it can be schematically represented as in fig. 1, where the connections a, b, c are essentially inactive in classical economic applications, while connections d, e, f are not considered in black-box technical applications.

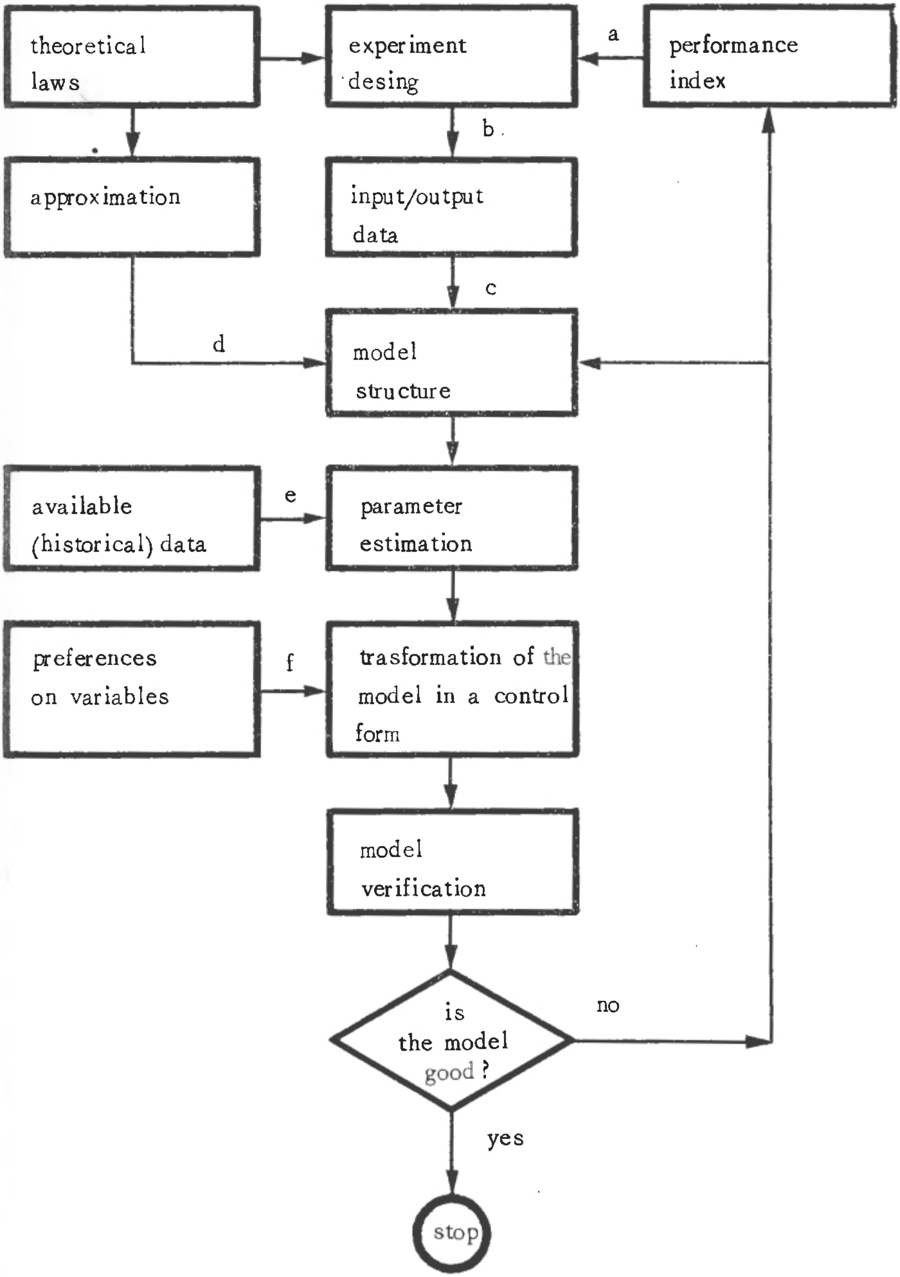
In sections 2, 3, 4 we first describe a set of results on the relations between internal models (i.e. state-space forms) and external models (i.e. input-output equations) which play a fundamental role for linear models building. In section 5 engineering examples are given to show the effectiveness of the resulting procedure starting from i/o experimental data. In section 6 the same results are applied to construct a model suitable for controlling the monetary sector from an *a-priori* available complex model of the italian economy [8], [9].

2. SOME BASIC RESULTS AND STATEMENT OF THE IDENTIFICATION PROBLEM

The situation considered in this section is represented in fig. 2. The system S is finite-dimensional time-invariant discrete and linear with input $u(k)= =[u_1(k)\dots u_r(k)]^T \in R^r$ and output $y(k)=[y_1(k)\dots y_m(k)]^T \in R^m$. The vectors $v(k)$ and $w(k)$ represent the additive zero-mean Gaussian noise affecting the available data $u^*(k)$, $y^*(k)$. The noise covariance matrix is

$$\mathrm{cov}\left(\begin{bmatrix} v(k) \\ w(k) \end{bmatrix}\right) = R\delta(k-j) \tag{2.1}$$

with R diagonal and $\delta(k-j)=1$ for $k=j$ and zero otherwise.


theoretical laws
experiment desing
a
performance index
b
approximation
input/output data
c
d
model structure
available (historical) data
e
parameter estimation
preferences on variables
f
trasformation of the model in a control form
model verification
is the model good ?
no
yes
stop
Fig. 1

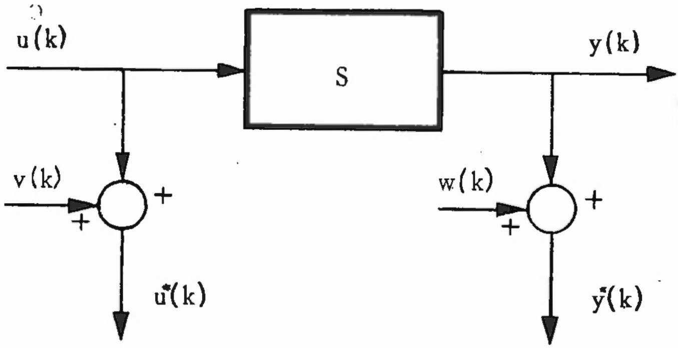


Fig. 2.

Without loss of generality, S is assumed to be completely observable and then [10] representable with the model

$$x(k+1) = Ax(k) + Bu(k) \tag{2.2.a}$$

$$y(k) = Cx(k)$$

where

$$A = \{A_{ij}\}$$

$$\dim A = n$$

$$\underset{(\nu_i \times \nu_i)}{A_{ii}} = \begin{bmatrix} 0 & \\ \vdots & I_{\nu_i - 1} \\ 0 & \\ a_{ii,1} & \dots \; a_{ii,\nu_i} \end{bmatrix} \tag{2.2.b}$$

$$\underset{(\nu_i \times \nu_j)}{A_{ij}} = \begin{bmatrix} 0 & \dots\dots\dots\dots & 0 \\ \vdots & & \vdots \\ 0 & \dots\dots\dots\dots & 0 \\ a_{ij,1} & \dots \; a_{ij,\nu_{ij}} \; 0 \dots & 0 \end{bmatrix} \tag{2.2.c}$$

$$\nu_{ij} = \begin{cases} \min(\nu_i + 1, \nu_j) & \text{if } j < i \\ \min(\nu_i, \nu_j) & \text{if } j > i \end{cases}$$

$$C = \begin{array}{c} \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & 1 & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & 0 & 1 & 0 & \dots \; 0 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \qquad \uparrow & \qquad \uparrow \\ 1 & \qquad (\nu_1+1) & \qquad (\nu_1+ \dots +\nu_{m-1}+1) \end{array} \end{array} \tag{2.2.d}$$

$$n = \sum_{i=1}^{m} \nu_i$$

while B has no special structural properties and only for notational convenience will be written in the following partitioned form

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix} \qquad \text{where } B_i = \begin{bmatrix} b_{i1}^T \\ b_{i2}^T \\ \vdots \\ b_{i\nu_i}^T \end{bmatrix} = \begin{bmatrix} b_{i1,1} & \dots & b_{i1,r} \\ b_{i2,1} & \dots & b_{i2,r} \\ \vdots & & \vdots \\ b_{i\nu_i,1} & \dots & b_{i\nu_i,r} \end{bmatrix} \tag{2.2.e}$$

The important features of the model (2.2) lie in the following basic results.

Theorem 2.1. — For every $i \in \{1, 2, \dots, m\}$ the dimension ν_i of A_{ii} is equal to the minimum integer n_i such that

$$(F^T)^{n_i} h_i^T \in \operatorname{span}\{Ant(n_i, i)\} \tag{2.3}$$

where $\operatorname{Ant}(p, q)$ denotes the set of all the vectors preceding $(F^T)^p h_q^T$ in the ordered sequence $h_1^T, h_2^T, \dots, h_q^T, \dots, h_m^T, F^T h_1^T, F^T h_2^T, \dots, F^T h_q^T, \dots$: h_q is the q-th row of the matrix $H = CT^{-1}$ and $F = TAT^{-1}$ for any nonsingular matrix T. $\lhd$

Theorem 2.2. — For any invariant index ν_i, the numbers $\{\alpha_{ij,k} : j = 1, \dots, m, k = 1, \dots, \nu_{ij} (\nu_{ii} \triangleq \nu_i)\}$, associated with the linear dependence relationship

$$(F^T)^{\nu_i} h_i^T \in \operatorname{span}\{Ant(\nu_i, i) \cap Reg\} \tag{2.4}$$

where Reg denotes the set of all the vectors $(F^T)^k h_j^T$, $k < \nu_{ij}$, $j = 1, \dots, m$, are equal to the corresponding parameters of A, i.e. $\alpha_{ij,k} = a_{ij,k}$. $\lhd$

The proof of these Theorems can be found in [11]. The number $\nu_i = \min_j \{n_j :$: (2.3) holds$\}$ is called the *i-th invariant index of the pair* (A, C). A further result which plays a basic role both in the position of the realization-identification problem and in its solution is the following:

Theorem 2.3. — The model (2.2) is equivalent to the input-output representation:

$$P(z)\, y(k) = Q(z)\, u(k) \tag{2.5}$$

where $P(z)=[p_{ij}(z)]$ $(i, j=1, \ldots, m)$, $Q(z)=[q_{ij}(z)]$ $(i=1, \ldots, m;\ j=1, \ldots, r)$ are polynomial matrices in the operator z, where z^{-1} is the unitary delay operator, with entries:

$$
\begin{aligned}
p_{ii}(z) &= z^{\nu_i} - a_{ii,\nu_i} z^{\nu_i - 1} - \ldots - a_{ii,2}\, z - a_{ii,1} \\
p_{ij}(z) &= -a_{ij,\nu_{ij}} z^{\nu_{ij}-1} - \ldots - a_{ij,2}\, z - a_{ij,1} \qquad (i \neq j) \\
q_{ij}(z) &= \beta_{i\nu_i,j}\, z^{\nu_i - 1} + \ldots + \beta_{i2,j}\, z + \beta_{i1,j}
\end{aligned}
$$

where the coefficients $\beta_{ik,j}$ ae the entries of the matrix $\bar{\boldsymbol{B}}=\boldsymbol{MB}$ in the same positions as the corresponding $b_{ik,j}$ in matrix $\boldsymbol{B}$, and the $(n \times n)$ transformation matrix $\boldsymbol{M}$ is the unitary-determinant matrix given by $\boldsymbol{M}=[\boldsymbol{M}_{ij}]$ $(i, j=$ $=1, \ldots, m)$ where:

$$
\underset{(\nu_i \times \nu_i)}{\boldsymbol{M}_{ii}} = \begin{bmatrix}
-a_{ii,2} & \cdots\cdots & -a_{ii,\nu_i} & 1 \\
-a_{ii,3} & \cdots & 1 & 0 \\
\vdots & & & \vdots \\
-a_{ii,\nu_i} & & & \vdots \\
1 & 0 \cdots\cdots & & 0
\end{bmatrix}
$$

$$
\underset{(\nu_i \times \nu_j)}{\boldsymbol{M}_{ij}} = \begin{bmatrix}
-a_{ij,2} & \cdots\cdots & -a_{ij,\nu_{ij}} \cdots & 0 \\
-a_{ij,3} & & & \vdots \\
\vdots & & & \vdots \\
-a_{ij,\nu_{ij}} & & & \vdots \\
0 & & & \vdots \\
0 & \cdots\cdots & & 0
\end{bmatrix} \qquad \Delta
$$

Outline of the proof.

Let us consider the canonical description (2.2). It can be noted that this representation partitions the system into m interconnected subsystems and that the j-th subsystem, because of the structure of the pair (A, C), is completely observable from the j-th component of the output vector.

It is in fact possible to write the state of the j-th subsystem as

$$
\begin{aligned}
x_{(\nu_1+\ldots+\nu_{j-1}+1)}(k) &= y_j(k) \\
x_{(\nu_1+\ldots+\nu_{j-1}+2)}(k) &= z y_j(k) - b_{j1}^T\, u(k) \\
x_{(\nu_1+\ldots+\nu_{j-1}+3)}(k) &= z^2 y_j(k) - b_{j2}^T\, u(k) - b_{j1}^T\, z u(k) \\
&\vdots \\
x_{(\nu_1+\ldots+\nu_j)}(k) &= z^{\nu_j - 1} y_j(k) - b_{j(\nu_j-1)}^T\, u(k) - \ldots - b_{j1}^T\, z^{\nu_j-2} u(k)
\end{aligned} \tag{2.6}
$$

Writing eq. (2.6) for $j=1, 2, \ldots, m$ the whole state of the system can be written as

$$x(k) = \boldsymbol{V}(z)\, y(k) - \boldsymbol{W}\boldsymbol{Z}(z)\, u(k) \tag{2.7}$$

where

$$\boldsymbol{V}(z) = \begin{bmatrix} 1 & \cdots & 0 \\ z & & \vdots \\ \vdots & & \vdots \\ z^{\nu_1 - 1} & & \vdots \\ 0 & & 0 \\ \vdots & & 1 \\ \vdots & & z \\ \vdots & & \vdots \\ 0 & \cdots & z^{\nu_m - 1} \end{bmatrix} \tag{2.8}$$

$$\boldsymbol{Z}(z) = \begin{bmatrix} I \\ zI \\ \vdots \\ z^{\nu_M - 1} I \end{bmatrix} \tag{2.9}$$

$\nu_M = \max_i(\nu_i)$

$$\boldsymbol{W} = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ b_{11}^T & & & & \vdots \\ \vdots & & & & \vdots \\ b_{1\nu_1-1}^T & \cdots & b_{11}^T & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & \cdots & \cdots & 0 \\ b_{m1}^T & & & & \vdots \\ \vdots & & & & \vdots \\ b_{m\nu_m-1}^T & \cdots & b_{m1}^T & 0 \cdots & 0 \end{bmatrix} \tag{2.10}$$

The substitution of (2.7) in the equation (2.2a) leads to the input-output description

$$[(z\boldsymbol{I} - \boldsymbol{A})\, \boldsymbol{V}(z)]\, y(k) = [(z\boldsymbol{I} - \boldsymbol{A})\, \boldsymbol{W}\boldsymbol{Z}(z) + \boldsymbol{B}]\, \boldsymbol{u}(k) \tag{2.11}$$

In equation (2.11) only the ν_1-th, $(\nu_1 + \nu_2)$-th, ..., n-th equations are significant; the remaining ones are simple identities. By selecting the significant equations in (2.11) the relation (2.5) follows by means of simple algebra.

The presence of an algebraical link between the input and the output ($D \neq \neq O$) simply modifies the equation (2.5) as

$$\boldsymbol{P}(z)\, y(k) = (\bar{\boldsymbol{Q}}(z) + \boldsymbol{P}(z)\, \boldsymbol{D})\, u(k) \tag{2.12}$$

where $\bar{\boldsymbol{Q}}(z)$ is a polynomial matrix whose degree is less than the degree of $\boldsymbol{P}(z)$.

Note that (2.7) allows for a simple calculation of the initial value of the state, given proper i/o sequences. It is useful to point out that the i-th equation from (2.5) can be written as

$$y_i(k+\nu_i) = \sum_{j=1}^{m} \sum_{l=1}^{\nu_{ij}} a_{ij,l}\, y_j(k+l-1) + \sum_{j=1}^{r} \sum_{l=1}^{\nu_i} \beta_{il,j}\, u_j(k+l-1) \tag{2.13}$$

$\triangle$

On the basis of the introduced results and notations it is possible to formalize the identification problem in the following way.

Assume that the set of numbers $\{\nu_i\}$ is *a priori* known. This hypothesis is in practice very limiting but will be relaxed in the following when the structure identification problem will be considered.

Given a noisy input-output sequence

$$\{u^*(k),\, y^*(k);\, k = 1, \dots, N_1\} \tag{2.14}$$

from S, let us consider the vector space R^{t_i} with dimension $t_i = 1 + \sum_{j=1}^{m} \nu_{ij} + r\nu_i$ $(\nu_{ii} \triangleq \nu_i)$ associated with the equation (2.13), for $i = 1, \dots, m$. The sequence (2.14) defines in R^{t_i} the points

$$\begin{aligned} \eta_i^*(k) = [&y_1^*(k) \dots y_1^*(k+\nu_{i1}-1); \dots; y_i^*(k) \dots y_i^*(k+\nu_i); \dots \\ &; y_m^*(k) \dots y_m^*(k+\nu_{im}-1); u_1^*(k) \dots u_1^*(k+\nu_i-1); \dots \\ &; u_r^*(k) \dots u_r^*(k+\nu_i-1)]^T \end{aligned} \tag{2.15}$$

$k = 1, \dots, N$ for a suitable $N < N_1$

Problem — Determine the i-th equation (2.13) parameter vector

$$\begin{aligned} \vartheta_i = (&a_{i1}, \dots, a_{i1,\nu i1} | \dots | a_{ii,1}, \dots, a_{i1,\nu ii} | \dots | a_{im,1}, \dots, a_{im,\nu im} | \\ &\beta_{i1,1}, \dots, \beta_{i1,r} | \dots | \beta_{i\nu_i,1}, \dots, \beta_{i\nu_i,r})^T \end{aligned} \tag{2.16}$$

which maximizes the limited information maximum likelihood (*LIML*) criterion

$$LIMLF = |\Psi|^{-N/2} \exp\left[-\frac{1}{2} \sum_{k=1}^{N} \varepsilon_i^T(k)\, \Psi^{-1} \varepsilon_i(k)\right]$$

where $\varepsilon_i(k)$ is the disturbance affecting the exact point

$\eta_i(k)$, i.e. $\eta_i^*(k) = \eta_i(k) + \varepsilon_i(k)$, and $\Psi = E\{\varepsilon_i(k)\, \varepsilon_i^T(k)\}$. Δ

In the noisy-free case the problem reduces to the determination of the parameter vectors ϑ_i such that:

$$\eta_i^T(k)\,\vartheta_i = 0 \qquad k = 1, \dots, N.$$

Note that the problem of the parameter identification of S has been decomposed into m independent problems, each supplying ϑ_i. The set $\{\vartheta_1, \vartheta_2, \dots, \vartheta_m\}$, if the indexes $\{\nu_i\}$ are known, completely identifies $\{A, B, C\}$, as it will be shown in the following section.

3. DETERMINISTIC CASE

The resolution of the identification problem previously stated mainly relies on the following realization algorithm from input-output sequences.

Given in R^{t_i} the noise-free points $\eta_i(k)$, $k=1, 2, \dots, N$, let us construct the matrix sequence:

$$\begin{aligned} &\boldsymbol{R}(\mu_1 \dots \mu_m | \mu_{m+1} \dots \mu_{m+r}) \triangleq [\boldsymbol{R}_1(\mu_1), \dots, \boldsymbol{R}_m(\mu_m), \dots, \boldsymbol{R}_{m+r}(\mu_{m+r})] \triangleq \\ &\triangleq [y_1(1) \dots y_1(\mu_1)| \dots |y_m(1) \dots y_m(\mu_m)|u_1(1) \dots u_1(\mu_{m+1})| \dots \\ &\dots |u_r(1) \dots u_r(\mu_{m+r})] \end{aligned} \tag{3.1}$$

where

$$y_i(k) = [y_i(k), y_i(k+1), \dots, y_i(k+N-1)]^T \tag{3.2}$$

$$u_i(k) = [u_i(k), u_i(k+1), \dots, u_i(k+N-1)]^T \tag{3.3}$$

and the indexes $(\mu_1, \dots, \mu_{m+r})$ are increased as follows $(1, 1, \dots, 1)$, $(2, 1, \dots, 1) \dots (2, 2, \dots, 2)$, $(3, 2, \dots, 2) \dots$

Notice that

$$\boldsymbol{R}(\nu_{i1}, \nu_{i2}, \dots, \mu_i = \nu_i+1, \dots, \nu_{im}|\nu_i, \dots, \nu_i) = \begin{bmatrix} \eta_i^T(1) \\ \eta_i^T(2) \\ \vdots \\ \eta_i^T(N) \end{bmatrix}$$

Because of the relation (2.13) the vector $y_i(\nu_i+1)$ is linearly dependent on the previously selected vectors and the dependence coefficients vector is just ϑ_i given in (2.16) where the normalized -1 component is associated with $y_i(\nu_i+1)$.

Therefore the determination of the invariant indexes $\{\nu_1, \dots, \nu_m\}$ and the parameter set $\{\vartheta_1, \dots, \vartheta_m\}$ can be performed by testing the linear dependence of the vectors (3.2) entering (3.1) in the following order:

$$y_1(1), y_2(1), \dots, y_m(1), u_1(1), u_2(1), \dots, u_r(1), y_1(2), \dots, y_m(2), u_1(2), \dots$$

For computational convenience, the dependence test can be carried out as a singularity test on the matrix sequence

$$S(1,1,\dots,1),\ S(2,1,\dots,1),\ \dots,\ S(2,2,\dots,2),\ S(3,2,\dots,2)\dots \tag{3.4}$$

where

$$S(\mu_1,\mu_2,\dots,\mu_{m+r}) \triangleq R^T(\mu_1,\mu_2,\dots,\mu_{m+r})\,R(\mu_1,\mu_2,\dots,\mu_{m+r}) \tag{3.5}$$

Summarizing, the previous discussion proves the following results [12], [14].

Theorem 3.1. — The maximal integers $\{\mu_1,\mu_2,\dots,\mu_m\}$, obtained according to the growth rule (3.4) and such that $S(\mu_1,\mu_2,\dots,\mu_m,\dots,\mu_{m+r})$ is non-singular, are the (A,C)-invariant indexes $\{\nu_1,\dots,\nu_m\}$ of Theorem 2.1. ◁

Theorem 3.2. — The linear dependence coefficients between the vectors $y_i(\nu_i+1)$ and the vectors in $R(\nu_{i1},\nu_{i2},\dots,\nu_{ii}\triangleq\nu_i,\dots,\nu_{im}|\nu_i,\dots,\nu_i)$, or equivalently between the corresponding vectors in the matrix S, are the components of the parameter vector (2.16). ◁

The complete realization algorithm (fig. 3) can be therefore formalized as follows:

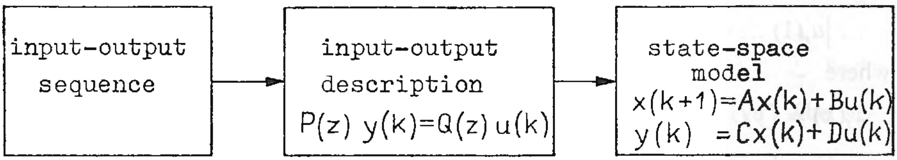


Fig. 3.

Step 1 (or structural determination): From $\{u(k),\ y(k),\ k=1,2,\dots\}$ compute the invariant indexes $\{\nu_1,\dots,\nu_m\}$ by testing the matrices (3.4). See fig. 4.

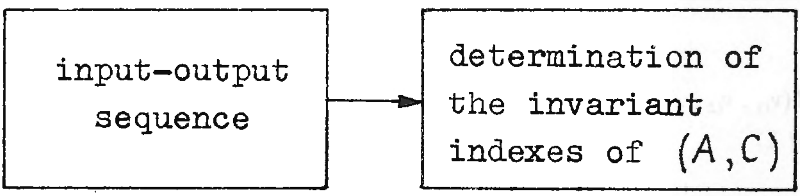


Fig. 4.

Step 2 (or parameters determination): From the knowledge of $\{\nu_1,\dots,\nu_m\}$ compute the dependence coefficient vectors $\{\vartheta_1,\dots,\vartheta_m\}$; i.e. the polynomial coefficients of $P(z)$ and $Q(z)$, by the formula:

$$\bar{\vartheta}_i = [S(\nu_{i1},\dots,\nu_{im}|\nu_i,\dots,\nu_i)]^{-1} R^T(\nu_{i1},\dots,\nu_{im}|\nu_i,\dots,\nu_i)\,y_i(\nu_i+1)$$

where $\bar{\vartheta}_i$, $i=1,2,\dots,m$, differs from ϑ_i only for the elimination of the normalized component -1. See fig. 5.

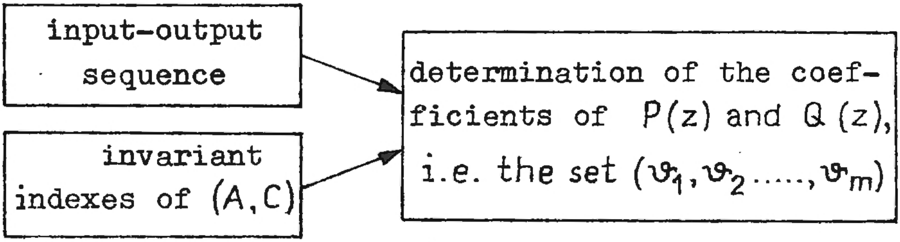


Fig. 5.

Step 3 (or state-space model construction): Directly write from $\boldsymbol{P}(z)$, $\boldsymbol{Q}(z)$ the matrices $\boldsymbol{A}$, $\boldsymbol{C}$, $\boldsymbol{M}$, $\overline{\boldsymbol{B}}$, $\boldsymbol{D}$*). Then compute $\boldsymbol{B}=\boldsymbol{M}^{-1}\overline{\boldsymbol{B}}$. See fig. 6.

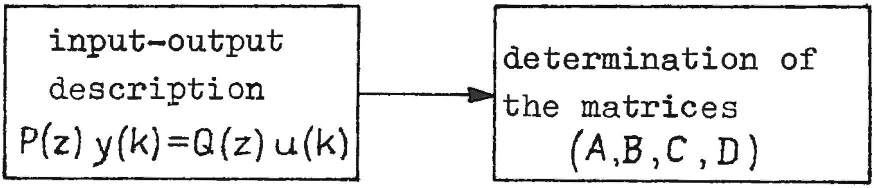


Fig. 6.

Step 4 (or initial state reconstruction): Compute $x(1)$ from (2.7), for $k=1$.

A more efficient even if more involved to deduce algorithm for executing Steps 3 and 4, based on a state-space model with a dual structure w.r.t. (2.2), is reported in [15].

4. SOLUTION OF THE IDENTIFICATION PROBLEM

In this section a computational solution to the Problem of Section 2 in the hypothesis of the presence of an equal amount of noise on the input and output components, i.e. $\boldsymbol{\Psi}=\sigma^2\boldsymbol{I}$, is given.

By virtue of the Koopmans-Levin theory [13] one has

$$\max_{\vartheta_i} LIMLF = \min_{\vartheta_i} \frac{\vartheta_i^T S_i^* \vartheta_i}{\vartheta_i^T \Psi \vartheta_i} \tag{4.1}$$

where

$$\boldsymbol{S}_i^* = \boldsymbol{R}_i^{*T}\boldsymbol{R}_i^*$$

*) If an algebraic link is present ($\boldsymbol{D}\neq\boldsymbol{O}$), from (2.12) we have

$\boldsymbol{Q}(z)=\overline{\boldsymbol{Q}}(z)+\boldsymbol{P}(z)\boldsymbol{D}$

so that

$\boldsymbol{P}^{-1}(z)\overline{\boldsymbol{Q}}(z)=\boldsymbol{D}+\boldsymbol{P}^{-1}(z)\overline{\boldsymbol{Q}}(z)$

and

$\boldsymbol{D}=\lim \boldsymbol{P}^{-1}(z)\boldsymbol{Q}(z)$.

$$R_i^* \triangleq R^*(\mu_1 = \nu_{i1}, \mu_2 = \nu_{i2}, \dots, \mu_i = \nu_i + 1, \dots, \mu_m = \nu_{im} | \mu_{m+1} = \\ = \nu_i, \dots \mu_{m+r} = \nu_i)$$

and the star denotes quantities constructed with noisy data.

As well known, the solution $\hat{\nu}_i$ of (4.1) can be obtained from the eigen-problem:

$$(S_i - \lambda_i I)\hat{v}_i = 0 \tag{4.2}$$

where λ_i is the minimum eigenvalue of S_i.

An explicit solution of (4.2) is given by the following Theorem.

Theorem 4.1. — The vector $\begin{bmatrix} -1 \\ \vartheta_i \end{bmatrix}$

where

$$\vartheta_i = [^{D*T}R_i^* - \lambda_i I]^{-1}\bar{R}_i^{*T}y_i^* \tag{4.3}$$

and $\bar{R}_i^*$ derives from R_i^* only by deleting the dependent vector $y_i^* \triangleq y_i^*(\nu_i+1)$, is the solution of (4.2).

Proof: By rewriting (4.2) as

$$\left\{\left[\begin{array}{c:c} y_i^{*T}y_i^* & y_i^{*T}\bar{R}_i^* \\ \hdashline \bar{R}_i^{*T}y_i^* & \bar{R}_i^{*T}\bar{R}_i^* \end{array}\right] - \lambda_i I\right\}\begin{bmatrix} -1 \\ \dots \\ \vartheta_i \end{bmatrix} = 0 \tag{4.4}$$

it follows

$$\begin{cases} -y_i^{*T}y_i^* + \lambda_i + y_i^{*T}\bar{R}_i^*\vartheta_i = 0 \\ -\bar{R}_i^{*T}y_i^* + (\bar{R}_i^{*T}\bar{R}_i^* - \lambda_i I)\,\vartheta_i = 0 \end{cases}$$

or

$$\begin{cases} y_i^{*T}[y_i^* - \bar{R}_i^*\vartheta_i] = \lambda_i \\ \vartheta_i = (\bar{R}_i^{*T}\bar{R}_i^* - \lambda_i I)^{-1}\bar{R}_i^{*T}y_i^* \end{cases} \tag{4.5}$$

Moreover, it can be noted that from the first relation of (4.5) it derives:

$$p\lim_{N\to\infty} \frac{1}{N}\lambda_i = \sigma^2 \tag{4.6}$$

so that, when σ^2 is *a priori* known, (4.3) reduces to

$$\vartheta_i = [\bar{R}_i^{*T}\bar{R}_i^* - N\sigma^2 I]^{-1}\bar{R}_i^{*T}y_i^*. \qquad \triangleleft$$

Remark — It can be interesting to note that it is possible to formulate the solution (4.3) in the following iterative way, starting from the simple least-squares solution quoted as $\vartheta_i\,(0)$:

$$\bar{\vartheta}_i^{(k+1)} = \bar{\vartheta}_i^{(0)} + (\bar{R}_i^{*T}\bar{R}_i^*)^{-1}\alpha^{(k)}\bar{\vartheta}_i^{(k)}$$

$$\alpha^{(k)} = \frac{(y_i^* - \bar{R}_i^*\bar{\vartheta}_i^{(k)})^T (y_i^* - \bar{R}_i^*\bar{\vartheta}_i^{(k)})}{1+\bar{\vartheta}_i^{(k)T}\bar{\vartheta}_i^{(k)}} \tag{4.7}$$

$$\bar{\vartheta}_i^{(0)} = (\bar{R}_i^{*T}\bar{R}_i^*)^{-1}\bar{R}_i^{*T}y_i^*$$

This formulation does not require the *a priori* knowledge of σ^2 or, in any case, the computation of the eigenvalue λ_i.

The derivation of the iterative formulas (4.7), here omitted for a simple exposition, is based on some manipulations of (4.4) and on the fact, stated by the Koopmans-Lewin theory, that λ_i is equal to the value of the right side of (4.1).

The solution just described requires the *a priori* knowledge of the system structure $\{\nu_1, \nu_2, \dots \nu_m\}$. The structural identification can be performed before the parametric identification on the basis of the results of Theorem (3.1) and observing that asymptotically

$$p\lim_{N\to\infty}\frac{1}{N}S(\mu_1, \dots, \mu_{m+r}) = \lim_{N\to\infty}\frac{1}{N}S(\mu_1, \dots, \mu_{m+r})+\sigma^2 I \tag{4.8}$$

and

$$\det[S^*(\mu_1, \dots, \mu_{m+r})-\lambda_i I] \neq 0 \Leftrightarrow \det S(\mu_1, \dots, \mu_{m+r}) \neq 0$$

$$\det[S^*(\mu_1, \dots, \mu_{m+r})-\lambda_i I] = 0 \Leftrightarrow \det S(\mu_1, \dots, \mu_{m+r}) = 0$$

Hence it follows that the singularity test of Theorem 3.1 must be performed on the sequence analogous to (3.4) where $S(\mu_1 \dots \mu_{m+r})$ is substituted by $S^*(\mu_1 \dots \mu_{m+r})-\lambda_i I]$.

5. TECHNICAL APPLICATIONS

This section is devoted to a brief review of two applications to industrial systems of the identification procedure described in previous sections.

In these cases the technique was employed completely, i.e. models were directly developed from experimental i/o data. More detailed results can be found in [16] and [17].

CASE 1: A distillation column

In the plant, schematically shown in fig. 7 the refinement of row benzol (benzene 87%, toluene 11,8%) into pure benzol (benzene 99,8%) is performed.

The only column B, supplied from the bottom of the column A by a constant flow, was the object of the experiments. The final goal was the dynamic regulation of the column behaviour by acting on the reboiler steam valve; as

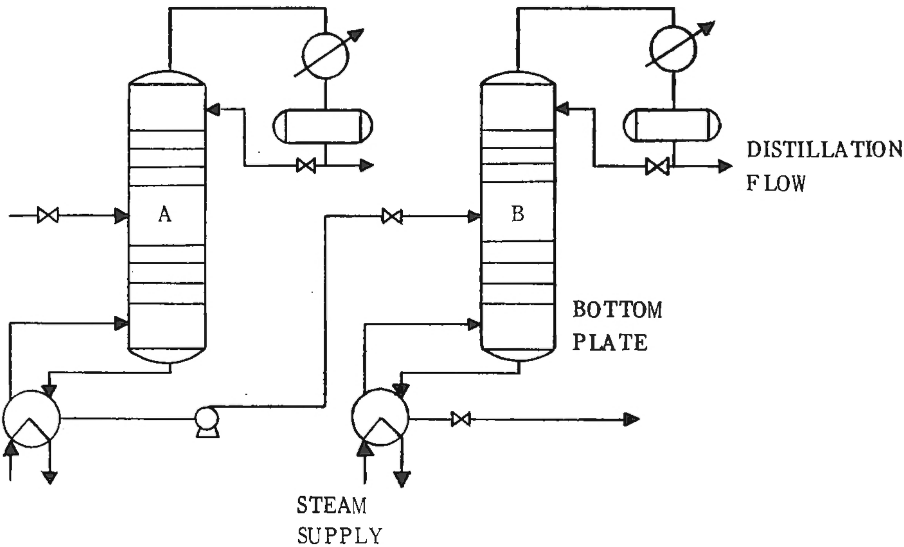


Fig. 7

controlled variables the top distillation flow and the bottom plate temperature were chosen. Starting from an equilibrium condition, an impulse-like perturbation (three minutes long) was given and output variables values were recorded at a sampling interval of 30 sec. for a period of 20 minutes.

Figures 8,9 show the drops for the structure determination (i.e. structural indexes) without and with eigenvalue compensation (see section 4).

On the basis of the resulting structural indexes $\nu_1 = \nu_2 = 2$ the parametric estimate gave the (A, B, C) state-space matrices.

Figures 10, 11 show the output variables patterns from real system and model, respectively.

CASE 2: A power station

The application was concerned with the termic part of a thermoelectric power plant. The scheme is plotted in fig. 12.

where

1-2-3	superheaters	6 condenser
4	postheater	7 drum
5	gas damper	8 burners

In this case the controlled variables were

T_s, P_s superheat steam temperature and pressure

T_r reheat steam temperature,

and the input manipulated variables were

q_F fuel flow

q_A air flow

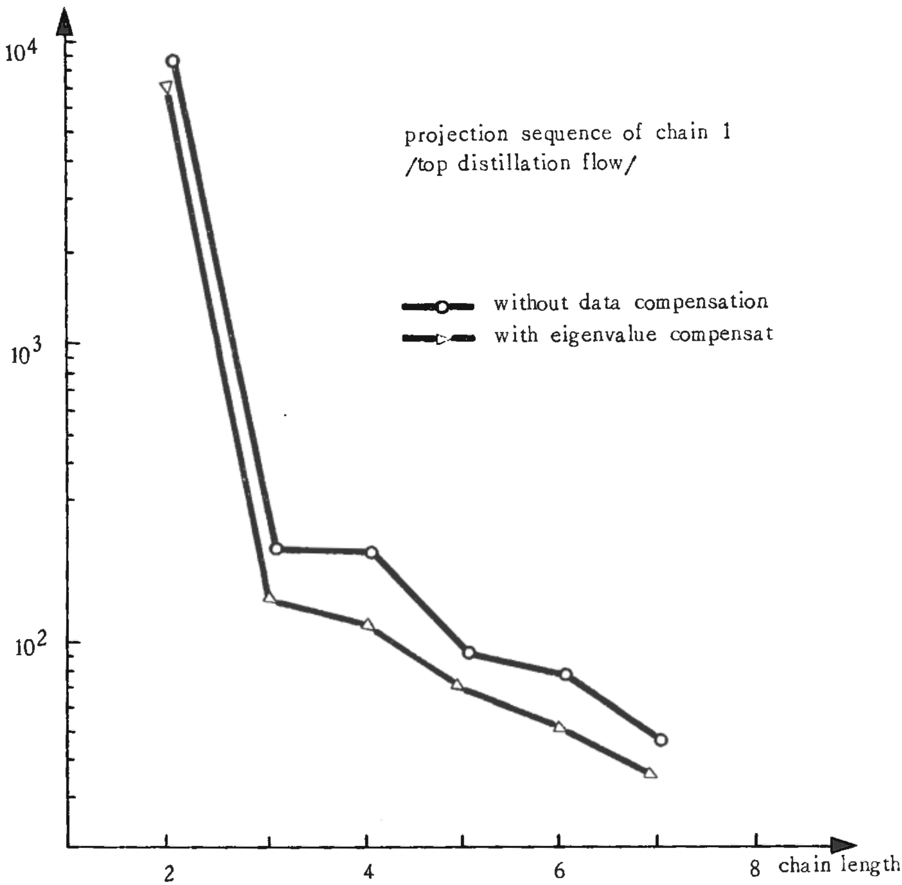


Fig. 8

q_d super heater flow
ϑ_T turbine valve opening
y_D dampers position.

The original data records consisted of sequences of normal operating values with a sampling period of 10 sec. However, the identification procedure was performed on a reduced data set obtained by substituting every consecutive 12 values with their mean so that an equivalent sampling period of 120 seconds was obtained; thus the samples were reduced to 183. Four steps were performed on the data: 1) noise variance estimate, 2) model structure identification, 3) parametric estimation, 4) initial state and output sequences reconstruction according to the formulas given in sects. 3, 4.

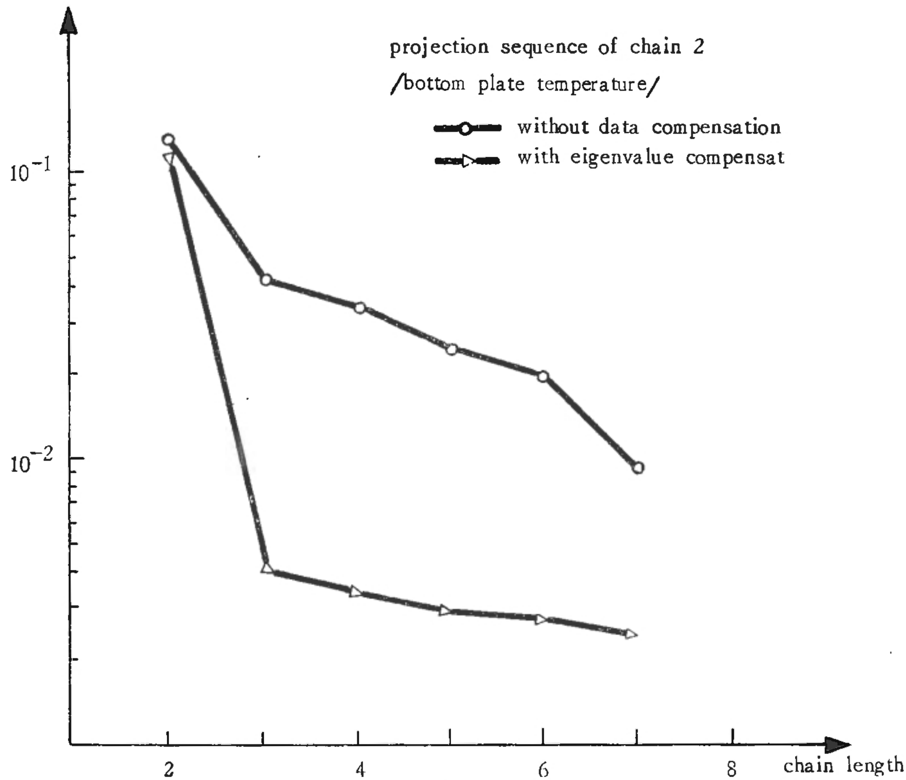


Fig. 9

Since no *a priori* knowledge was available about the noise level on each variable, the assumption of the same amount of additive noise on each variable (i.e. the same standard deviation) was made.

The model structure resulted in an interconnection of three 3rd order dynamic subsystems relative to P_s, T_s and T_r. Moreover the data analysis suggested the introduction of an algebraic input-output link, so that the model outcame not purely dynamic.

The results of the output sequences reconstruction (solid lines) are shown in figs. 13—14—15 in a normalized scale.

6. AN ECONOMIC APPLICATION

In macroeconomic applications the black-box identification approach suffers some difficulties and limitations fundamentally because the available records are extremely short if compared with the relative necessary detail of any model useful for a concrete policy implementation.

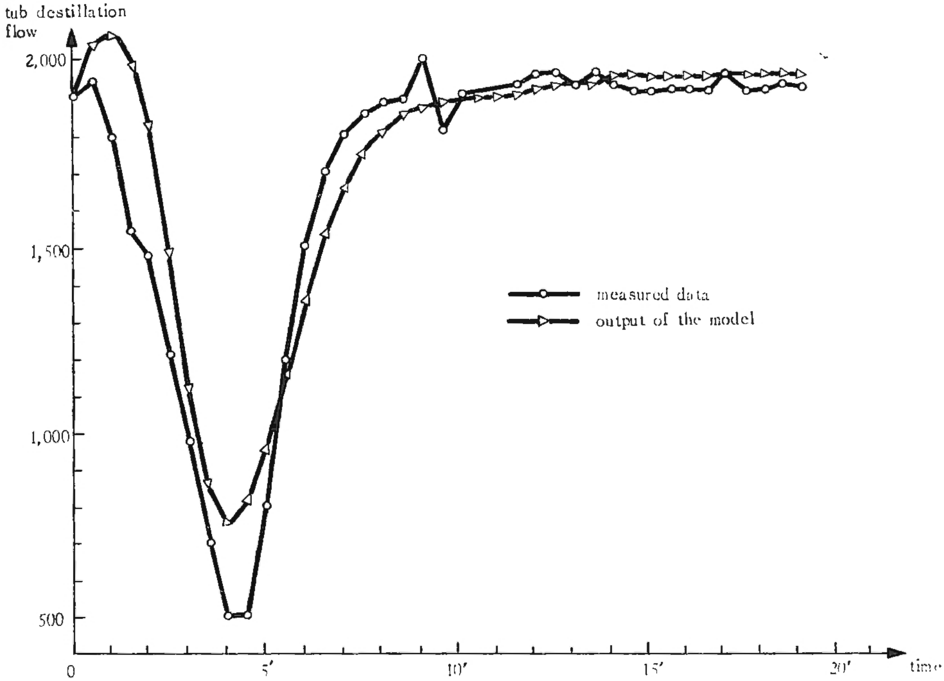

tub destillation flow
2,000
1,500
1,000
500
0
5'
10'
15'
20'
time
measured data
output of the model

Fig. 10

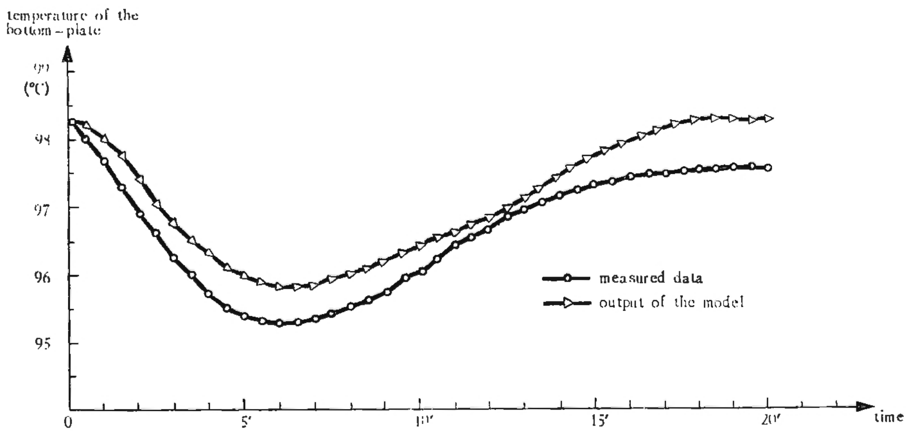
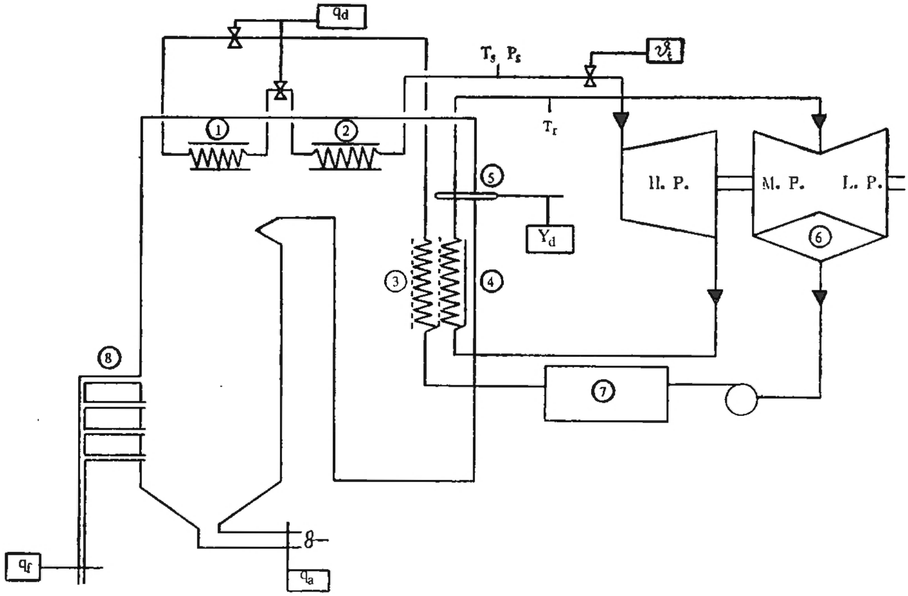
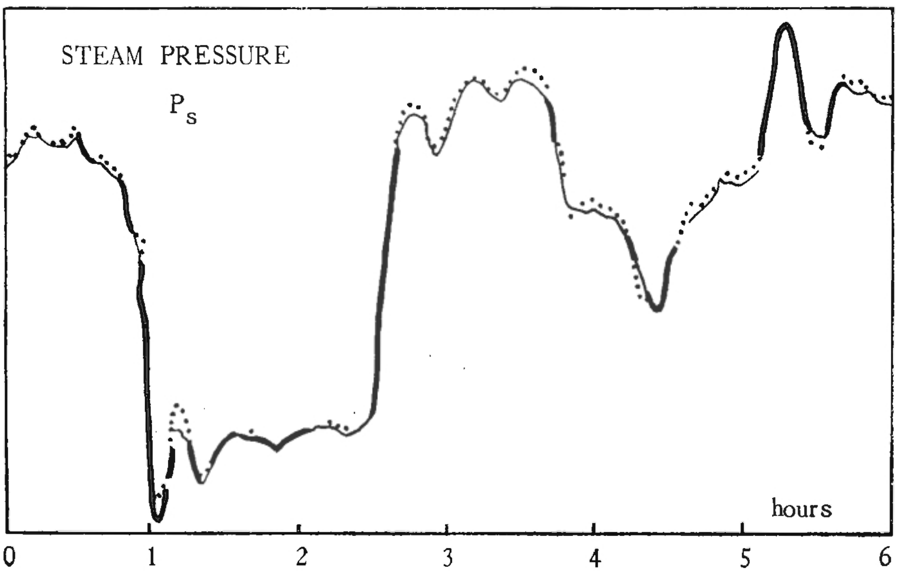

temperature of the bottom-plate
(°C)
97
96
95
0
5'
15'
time
measured data
output of the model

Fig. 11



q_d
T_s P_s
ϑ_t
T_r
1
2
5
H. P.
M. P.
L. P.
Y_d
6
3
4
8
7
q_f
q_a

Fig. 12



STEAM PRESSURE
P_s
hours
0
1
2
3
4
5
6

Fig. 13

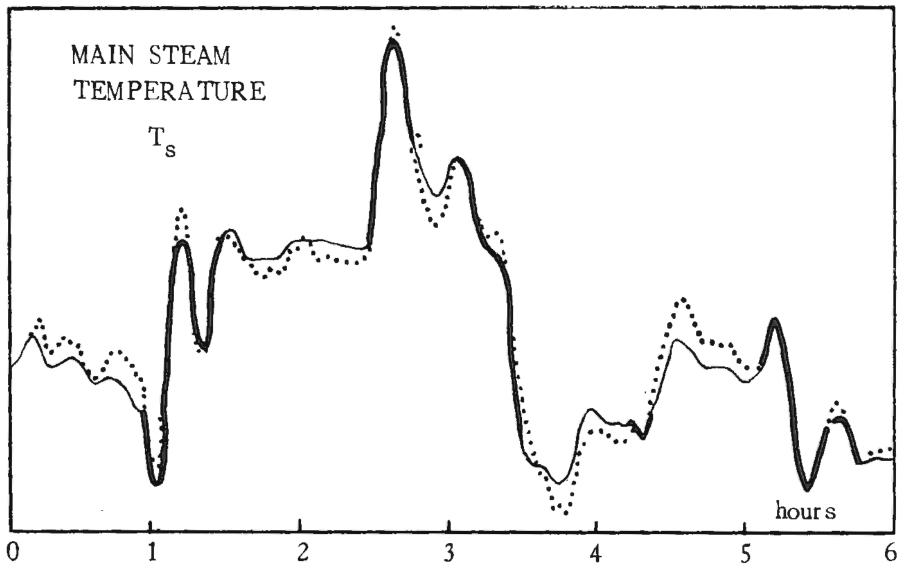

MAIN STEAM
TEMPERATURE
T_s
hours
0
1
2
3
4
5
6

Fig. 14

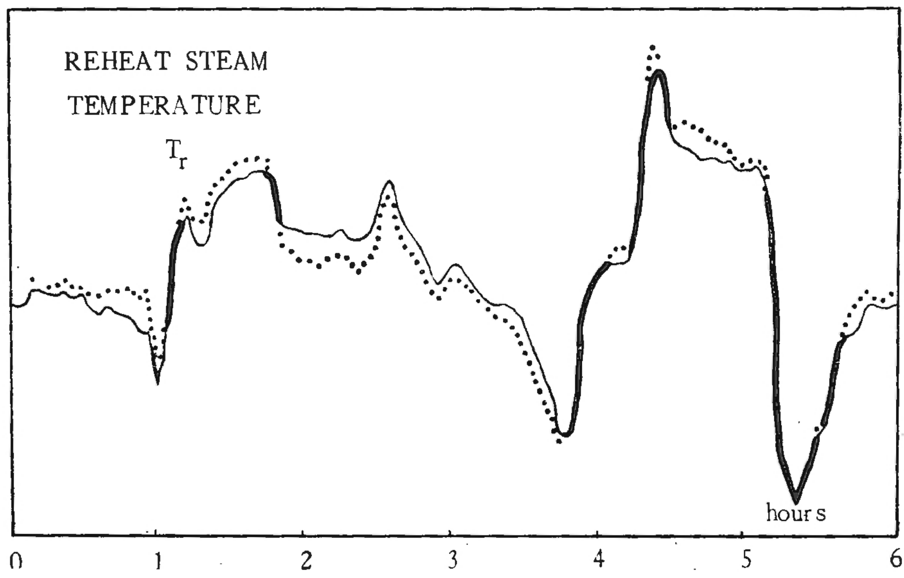

REHEAT STEAM
TEMPERATURE
T_r
hours
0
1
2
3
4
5
6

Fig. 15

On the other hand, even if data sequences of remarkable length are disposable, they are produced over such a number of years that one can't be sure about model structure and/or parameters durability (i.e. time invariance) because of possible institutional modifications.

Consequently one is compelled to use carefully every *a priori* knowledge about the economic system in order to come to a realistic model which justifies historical data.

In the following application, a forecasting quarterly model of the Italian Economy (the *LINK* project model) was the starting point. Since our ultimate goal was an optimal tracking algorithm synthesis for some monetary variables, the equations of the monetary sector were selected by means of a proper exogenization procedure. The form of such (nonlinear) equations is the following:

Behaviour equations

$$CIR = f_1(CIR_{-1}, GNPP, Q_1, Q_2, Q_3)$$
$$RLTB = f_2(RB_{-i}, RLTB_{-i}, DPOT/D_{-i})$$
$$DCC = f_3(GNPP_{-i}, RLTB_{-i})$$
$$DCCCR = f_4(GNPP_{-i}, RLTB_{-i})$$
$$DR = f_5(GNPP_{-i}, RLTB_{-i})$$
$$DRCR = f_6(GNPP_{-i}, RLTB_{-i})$$
$$RES = f_7(DCC, DCC_{-1})$$

Definition equations

$$D = DR + DCC$$
$$UR = UBAS - CIR$$
$$RR = RR_{-1} + 0.225(DCC - DCC_{-1} - DCCCR + DCCCR_{-1}) + RES$$
$$DPOT = DCC + (1/0.225)(UR - RR)$$

where subscripts are for time-lags.

Endogenous variables are:

CIR	currency outside banks
RLTB	interest rate on long-term bonds
DCC	demand deposits with Commercial Banks
DCCCR	demand deposits with "Casse Risparmio"
D	total deposits
RR	required reserve of Commercial Banks
DPOT	potential deposits
DRCR	saving deposits with "Casse Risparmio"
DR	time deposits with Commercial Banks
RES	residual required reserves over the sample period
UR	unborrowed reserves

Exogenous variables are:

GNPP	inflated gross national product at market price
RB	discount rate
UBAS	unborrowed monetary base
Q1	seasonal dummy for the 1st quarter
Q2	seasonal dummy for the 2nd quarter
Q3	seasonal dummy for the 3rd quarter.

In order to overcome the above mentioned difficulties, a model suitable for control was obtained according to the following steps:

1) original (nonlinear) model linearization,

2) parameter estimation from historical quarterly data covering the period 1962.1—1973.4. A first difference model was obtained,

3) performance index and nominal paths (i.e. desired trajectories) choice,

4) linear model re-ordering to obtain an "essential" input-output submodel,

5) essential i/o submodel transformation into a state-space form.

Steps 1 and 2 led to the usual structural equations:

$$\Omega \boldsymbol{y}(k) = \Gamma w(k) \tag{6.1}$$

where $\boldsymbol{y}(k)$ is the vector of the endogenous variables first differences at time t and $\boldsymbol{w}(k)$ is the vector of the predetermined (lagged endogenous and current and lagged exogenous) variables first differences. $\boldsymbol{\Omega}$ and $\boldsymbol{\Gamma}$ are real matrices, $\boldsymbol{\Omega}$ being non singular.

Step 3 is a very crucial one. From the economic point of view, since a quadratic function of a selection of exogenous and endogenous variables (i.e. instruments and intermediate objectives) was assumed as performance criterium, careful choice of the weighting matrices is to be done to take into account the policy-maker priorities.

From the computational point of view, complexity was reduced keeping in mind that only the interconnected part of the model affected by variables appearing also in the performance index has to be considered for control implementation. This part is what we mean for "essential" submodel and, in our case, it was obtained by an ordering procedure (Step 4) of equations (6.1) which led to the scheme shown in fig. 16.

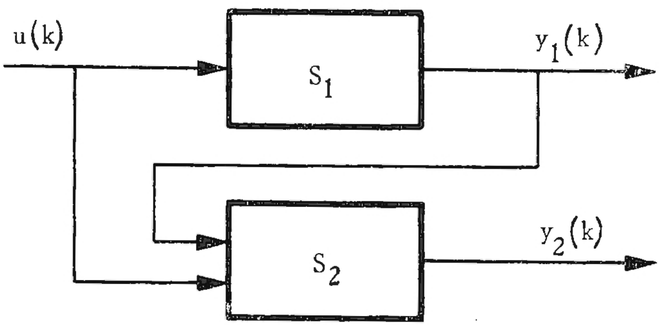


Fig. 16

where

$$y_1 = [CIR, RLTB, DCC, DCCCR, D, RR, DPOT]^T$$

$$y_2 = [DRCR, DR, RES, UR]^T$$

Since DCC (or D) and $RLTB$ were chosen as controlled variables, the only essential part S_1 was considered in next steps. In particular step 5 reduces to the application of procedure of sect. 3 to a polynomial form

$$P(z)\, y_1(k) = [P(z)\, D + \bar{Q}(z)]\, u(k) \tag{6.2}$$

following from (6.1) by simple algebra.

The final state space model resulted in a 42th order quadruplet (A, B, C, D) with structural indexes $\nu_i = 6$, $i = 1, \ldots, 7$.

The variables $UBAS$ and RB were considered as manipulated variables, i.e. as monetary instruments. Some computational results for different values of weighting matrices in the performance index

$$J = \sum_{i=1}^{N} \|z_i - \hat{z}_i\|_Q + \sum_{i=0}^{N} \|s_i - \hat{s}_i\|_R, \tag{6.3}$$

where

$Q \geqslant 0$, $R > 0$, $s = (RB, UBAS)^T$, $z = (RLTB, D)^T$

$\hat{z}_i$, $\hat{s}_i$ nominal values,

are collected in the trade-off curve of fig. 17.

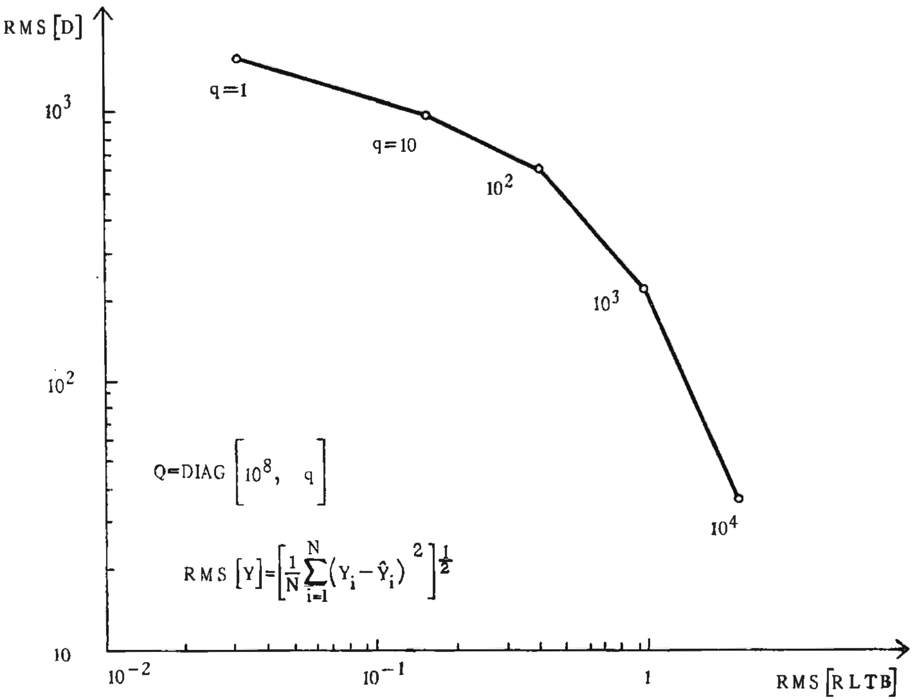


Fig. 17

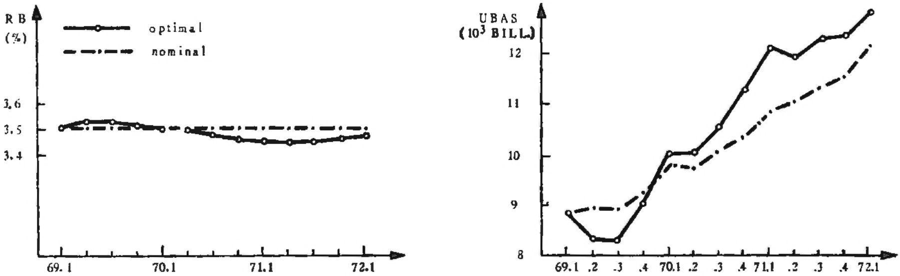


Fig. 18

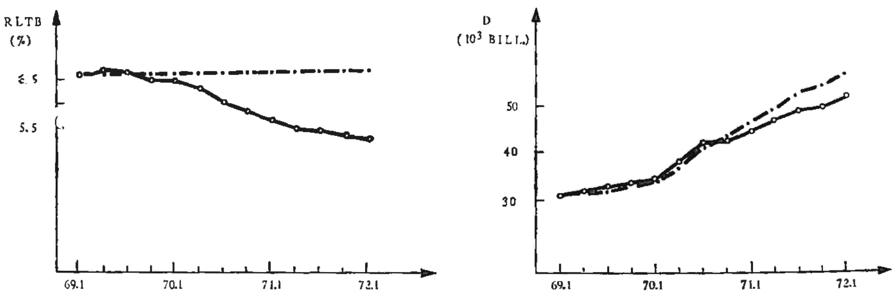


Fig. 19

In figs. 18—19 nominal and optimal trajectories of instruments and target variables are reported for $Q = \text{diag}(10^8, 10)$, $R = \text{diag}(10^8, 1)$.

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SUMMARY

Mathematical system description has become a necessary tool for technicians and economists for developing efficient analysis criteria and designing suitable interventions. Because of the increasing complexity of the systems on which one must operate, methodological approach to model building, such as identification techniques, are playing a fundamental role.

Although the model purposes in technological and economic areas are essentially the same (i.e. simulation, forecasting and synthesis of control strategies), some important differences are present in the application of a given method. In particular the amount of available experimental data, the experiment design possibility, the easiness of specifying the correct control objectives are relevant elements for suggesting the way how a given set of theoretical results can be usefully applied.

To contribute to focus this aspect, in the paper a unified identification technique for linear time-invariant systems is used for modelling both some chemical processes and a monetary sector of the Italian macroeconomic system.

Some experimental results are evaluated and possible further developments are indicated.

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