POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

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PREFACE

The present volume comprises the papers from the 3rd Italian-Polish Conference which was held in [illegible] in [illegible] 26 to 31. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Kraków (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis enabled by the economic, management, data processing and technological systems.

From the past, each meeting enrich the exchange of views and experiences between the specialists from both countries what is the Italian [illegible] to both sides, shows the natural path, the concentration of the systems and control theory specialists in the [illegible] the essential development of the quantitative methods and the [illegible] of large complex systems.

The meaning of such meetings, however, goes far beyond that, what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — begin the friendly links between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

P. Mazzoleni
University of Venice

INFLUENCE OF INFLATION ON INTERNATIONAL COMMERCIAL BANKING*)

The evolution of a banking system that trascends national frontiers and which occupies a central place in the financing of the world economy clearly calls for a more uniform approach to bank management. However, the attainment of this is long way off and in the meantime management practices will continue to reflect the diversity of national banking markets and national policy interests. For instance, United States regulations have been extended to the international activities of American banks, while Canada and England are two examples of a "hand off" regulatory posture towards international operations. In Italy the regulatory policy does not concern activities in other countries, but it puts some bounds on bank's foreign investments as for ordinary investors. Branches are the most important instrument for the conduct of a bank's business abroad and one way to analyze their performance is to look at balance sheet data which highlights sources and uses of funds. Some of these foreign branches compete for business in local foreign markets, but the overwhelming part of the foreign branches business is concentrated in the inter-bank market. This market is a network of foreign commercial banks which hold each other's deposits and extend credits to each other. The dominant reason under laying the initial expansion of foreign branches was the bank's desire to finance its foreign customers. Afterwards, this demand was reduced appreciably and foreign branches apply themselves to financial rather than banking activities. It is important to notice how domestic money and capital markets are now more closely linked with financial markets abroad and this has led to a more efficient allocation of savings and credit to productive activities. This development of banks in international banking has added a new dimension to banking policy, because it is no longer appropriate for public authorities to look exclusively to their domestic operations. And it is particularly important because of new risks introduced by world inflation, floating exchange rates, oil and other commodity prices.

Thus a number of public policy issues are still present and unresolved: the effect of international operations on the integrity of the domestic banking

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organization, the limits to be placed on the types of activities engaged abroad, the treatment of foreign joint ventures, the overlay of foreign activities conducted with domestic activities.

Many of the existing models on international finance are based on segmented market approach, assuming that different currency areas, separated political organizations and trade barriers justify to consider the different national capital markets as independent entities. But preceeding considerations show how these models are inadequate to describe the reality of international activities. A first model of international capital asset pricing has been introduced by B. Solnik [3] and is based on Merton's approach with the indroduction of exchange risk.

Under a certain number of assumptions

1. The capital markets are always in equilibrium.
2. Capital markets are perfect, with no transaction costs, taxes or capital controls. Investors are price takers.
3. Assets can be sold short.
4. In each country there exists a market for borrowing and lending at the same rate.
5. Trading in assets and currencies take place continuously in time and this implies a world of flexible exchange rates.
6. Investors hold homogeneous expectations about exchange rate variations and the distribution of returns in terms of the asset currency.
7. There are no constraints on international capital flows.
8. Investors consumption is limited to their home country.

Solnik proved a mutual fund theorem which states that all investors will be indifferent between choosing portfolios from the original assets or from three funds:

- a portfolio of stocks hedged against exchange risk,
- a portfolio of bonds, speculative in the exchange risk dimension,
- the risk free asset of their own country, of which only the third fund will change according to the investor's nationality. Solnik's model is used here in order to extend a preceeding study for bank portfolio, veloped in a domestic and static framework [4], to a dynamic formulation.

In this paper we shall introduce transaction costs, different interest rates on borrowing and lending, which will be considered as random entities and the utility function will depend not on domestic consumption level, but on capital investments.

These new conditions allow us to derive a mutual fund theorem for a noninflationary market which separates domestic and foreign maganement decisions according to their degree of correlation.

In a further section we try to take into account the most evident structural reasons of inflation and their influence on bank's deposits, loans and investments.

Referring to Italian situation, treasury deficit and wage policy are recognized as the fundamental reasons of inflation. In correspondence, labour cost alters competition between Italian and foreign firms and escalator clauses in wage contracts don't allow to adjust the consumption level to the real situation. Under these conditions we shall prove how it is not possible to manage domestic and foreign activities separately in an inflationatory framework and how it is desirable that regulatory policy controls the whole bank's portfolio and through it the market situation. These results are also extended to an opportunity set varying in time.

1. RATES OF RETURN DYNAMICS AND BANK'S MANAGEMENT WITHOUT INFLATION

The way in which management looks at its business and attempts to balance certain components with others is heavily influenced by its attitude to risk, feeling of national responsibility, size of the bank and other factors which vary from bank to bank and country to country. The loan and deposit portfolios of a bank are traditionally the heart of the bank's business, but, according to our previous discussion, management would like to balance not only domestic deposits with domestic loans and assets, but also funds collected abroad with foreign investments.

For simplicity, we shall consider relations with only one country, but results can be extended to any number of foreign countries.

Let $X_1(t)$ be the domestic deposit level at time t from ordinary residents, $X_2(t)$ the domestic loan level at time t and $R_1(t), R_2(t)$ the corresponding interest rates. As deposit and loan demand varies according to market conditions and public behaviour, the dynamic equations for corresponding interest rates are described by the continuous-time stochastic processes

$$dR_1 = r_1\, dt + \vartheta_1\, dz_1 \tag{1.1}$$

$$dR_2 = r_2\, dt + \vartheta_2\, dz_2 \tag{1.2}$$

where r_1 and r_2 are the instantaneous expected value for deposit and loan interest rates, z_1 and z_2 are Wiener processes describing random market conditions.

Let $X_3(t)$ be the level of domestic alternative investments at time t and $V_x(t)$ the corresponding level of transactions. Then we can control changes in the interest rate $R_3(t)$ by modifying $V_x(t)$, but they are also influenced by other investors behaviour and we get

$$dR_3 = l\, V_x(t)\, dt + \vartheta_3\, dz_3 \tag{1.3}$$

Assume that unit transaction cost varies according to $dc_x = \gamma_x\, dt$ and the interest rate on capital investments $K(t)$ follows the relation $dR_k = r_k dt$. In order to keep the customer's confidence, Italian banks operate under some restrictions, i.e. the usual conditions on statutory reserve in cash

$$R_0(t) = uX_1(t), \quad 0 < u < 1 \tag{1.4}$$

as well as in fixed return assets. This latter amount is linked to the level of deposits in the domestic currency by the equation

$$F(t) = vX_1(t), \quad 0 < v < 1, \quad u+v < 1 \tag{1.5}$$

The corresponding interest rates vary according to $dR_R = r_R\,dt$, $dR_F = r_F\,dt$. Let $Y_1(t)$ be the level of foreign funds collected from non residents (including emigrants) expressed in the local currency, $Y_2(t)$ be the level of foreign investments at time t. Interest rates on foreign funds and investments depend on the variation of exchanges rate and on relations between capital markets so that their dynamics can be described as follows

$$dS_1 = s_1\,dt + \phi_1\,dw_1 \tag{1.6}$$

$$dS_2 = s_2\,dt + \phi_2\,dw_2 \tag{1.7}$$

As the exchange rate f between the two currences in exam varies according to the stochastic process

$$df = \mu\,dt + v\,dw_f \tag{1.8}$$

the bank earns fS_2 with

$$d(fS_2) = (\mu + s_2 + \varrho_{f,\,s_2} v\phi_2)\,dt + v\,dw_f + \phi_2\,dw_2 \tag{1.9}$$

where ϱ_{f,s_2} is the instantaneous correlation coefficient between the two Wiener processes that regulate the evolution of the exchange rate f and the interest rate on the foreign investments $S_2(t)$.

As further regulations are introduced in order to discourage the speculative movement of capital into foreign currencies, we introduce the simple constraints

$$Y_2(t) < L, \quad |V_y(t)| < M \tag{1.10}$$

where $V_y(t)$ represents the level of foreign transactions.

Moreover, we shall assume that the unit transaction cost is given by $dc_y = \gamma_y\,dt$. The budget constraint can be written as

$$K(t) + (1-u-v)X_1(t) + Y_1(t) = X_2(t) + X_3(t) + c_x|V_x(t)| + \\ + Y_2(t) + c_y|V_y(t)| \tag{1.11}$$

If we express $|V_y(t)|$ in terms of other control variables

$$|V_y(t)| = \frac{1}{c_y}[K(t) + (1-u-v)X_1(t) + Y_1(t) - X_2(t) - X_3(t) - c_x|V_x(t)| - Y_2(t)] \tag{1.12}$$

the accumulation equation for the return π is

$$\frac{d\pi}{\pi}=\left\{\left[ur_R+vr_F-r_1-(1-u-v)\frac{\gamma_y}{c_y}\right]x_1+\left(r_2+\frac{\gamma_y}{c_y}\right)x_2+\right.$$
$$+\left(1V_x+\frac{\gamma_y}{c_y}\right)x_3-\left(r_k+\frac{\gamma_y}{c_y}\right)k-\left(s_1+\frac{\gamma_y}{c_y}\right)y_1+$$
$$\left.+\left(\mu+s_2+\varrho_{f,s_2}v\phi_2+\frac{\gamma_y}{c_y}\right)y_2-\left(\gamma_x-\gamma_y\frac{c_x}{c_y}\right)|v_x|\right\}dt+$$
$$+\sum_{2,3}\vartheta_i x_i\,dz_i+\phi_2\,y_2\,dw_2+vy_2\,dw_f-\vartheta_1 x_1\,dz_1-\phi_1\,y_1\,dw_1 \tag{1.13}$$

where we express all the levels as a fraction of π. Without inflation, we can assume domestic deposits, loans and assets to be correlated, while exchange rate is supposed to be independent from these activities ($\varrho_{f,R_i}=0$, $i=1, 2, 3$). Analogously domestic activities are assumed to be uncorrelated with foreign ones ($\varrho_{R_i,S_j}=0$, $=1, 2, 3, j=1, 2$).

The bank must choose capital investments $K(t)$ in order to solve the following maximization problem:

$$\max E(0)\left\{\int^{\infty} U[K(\tau),\tau]\,d\tau\right\} \tag{1.14}$$

where $E(0)$ is the conditional expectation operator with $\pi(0)=\pi_0$ and the constraint 1.13.

It can be shown [4] that the necessary optimilaty conditions for a bank which acts according to 1.13 in choosing its deposit, loan and investment program are that, at each point of time,

$$0=\max\left\{U[K(t),t]+J_t+J_\pi E[d\pi]+\frac{1}{2}J_{\pi\pi}E[(d\pi)^2]\right\} \tag{1.15}$$

where we define

$$J=J[\pi(t),t]=\max E(t)\left\{\int_t^{\infty} U[K(\tau),\tau]\,d\tau\right\} \tag{1.16}$$

In order to derive the optimal policy we can write the control variable $V_x(t)$ as

$$V_x(t)=V_x^1(t)-V_x^2(t),\qquad \text{with}\qquad V_x^1(t)>0,\ V_x^2(t)>0 \tag{1.17}$$

We now require $V_x(t)=V_x^1(t)$ when $V_x(t)$ is positive and $V_x(t)=V_x^2(t)$, when it is negative. Therefore the following constraint is imposed

$$V_x^1(t)\cdot V_x^2(t)=0 \tag{1.18}$$

From 1.17—1.18 it follows

$$|V_x(t)| = V_x^1(t) + V_x^2(t) \tag{1.19}$$

Necessary conditions for 1.15 are

$$0 = J_\pi\left[ur_R + vr_F - r_1 - (1-u-v)\frac{\gamma_y}{c_y}\right] +$$

$$+ J_{\pi\pi}\pi\left[\vartheta_1^2 x_1 - \varrho_{R_1,R_2}\vartheta_1\vartheta_2 x_2 - \varrho_{R_1,R_3}\vartheta_1\vartheta_3 x_3\right] \tag{1.20a}$$

$$0 = J_\pi\left(r_2 + \frac{\gamma_y}{c_y}\right) + J_{\pi\pi}\pi\left[-\varrho_{R_1,R_2}\vartheta_1\vartheta_2 x_1 + \vartheta_2^2 x_2 + \varrho_{R_2,R_3}\vartheta_2\vartheta_3 x_3\right] \tag{1.20b}$$

$$0 = J_\pi\left(1V_x + \frac{\gamma_y}{c_y}\right) + J_{\pi\pi}\pi\left[-\varrho_{R_1,R_3}\vartheta_1\vartheta_3 x_1 + \varrho_{R_2,R_3}\vartheta_2\vartheta_3 x_2 + \vartheta_3^2 x_3\right] \tag{1.20c}$$

$$0 = J_\pi\left[-\left(s_1 + \frac{\gamma_y}{c_y}\right)\right] + J_{\pi\pi}\pi\left[\phi_1^2 y_1 - (\varrho_{S_1,S_2}\phi_1\phi_2 + \varrho_{f,S_1}\phi_1 v)y_2\right] \tag{1.20d}$$

$$0 = J_\pi\left(\mu + s_2 + \varrho_{f,S_2} v\phi_2 + \frac{\gamma_y}{c_y}\right) + J_{\pi\pi}\pi\left[-(\varrho_{S_1,S_2}\phi_1\phi_2 + \varrho_{f,S_1}\phi_1 v)y_1 +\right.$$

$$\left. + (\phi_2^2 + v^2 + 2\varrho_{f,S_2} v\phi_2)y_2\right] \tag{1.20e}$$

$$0 = U_k - J_\pi\left(r_k + \frac{\gamma_y}{c_y}\right) \tag{1.20f}$$

Let us set

$$1/A = -(J_{\pi\pi}\pi)/J_\pi$$

$$R = [ur_R - vr_F + r_1,\, r_2,\, 1V_x]^T$$

$$e = [1-u-v,\, 1,\, 1]^R$$

$$\Sigma_1 = \begin{bmatrix} \vartheta_1^2 & \varrho_{R_1,R_2}\vartheta_1\vartheta_2 & \varrho_{R_1,R_3}\vartheta_1\vartheta_3 \\ \varrho_{R_1,R_2}\vartheta_1\vartheta_2 & \vartheta_2^2 & \varrho_{R_2,R_3}\vartheta_2\vartheta_3 \\ \varrho_{R_1,R_3}\vartheta_1\vartheta_3 & \varrho_{R_2,R_3}\vartheta_2\vartheta_3 & \vartheta_3^2 \end{bmatrix}$$

$$X = [-x_1,\, x_2,\, x_3]^T$$

Then we obtain

$$X = A\,\Sigma_1^{-1}\left(R + e\,\frac{\gamma_y}{c_y}\right) \tag{1.21}$$

and deposit and loan supply is sum of two terms of which the first one is linked to market conditions, the second one is hedged against transaction costs.

Moreover, if we write

$$T = [s_1, s_2]^T \qquad e' = [1, 1]^T$$

$$R(f) = [0, \mu + \varrho_{f,s_2} v\phi_2]^T$$

$$\sum_2 = \begin{bmatrix} \phi_1^2 & \varrho_{s_1,s_2}\phi_1\phi_2 + \varrho_{f,s_1}\phi_1 v \\ \varrho_{s_1,s_2}\phi_1\phi_2 + \varrho_{f,s_1}\phi_1 v & \phi_2^2 + v^2 + 2\varrho_{f,s_2} v\phi_2 \end{bmatrix}$$

$$Y = [-y_1, y_2]^T$$

then the critical point is given by

$$Y = A\sum_2^{-1}\left(T + e'\frac{\gamma_y}{c_y} + R(f)\right), \tag{1.22}$$

the first term depends on market conditions, the second one is hedged against transaction costs, the third one is hedged against exchange rate risk.

As the objective function 1.15 constains the term

$$J_\pi\left(1V_x + \frac{\gamma_y}{c_y}\right)x_3\,\pi - \left(\gamma_x - \gamma_y\frac{c_x}{c_y}\right]|V_x|$$

if we have derived the critical value for x_3, we obtain the corresponding value for V_x^1 and V_x^2

$$\hat{V}_x^1 = -\frac{1}{2\vartheta_3}[-\varrho_{R_1,R_3}\vartheta_1(ur_R + vr_F - r_1) + \varrho_{R_2,R_3}\vartheta_2 r_2] -$$

$$-\frac{1}{2\vartheta_3}[\varrho_{R_1,R_3}\vartheta_1(1-u-v) + \varrho_{R_2,R_3}\vartheta_2 + 2\vartheta_3]\frac{\gamma_y}{c_y} + \frac{1/A}{2J_\pi\vartheta_3^2 1\pi}\left(\gamma_x - \gamma_y\frac{c_x}{c_y}\right)$$

$$\hat{V}_x^2 = \frac{1}{2\vartheta_3}[-\varrho_{R_1,R_3}\vartheta_1(ur_R + vr_F - r_1) + \varrho_{R_2,R_3}\vartheta_2 r_2] +$$

$$+\frac{1}{2\vartheta_3}[\varrho_{R_1,R_3}\vartheta_1(1-u-v) + \varrho_{R_2,R_3}\vartheta_2\, 2\vartheta_3]\frac{\gamma_y}{c_y} + \frac{1/A}{2J_\pi\vartheta_3^2 1\pi}\left(\gamma_x - \gamma_y\frac{c_x}{c_y}\right)$$

Thus, comparing feasibility of the critical point with respect to intervals $0 \leqslant V_x^1 \leqslant M$, $-M \leqslant V_x^2 \leqslant 0$, we get the optimal level for transactions

$$\hat{V}_x = \begin{cases} \hat{V}_x^1 & \text{if } 0 \leqslant V_x^1 \leqslant M \\ M & \text{if } M < \hat{V}_x^1 \end{cases} \tag{1.23}$$

$$\hat{V}_x = \begin{cases} \hat{V}_x^2 & \text{if } -M \leqslant \hat{V}_x^2 \leqslant 0 \\ -M & \text{if } -M > \hat{V}_x^2 \end{cases} \tag{1.24}$$

As the objective function is quadratic in y_2, we can examin feasibility of the critical point: otherwise bank invests abroad the amount $p_2 = L$.

The critical point in K is obtained from equation

$$U_k = J_\pi \left(r_k + \frac{\gamma_y}{c_y} \right)$$

Let U be a strictly convcae function with constant relative risk aversion

$$U[K(t), t] = \frac{1}{\eta} [K(t)]^\eta \exp(-\delta t) \quad \text{with } \eta < 1 \tag{1.25}$$

δ being the discount rate.

The "derived" utility function

$$J = J[\pi(t), t] = \frac{h}{\eta} \pi^\eta \text{esp}(-\delta t) \tag{1.26}$$

allows us to find

$$K(t) = \left[h \left(r_k + \frac{\gamma_y}{c_y} \right) \right]^{\frac{1}{\eta - 1}} \tag{1.27}$$

h being determined by solving equation 1.15.

We can conclude that bank portfolio can be chosen by means of separate decisions on domestic and foreign activities in absence of inflation.

Separation theorem

Without inflation, bank management can choose its portfolio indifferently from all the activities or from two funds of which one contains the domestic activities, the other one contains the foreign ones.

2. WHEN THE OPPORTUNITY SET VARIES IN TIME

More generally, we can take into account the variations of mean value and variance of interest rates. Let us assume that the demands for domestic deposits and loans, $D(x_1)$, $D(x_2)$ and for foreign funds $D(y_1)$ vary according to the continuous time stochastic processes

$$dD(x_1) = E(x_1)\, dt + G(x_1)\, dg(x_1) \tag{2.1}$$

$$dD(x_2) = E(x_2)\, dt + G(x_2)\, dg(x_2) \tag{2.2}$$

$$dD(y_1) = E(y_1)\, dt + G(w_1)\, dg(y_1) \tag{2.3}$$

where $E(x_1)$, $E(x_2)$, $E(y_1)$ are the expected value of the demand for domestic deposits and loans and for foreign funds, respectively.

Let us link the mean value of interest rates with the demand levels through the linear equations [4]

$$r_1 = a_1 D(x_1) + a_2 D(x_2) \tag{2.4}$$

$$r_2 = a_3 D(x_2) \tag{2.5}$$

$$s_1 = b_1 D(y_1) \tag{2.6}$$

where the sign of coefficients is positive or negative according to the forecasted behaviour of the demand. Then we have

$$dr_1 = [a_1 E(x_1) + a_2 E(x_2)]\, dt + a_1 G(x_1)\, dg(x_1) + a_2 G(x_2)\, dg(x_2) = \\ = a(x_1)\, dt + b(x_1)\, dg(x_1) \tag{2.7}$$

$$dr_2 = a_3 E(x_2)\, dt + a_3 G(x_2)\, dg(x_2) = a(x_2)\, dt + b(x_2)\, dg(x_2) \tag{2.8}$$

$$ds_1 = b_1 E(y_1)\, dt + b_1 G(y_1)\, dg(y_1) = a(y_1)\, dt + b(y_1)\, dg(y_1) \tag{2.9}$$

As the interest rate on domestic and foreign investments is affected by all investors' behaviour, we shall give more general relations

$$dr_3 = a(x_3)\, dt + b(x_3)\, dg(x_3) \tag{2.10}$$

$$ds_2 = a(y_2)\, dt + b(y_2)\, dg(y_2) \tag{2.11}$$

Analogously, let us assume variances to follow the stochastic processes

$$d\vartheta_i = m(x_i)\, dt + n(x_i)\, dh(x_i) \qquad i = 1, 2, 3 \tag{2.12}$$

$$d\phi_j = m(y_j)\, dt + n(y_j)\, dh(y_j) \qquad j = 1, 2 \tag{2.13}$$

so that 1.1—1.3, 1.6—1.7, 1.9, 2.7—2.13 form a Markov system.

In order to write such a system in a more compact form, let us introduce a vector ξ whose components ξ_t denote the current levels of R_i, S_j, f, r_i, s_j, μ, and v_i, φ_j; v and diagonal matrices $\Phi(\xi)$ and $\psi(\xi)$, so that the whole system can be written as

$$d\xi = \Phi(\xi)\, dt + \Psi(\xi)\, dZ \tag{2.14}$$

Under these assumptions, problem 1.15 becomes

$$0 = \max\Big\{ U[K(t), t] + J_t + J_\pi[d\pi] + \frac{1}{2} J_{\pi\pi} E[(d\pi)^2] + \tag{2.15} \\ + \sum_i J_i \Phi(\xi_i) - \sum_j J_{\pi j}\, \varrho_{R_1,Z_j}\, \vartheta_1\, \Psi(\xi_j)\, x_1\, \pi + \\ + \sum_{2,3} \sum_j J_{\pi j}\, \varrho_{R_i,F_j}\, \vartheta_i\, \Psi(\xi_j)\, x_i\, \pi - \sum_j J_{\pi j}\, \varrho_{S_1,Z_j}\, \phi_1\, \Psi(\xi_j)\, y_1\, \pi + \\ + \sum_j J_{\pi j} (\varrho_{f,Z_j}\, v + \varrho_{S_2,Z_j}\, \Phi_2)\, \Psi(\xi_j)\, y_2\, \pi + \\ + \frac{1}{2} \sum_i \sum_j J_{ij}\, \varrho_{Z_i,Z_j}\, \Psi(\xi_i)\, \Psi(\xi_j) \Big\}$$

where ϱ_{R_i,Z_j}, ϱ_{S_i,Z_j}, ϱ_{f,Z_i}, ϱ_{Z_i,Z_j} are the instantaneous correlation coefficient between the Wiener processes Z_i and Z_j, w_i and Z_j, w_f and Z_j, Z_i and Z_j.

Let us set

$$N_j = J_{\pi j}/(J_{\pi\pi}\pi)$$

$$\Delta_x = \sum_j N_j \Psi(\xi_j) [\varrho_{R_1,Z_j} \vartheta_1 , \varrho_{R_2,Z_j} \vartheta_2 , \varrho_{R_3,Z_j} \vartheta_3]$$

$$\Delta_y = \sum_j N_j \Psi(\xi_j) [\varrho_{S_1,Z_j} \phi_1 , \varrho_{S_2,Z_j} \phi_2 + \varrho_{f,Z_j} v]$$

and the critical point for 2.15 becomes

$$X = \textstyle\sum_1^{-1} \left[A \left(R + e \frac{\gamma_y}{c_y} \right) + \Delta_x \right] \tag{2.16}$$

$$Y = \textstyle\sum_2^{-1} \left[A \left(T + e' \frac{\gamma_y}{c_y} + R(f) \right) + \Delta_y \right] \tag{2.17}$$

The transaction levels will be modified in correspondence; the level for capital investments will be determined from equation

$$0 \doteq U_k - J_\pi(\pi, t, \xi) \left(r_k + \frac{\gamma_y}{c_y} \right)$$

Thus, we can see that the separation property is still true, but variations in the opportunity set imply a further fraction of deposits, loans and investments against such a risk.

3. INFLUENCE OF INFLATION ON BANK PORTFOLIO MANAGEMENT

Treasury deficit and wage policy are considered as the main reasons of inflation. Certainly public deficit creates liquidity and firms accept wage requests more easily. Thus competition between Italian and foreign firms is altered with negative influence on balance of payments. A great part of savings is devolved to balance the current deficit of public administration, while firms' savings decline because of the higher labour cost.

Thus changes on deposit rate are due to wage policy, while loan rate depends on both escalator clauses for wage contracts and prices of raw materials. Indeed, firms require more and more financing from the banking sector. Let λ be the local wholesale price level whose variations depend on public deficit PD, on wage changes dw and on mean variations of raw material prices dp_m according to the linear behavioral equation

$$d\lambda = a\, dw + b\, dPD + c\, dp_m \tag{3.1}$$

where public deficit varies according to the relation $dPD = PD^* dt$, w and p_m follow the continuous time stochastic processes

$$dw = w^* dt + \vartheta_w dz_w \tag{3.2}$$

$$dp_m = p_m^* dt + \vartheta_{p_m} dz_{p_m}: \tag{3.3}$$

Wiener processes z_w and z_{pm} are assumed to be uncorrelated ($p_{w,pm} = 0$). As wage policy affects the depositors' behaviour, while both wage policy and raw material prices affect the loan demand, deposit and loan interest rates will vary according to equations

$$dR_1 = (r_1 + m_1 w^*) dt + m_1 \vartheta_w dz_w \tag{3.4}$$

$$dR_2 = (r_2 + m_2 w^* + n_2 p_m^*) dt + m_2 \vartheta_w dz_w + n_2 \vartheta_{p_m} dz_{p_m} \tag{3.5}$$

Even if we don't analyze the detailed structure of assets $X_3(t)$, different from statutory ones $F(t)$, the interest rate R_3 depends on inflation, on firms' budget and speculative movements of capital. Then we can assign the following equation

$$dR_3 = [1V_x(t) + m_3(aw^* + bPD^* + cp^*)] dt + m_3(a\vartheta_w dz_w + c\vartheta_{p_m} dz_{p_m}) \tag{3.6}$$

Currency situation is affected by the foreign debt and the consequent reduced confidence for new loans, by the speculative movements of capital that take place not only because of liquidity of the whole banking system, but also because of inter-bank deposits and short-term Treasury bills. Thus exchange rate f can be expressed as a function of the foreign debt $FD(t)$ and through it, f results to be correlated to mean variations of imported goods' prices and to wage policy, that affects prices of exported goods. Consequently, we can assign an evolution equation

$$df = \mu dt + v dw_f \tag{3.7}$$

with

$$dz_f dw = \varrho_{f,w} dt, \; dz_f dz_{p_m} = \varrho_{f,p_m} dt \tag{3.8}$$

and the bank earns

$$d(fS_2) = (\mu + s_2 + \varrho_{f,s_2} v\phi_2) dt + v dw_f + \phi_2 dw_2 \tag{3.8}$$

on foreign investments.

For S_1 we shall assume equation 1.6.

Now, the accumulation equation can be written as follows

$$\frac{d\pi}{\pi} = \left\{ \left[ur_R + vr_F - (r_1 + m_1 w^*) - (1-u-v)\frac{\gamma_y}{c_y} \right] x_1 + \right.$$

$$+ \left(r_2 + m_2 w^* + n_2 p_m^* + \frac{\gamma_y}{c_y} \right) x_2 + \left[1V_x + m_3(aw^* + bPD^* + cp_m^*) + \frac{\gamma_y}{c_y} \right] x_3 -$$

$$-\left(r_k+\frac{\gamma_y}{c_y}\right)k-\left(s_1+\frac{\gamma_y}{c_y}\right)y_1+\left(\mu+s_2+\varrho_{f,s_2}\nu\phi_2+\frac{\gamma_y}{c_y}\right)y_2-$$

$$-\left(\gamma_x-\gamma_y\frac{c_x}{c_y}\right)|v_x|\Big\}\,dt+(-m_1x_1+m_2x_2+am_3x_3)\vartheta_w\,dz_w+$$

$$+(n_2x_2+cm_3x_3)\vartheta_{p_m}\,dz_{p_m}-\phi_1y_1\,dw_1+(\nu\,dw_f+\phi_2\,dw_2)y_2 \qquad (3.10)$$

where we express all the levels as a fraction of π.

If we leave the utility criterion unchanged, the optimality conditions 1.20 become

$$0=J_\pi\left[ur_R+vr_F-(r_1+m_1w^*)-(1-u-v)\frac{\gamma_y}{c_y}\right]+ \qquad (3.11a)$$

$$+J_{\pi\pi}\pi\left[m_1^2\vartheta_wx_1-m_1m_2\vartheta_w^2x_2-am_1m_3\vartheta_w^2x_3-m_1\varrho_{f,w}\nu\vartheta_wy_2\right]$$

$$0=J_\pi\left(r_2+m_2w^*+n_2p_m^*+\frac{\gamma_y}{c_y}\right)+$$

$$+J_{\pi\pi}\pi\left[-m_1m_2\vartheta_w^2x_1+(m_2^2\vartheta_w^2+n_2\vartheta_{p_m}^2)x_2+\right.$$

$$\left.+(am_2m_3\vartheta_w^2+cn_2m_3\vartheta_{p_m}^2)x_3+(m_2\varrho_{f,w}\nu\vartheta_w+n_2\varrho_{f,p_m}\nu\vartheta_{p_m})y_2\right] \qquad (3.11\boldsymbol{d})$$

$$0=J_\pi\left[1V_x+m_3(aw^*+bPD^*+cp^*)+\frac{\gamma_y}{c_y}\right]+J_{\pi\pi}\pi\left[-am_1m_3\vartheta_w^2x_1+\right.$$

$$+(am_2m_3\vartheta_w^2+cn_2m_3\vartheta_{p_m}^2)x_2+(a^2m_3^2\vartheta_w^2+c^2m_3^2\vartheta_{p_m}^2)x_3+$$

$$\left.+(am_3\varrho_{f,w}\nu\vartheta_w+cm_3\varrho_{f,p_m}\nu\vartheta_{p_m})y_2\right] \qquad (3.11c)$$

$$0=J_\pi\left[-\left(s_1+\frac{\gamma_y}{c_y}\right)\right]+J_{\pi\pi}\pi\left[\phi_1^2y_1-(\varrho_{f,s_1}\nu\phi_1+\varrho_{s_1,s_2}\phi_1\phi_2)y_2\right] \qquad (3.11d)$$

$$0=J_\pi\left(\mu+s_2+\varrho_{f,s_2}\nu\phi_2+\frac{\gamma_y}{c_y}\right)+$$

$$+J_{\pi\pi}\pi\left[-(\varrho_{f,s_1}\nu\phi_1+\varrho_{s_1,s_2}\phi_1\phi_2)y_1+(\nu^2+\phi_2^2+2\varrho_{f,s_2}\nu\phi_2)y_2-\right.$$

$$-m_1\varrho_{f,w}\nu\vartheta_wx_1+(m_2\varrho_{f,w}\nu\vartheta_w+n_2\varrho_{f,p_m}\nu\vartheta_{p_m})x_2+$$

$$\left.+(am_3\varrho_{f,w}\nu\vartheta_w+cm_3\varrho_{f,p_m}\nu\vartheta_{p_m})x_3\right] \qquad (3.11e)$$

Let us set

$$\boldsymbol{R}_x=[-ur_R-vr_F+r_1,\,r_2,\,1V_x+bPD^*]^T,\qquad \boldsymbol{R}_y=[s_1,\,s_2]^T$$

$$\boldsymbol{e}=[1-u-v,\,1,\,1]^T,\qquad \boldsymbol{e}'=[1,\,1]$$

$$R(w)=[m_1, m_2, am_3]^T, \quad R(p)_m=[0, n_2, cm_3]^T$$

$$R_x(f)=\varrho_{f,w}\, v\vartheta_w\, R(w)+\varrho_{f,p_m}\, v\vartheta_{p_m}\, R(p_m)$$

$$R_y(f)=[0, \mu+\varrho_{f,s_2}\, v\phi_2]^T$$

$$R_y(X)=\left[0, R_x(f)^T \textstyle\sum_3^{-1}\left(R_x+R(w)\,w^*+R(p_m)\,p_m^*+e\,\frac{\gamma_y}{c_y}\right)\right]^T$$

$$\sum\nolimits_3=\begin{bmatrix} m_1\vartheta_w^2 & m_1 m_2\vartheta_w^2 & am_1 m_3\vartheta_w^2 \\ m_1 m_2\vartheta_w^2 & m_2^2\vartheta_w^2+n_2^2\vartheta_{p_m}^2 & am_2 m_3\vartheta_w^2+cn_2 m_3\vartheta_{p_m}^2 \\ am_1 m_3 v_m^2 & am_2 m_3\vartheta_w^2+cn_2 m_3\vartheta_{p_m}^2 & a^2 m_3^2\vartheta_w^2+c^2 m_3^2\vartheta_{p_m}^2 \end{bmatrix}$$

$$\sum\nolimits_4=\begin{bmatrix} \phi_1^2 & \varrho_{f,s_1} v\phi_1+\varrho_{s_1,s_2}\phi_1\phi_2 \\ \varrho_{f,s_1} v\phi_1+\varrho_{s_1,s_2}\phi_1\phi_2 & v^2+\phi^2+2\varrho_{f,s_2} v\phi_2+R_x(f)^T\sum_3^{-1} R_x(f) \end{bmatrix}$$

Then we get

$$X=\textstyle\sum_3^{-1}\left\{A\left[R_x+R(w)\,w^*+R(p_m)\,p_m^*+e\,\frac{\gamma_y}{c_y}\right]-R_x(f)\,y_2\right\} \tag{3.12}$$

$$Y=\textstyle\sum_4^{-1} A\left[R_y+R_y(f)+e'\,\frac{\gamma_y}{c_y}-R_y(X)\right] \tag{3.13}$$

These relations show that domestic deposit, loan supply and alternative investment demand depend on market conditions, wage policy, imported goods prices, transaction costs and on the level of foreign investments. In correspondence, foreign activities depend on market mutual conditions, exchange rate, transaction costs and on previous decisions on domestic activities.

Transaction decisions and capital investments will be modified according to these new values.

Also this model can be extended to an opportunity set varying in time as have seen in § 2.

CONCLUDING REMARKS

The rapid evolution of international activities induces a bank to consider not only the domectic balance, but to take into account and to control the whole portfolio. In this paper we extended the dynamic capital asset pricing model to the bank management and we compared two situations with respect to inflation. The analytical solution, we obtained, suggests that it is no more possible to separate domestic and foreign activities and regulations have to be extended to the whole portfolio.

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SUMMARY

In this paper the dynamic capital asset pricing model is extended to the bank management. There were introduced transaction costs, different interest rates on borrowing and leading, which are considered as random entities. The utility function depends not only a domestic consumption level, but also on capital investment.

These new conditions allows to derive a mutual fund theorem for a noninflationery market which separates domestic and foreign management decisions according to their degree of correlation.

A model of an international banking activities are considenal in the paper. The model takes into account transaction costs, different interest rates on borrowing and lending, which are considered as random entities. The utility function depend not only on domestic consumption levels but also on capital investments.

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