POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

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PREFACE

The present volume comprises the papers from the 3rd Italian-Polish Conference which was held in Mądralin (Poland) from 26 to 31. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Kraków (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis applied to the economic management, data processing and technological systems.

From the past, such meetings enrich the exchange of views and experiences between the specialists from both countries what is the useful to both sides from the general point, the investigations of the systems and control theory especially, as well as the contributions towards the essential development of the quantitative methods and the particular application of large complex systems.

The meaning of such meetings, however, goes far beyond that; what is happening during the conference between the established contacts are also fruitful in form of common scientific research works and — last but not least — in the friendly links between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

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GLOBAL IDENTIFICATION OF COMPLEX SYSTEMS

1. INTRODUCTION

Let us consider the complex input-output system with the known structure which is to be identified. The usual way of the identification of the system is the following: We use the data of the observations of the input and output for the separate subsystem and determine the optimal model (the optimal parameters for the known form of the model) for this subsystem. In this way we obtain the locally optimal submodel for the considered subsystem and the composition of the submodels according to the known structure does not give us (in general case) the optimal model for the whole system.

The problem is how to use the data of the input and output observations of each subsystems for the determination of the submodels in such a way that after the composition the model of the whole system will be globally optimal in some sense. The identification which leads to the globally optimal model will be called the global identification of the complex system.

Some first results concerning the global identification were presented in [1, 2]. In this paper the general problem is formulated, the identification of static and dynamic system with cascade structure is considered and the results for the linear case are given.

2. GENERAL PROBLEM STATEMENT

The problem is described here for the static system; the formulation for the dynamic system is analogous.

Consider the i-th subsystem of the complex system with the given structure, with input and output x_{in}, y_{in} respectively and assume that x_{in} and y_{in} ar the results of the independent observations of the random variables $\underline{x}_i$ and $\underline{y}_i$.

Let

$$\bar{y}_i = \Phi_i(\bar{x}_i, a_i) \tag{1}$$

be submodel of the i-th subsystem with the known function φ_i and the vector of parameters a_i is to be determined.

Definition 1

The submodel (1) is called locally optimal, if

$$a_i = \bar{a}_i$$

where $\bar{a}_i$ minimizes the criterion

$$Q_i = \underset{\underline{x}_i \underline{y}_i}{E}\left[\varphi(\underline{y}_i, \bar{\underline{y}}_i)\right] \tag{2}$$

with the known function φ, for example

$$\varphi(y_i, \bar{y}_i) = \left\| y_i - \bar{y}_i \right\|^2 \tag{3}$$

and for

$$x_i = \bar{x}_i$$

Definition 2

The model of the whole system with m subsystems is called globally optimal, if

$$a_i = a_i^* \qquad i = 1, 2, \dots, m$$

where $a_1^*, a_2^*, \dots, a_m^*$ minimize the global criterion

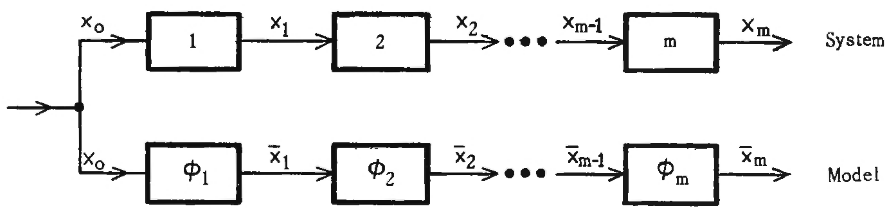
$$Q = F(Q_1, Q_2, \dots, Q_m)$$

for the known function F, for example

$$Q = \sum_{i=1}^{m} \gamma_i Q_i$$

3. STATIC SYSTEM WITH CASCADE STRUCTURE

Some analytical results concerning the global identification can be obtained for the system with cascade structure (Fig.).



Now

$$\left.\begin{aligned} \bar{x}_1 &= \varphi_1(x_0, a_1) \\ \bar{x}_i &= \varphi_i(\bar{x}_{i-1}, a_i) \end{aligned}\right\} \tag{4}$$

$$Q_i = E[\varphi(\underline{x}_i, \overline{\underline{x}}_i)] \tag{5}$$

and

$$Q = \sum_{i=1}^{m} Q_i$$

Assume that $x_0, x_1, \ldots, x_m$ are continuous random variables with the joint probability density

$$f(x_0, x_1, \ldots, x_m).$$

Assume also that $\underline{x}_i$ is a simple Marcov chain. Then the full probabilistic knowledge of the system is described by the probability densities

$$\bar{f}_1(x_1, x_0), \bar{f}_2(x_2, x_0), \ldots, \bar{f}_m(x_m, x_0). \tag{6}$$

Theorem

For the globally optimal model a_i^* is fully determined by the joint probability densities

$$f_{m,i-1}, f_{m-1,i-1}, \ldots, f_{i,i-1}$$

where

$$f_{l,k} = f_{l,k}(x_l, \overline{x}_k)$$

is the joint probability density of the outputs of l-th subsystem and k-th submodel.

Proof

The proof can be based on the dynamic programming approach which gives also the way to obtain the results.

From (4) and (5) we obtain

$$Q = \sum_{i=1}^{m} E[g(\underline{x}_i, \overline{\underline{x}}_{i-1}, a_i)]$$

where

$$g(\underline{x}_i, \overline{\underline{x}}_{i-1}, a_i) = \varphi[\underline{x}_i, \Phi_i(\overline{\underline{x}}_{i-1}, a_i)],$$

Denote

$$V_1 = \min_{a_1, a_2, \ldots, a_m} Q.$$

Starting from the last subsystem we obtain

$$V_2 = \min_{a_m} \underset{\underline{x}_m, \overline{\underline{x}}_{m-1}}{E} [g(\underline{x}_m, \overline{\underline{x}}_{m-1}, a_m)].$$

The value a_m^* minimizing V_1 depends on the $f_{m,\, m-1}$. The respective functional may be denoted as

$$a_m^* = F_m[f_{m,m-1}(\underline{x}_m, \overline{\underline{x}}_{m-1})].$$

The density $f_{m,\,m-1}$ depends on $f_{m,\,m-2}$ and Q_{m-1}. The respective relationship is determined by (4). Then

$$V_1 = V_1[f_{m,m-2}(x_m, \bar{x}_{m-2}), a_{m-1}].$$

In the next step

$$V_2 = \min_{a_{m-1}} E\{g(\underline{x}_{m-1}, \underline{\bar{x}}_{m-2}, a_{m-1}) + V_1[f_{m,m-2}(x_m, \bar{x}_{m-2}), a_{m-1}]\}.$$

The value a^*_{m-1} is then determined by $f_{m,\,m-2}$ and $f_{m,\,m-1}$

$$a^*_{m-1} = F_{m-1}[f_{m,m-2}, f_{m-1,m-2}].$$

Continuing this procedure we obtain

$$a^*_{m-i+1} = F_{m-i+1}[f_{m,m-i}, f_{m-1,m-i}, \ldots, f_{m-i+1,m-i}]$$

or

$$a^*_i = F_i[f_{m,i-1}, f_{m-1,i-1}, \ldots, f_{i,i-1}] \qquad i = 1, 2, \ldots, m$$

what should be proved.

Knowing (6) we can determine

$$a^*_1 = F_1[\bar{f}_m, \bar{f}_{m-1}, \ldots, \bar{f}_1].$$

Then for the known a^*_1 and (6) the densities

$$f_{m-1}, f_{m-1,1}, \ldots, f_{21}$$

can be determined and, consequently the a^*_2 can be obtained etc. In the practical situations the final result may be impossible to obtain because of the great analytical and computational difficulties. The analytical results can be obtained for the linear case.

4. IDENTIFICATION ALGORITHMS FOR THE LINEAR CASE

Let

$$\bar{x}_i = A_i \bar{x}_{i-1}, \qquad i = 1, 2, \ldots, m,$$

where the vectors x_i, x_{i-1} have the same dimension and

$$Q_i = E[(\underline{x}_i - \underline{\bar{x}}_i)^T (\underline{x}_i - \underline{\bar{x}}_i)].$$

It is well known that in this case for the locally optimal model the matrix A_i is the following

$$A_i = E(\underline{x}_i \underline{x}^T_{i-1})[E(\underline{x}_{i-1} \underline{x}^T_{i-1})]^{-1}.$$

Using the presented approach it may be easily shown that for the globally optimal model of the whole system

$$A^*_i = E(\underline{x}_i \underline{x}^T_0)[E(\underline{x}_{i-1} \underline{x}^T_0)]^{-1}. \tag{7}$$

The algorithm of the global identification is then the following

$$A_{in} = X_{in} X_{on}^T (X_{i-1,n} X_{0n}^T)^{-1}$$

where

$$X_{in} = [x_{i1}\, x_{i2} \ldots x_{in}]$$

is the matrix of the observations of x_i.

For the evaluation of the parameters for i-th submodel the results of the observations of the input and output of i-th subsystem and the input of the whole system must be used.

5. EXTENSION FOR DYNAMIC SYSTEM

The presented approach can be easily extended for the dynamic system, but the analogous consideretions are now much more complicated. Particularly, for the linear case

$$\bar{x}_i = \int_0^t K_i(t-\tau)\, \bar{x}_{i-1}(\tau)\, d\tau$$

where K is a matrix of the weight functions and

$$Q_i = E[(\underline{x}_i - \bar{x}_i)^T (\underline{x}_i - \bar{x}_i)]$$

(X_{in} are assumed to be stationary stochastic processes) — it is known that for the locally optimal submodel $K_i(t)$ must satisfied the following equation

$$R_{x_i x_{i-1}}(\Theta) = \int_0^\infty K(\lambda)\, R_{x_{i-1}, x_{i-1}}(\Theta - \lambda)\, d\lambda$$

where

$$R_{x_i x_{i-1}},\ R_{x_{i-1} x_{i-1}}$$

are the matrices of the correlation functions.

It can be shown by the presented approach that for the globally optimal model $K_i^*(t)$ must satisfied the equation

$$R_{x_i x_0}(\Theta) = \int_0^\infty K_i(\lambda)\, R_{x_{i-1}\, x_0}(\Theta - \lambda)\, d\lambda$$

which is analogous to (7) and may be used for the determination of the identification algorithm in which we use the empirical correlation functions obtained from the data of observations.

6. FINAL REMARKS

The general approach to the global identification of the complex system and some results for the linear static and dynamic sdstems with cascade structure are given. The presented approach can be extended for the case in which there are the externel measurable disturbances in each subsystem, i.e.

$$\bar{x}_i = \Phi_i(\bar{x}_{i-1}, z_i, a_i)$$

where z_i is the vector of the disturbances.

The further extension of the dynamic programming approach may be done for some cases of more complicated structure (particularly, for cascade-parallel and for hierarchical structure).

It should be noted that the global approach to the identification may lead to the great computational difficulties and the identification algorithms may be more complicated than in the local approach, but using the global identification method we can obtain the decrease of the value of the global criterion Q, i.e. the better model for the whole system. The presented idea was applied for the identification of the cascade of two reactors in some chemical process [3] and the value of Q was reduced about 10% by the application of the global approach.

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SUMMARY

The complex "input-output" system with a given structure is considered in the paper. For each element of the system the class of submodels is given. For each class the local criterion and for the whole system the global criterion are determined. The problem consits in finding such submodels from the given classes which minimize the global criterion, i.e. to find such submodels to obtain the globally optimal model for the whole system after composing the submodels according to the given structure.

The general problem of the global optimal model is formulated and fome special problems for the special cases concerning the form of the structure are considered.

The second part of the paper is concerned with the global identification problem for the complex system. The general approach to the solution for cascade structure using dynamical programming method is given and the algorithms of global identification for static and dynamical models and the cascade structure are presented.

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