POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

Redaktor techniczny
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BIAŁOWIEŻA, POLAND
MAY 26—31, 1976

EDITED BY J. GUTENBAUM

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PREFACE

The present volume comprises the papers from the first Italian-Polish Conference which was held in Warsaw in [illegible] 10 to 11. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Krakow (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis applied to the economic, management, data processing and technological systems.

From the past, such meetings enrich the exchange of views and experiences between the specialists from both countries what is the [illegible] to both sides, shows the several parts the [illegible] of the systems and control theory useful [illegible] the essential development of the quantitative methods and the possibilities [illegible] of large complex systems.

The meaning of such meetings, however, goes far beyond that, what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — begin the friendly links between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

G. P. Szegö
University of Bergamo
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ON THE OPTIMAL DESIGN OF MANAGEMENT INFORMATION SYSTEMS*)

1. INTRODUCTION

In recent years, management information systems based upon an extensive use of electronic computers has been applied to an ever expanding range of applications ranging from the financial management of firms, to the management of insurance companies and banks and to the management of local governments and of income tax offices.

The shift of areas of application from the industrial area into the areas of banking, insurance and governments has increased dramatically the "cost" associated with a possible malfunction or breakdown of the system. The possibilities of a fault in an information system are due to many causes and may take many different forms ranging from a short interruption of the service, to errors in the transcription of the data, from the loss or disruption of some files due to technical reasons to the loss of the whole data bank due either to natural calamities or to criminal cats.

Even if the possibilities of such disasterous events may be rather small, the "cost" involved in their occurrence is so large that in the design of the information system one must try to find ways and we may add the most economical ways, of reducing the damage due to some fault in the system.

Without discussing the elementary precautions that one can take, like the storage, in some form, of duplicates of the contents of the files, we notice that a "fail safe" system requires a certain set of duplication and redundancy at each level in order to increase the "reliability" of the system as a whole.

A second component which effects the overall cost of the management information system is its structure, i.e. the allocation of the memory and of the arithmetical capabilities among the various levels arranged in a hierarchical order versus a centralised solution, utilising a single large computer.

While the unitary cost of increasing the reliability of a unit of the system increases with the rise of the unit and it is therefore higher for a centralised system, other economic and organisational factors involved in the installation have so far been in favour of large systems. Only recently the introduction of

*) This research has been partially supported by CNR.

very conveniently priced mini-computers* has suggested the economic feasibility of designing management information systems based upon a network of mini-computers as opposed to a system based upon a single large computer with a single centralised memory and a number of input-output units. Notice that this solution of using a network of computers instead of a single centralised computer has been widely used in the control and scheduling of industrial production (see, for instance, S. E. Herson & R. G. Massey, proceedings IFAC Conference, Basle (1963)).

A clever design must attempt to optimise the structure of the system and to find the optimal trade off between the "costs" associated with the probability of a fault and the cost connected to the duplications and redundances required by the desired degree of reliability.

The aim of this paper is to formalise an optimisation problem which for a given problem and the relative "costs" associated to a fault in the system will enable the optimal structure of a multi-level management information system to be found.

2. THE OPTIMISATION MODEL

We shall present next an optimisation model for the identification of the optimal structure of a multi-level management information system (Fig. 1). We assume that the maximum number of levels in the system and the number of input-output lower lever units is fixed. The units at each level are all identical and are characterised by its computational capability and by the size of its memory. We assume that the input-output capabilities are fixed for each unit. At each unit we associate an economical cost associated to its operation and purchasing costs and another "cost" associated with the probability of its malfunction or breakdown. Clearly this second cost depends upon the particular **application** of the system. From this model it is possible to give a mathematical formulation of the problem of the optimisation of the information system, while the actual solution of the problem for each particular case requires the analytical identification of some functions which are assumed to be known in the model. In spite of the basic indeterminary of such function, we shall make some assumptions on their qualitative behaviour and these assumptions allow us to prove an existance theorem for the optimal solution of the problem.

In the following we shall denote by k **the maximum number of levels** in the system. We shall define the structure of the system by associating to it a certain number of k-dimensional column vectors, the i-th component of which characterises the corresponding property of i-th level. We shall denote

*) Clearly an exact definition of mini-computer is impossible, for the sake of comprehension we shall however call mini-computer the class of programmable digital computers which are now priced between $5,000 and $50,000, have memories ranging from a few K to around 128 K, and in ideal classification of digital calculating machines are placed between the calculators anf the computers.

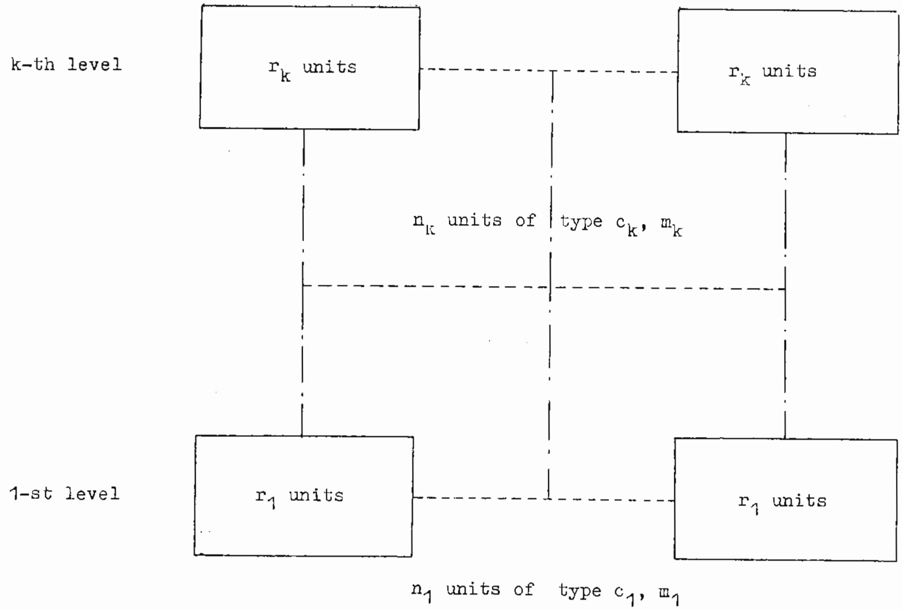

Fig. 1. Structure of the system

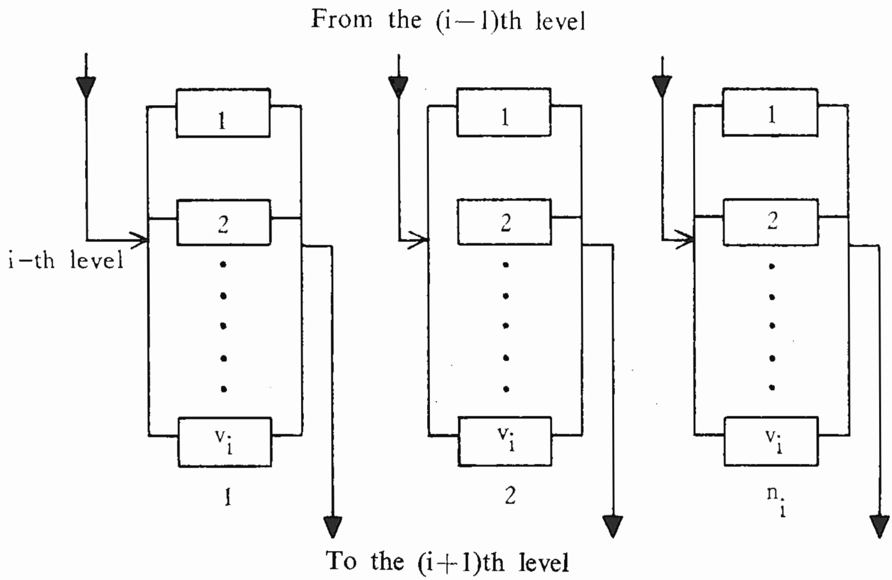

Fig. 2. The structure of the *i*-th level

by the k-dimensional column vector $\boldsymbol{n}$ the number of units used, by $\boldsymbol{m}$ the total capacity of the memory of the system, by $\boldsymbol{c}$ the computational capability of the system and with v the redundancy of the system. Thus the i-th components n_i, m_i, c_i and v_i of the k-dimensional vectors $\boldsymbol{n}$, $\boldsymbol{m}$, $\boldsymbol{c}$ and v will denote the number of units, the total capacity of memory, the computational capacibility and the redundancy of the i-th level, respectively, while m_i/n_i and c_i/n_i denote the memory capacity and the computational capability of each unit of the i-th level, respectively.

While the first three properties are very well defined, the fourth property, redundancy, must be properly defined. In order to simplify the problem, we shall denote by v_i the number of units which are parallel at the i-th level (Fig. 2). Thus the i-th level is composed of $v_i\ n_i$ unit of which only n_i can operate simultaneously.

If we then denote by $\boldsymbol{e}$ the k-dimensional unit vector, the total memory capacity of the whole system μ and the total computational capability γ are represented respectively by the expression

$$\mu = \boldsymbol{e}^T \boldsymbol{m} \tag{2.1}$$

and

$$\gamma = \boldsymbol{e}^T \boldsymbol{c} \tag{2.2}$$

If we respectively denote with μ_0 and γ_0 the minimum memory capacity and computational capability required by the system, then the following constraints must be satisfied

$$\mu \geqslant \mu_0 \tag{2.3}$$

and

$$\gamma \geqslant \gamma_0 \tag{2.4}$$

Define next two matrices $\boldsymbol{P}$ and $\boldsymbol{S}$ the first of which defines the probability of a fault in the system and the second defines the corresponding cost.

Thus

$$\boldsymbol{P} = [p_{ij}] \quad (i = 1, \ldots, k; \quad j = 1, \ldots, h) \tag{2.5}$$

where each element of the matrix

$$p_{ij} = p_{ij}(m_i, c_i, n_i, v_i, t) \tag{2.6}$$

represents the probability that at time t, a fault of type j will occur in any unit of the i-th level of the system.

The matrix $\boldsymbol{S}$ is defined by

$$\boldsymbol{S} = [s_{ij}] \quad (i = 1, \ldots, k; \quad j = 1, \ldots, h) \tag{2.7}$$

where each element of the matrix

$$s_{ij} = s_{ij}(m_i, c_i, n_i, v_i) \tag{2.8}$$

represents the cost corresponding to the fault ij. These costs include both the costs of repairing the unit in which the fault occurred as well as the costs connected with impossibility of using this unit.

The costs of purchasing, operating and up-keeping the system are represented by the k-dimensional vector a, the elements of which, a_i, have the form:

$$a_i = a_i(m_i, c_i) \quad (i = 1, \ldots, k) \tag{2.9}$$

and clearly depend on the memory capacity and on the computational capability of each unit of the i-th level only. The total purchasing and operating cost of the system is defined by the scalar α which has the form:

$$\alpha = \alpha(n_i, v_i, m_i, c_i) = \sum_{i=1}^{n} n_i v_i a_i \tag{2.10}$$

since the total number of units belonging to the i-th level is $n_i v_i$.

We can define next an efficiency function

$$\varepsilon = \varepsilon(\alpha, \bar{S}) \tag{2.11}$$

where α is defined by 2.10 and $\bar{S}$, the expected value of the costs associated with the faults, is given by the expression

$$\bar{S} = E(\sum_{i=1}^{k} \sum_{j=1}^{k} n_i s_{ij} p_{ij}) \tag{2.12}$$

We shall assume that the efficiency function ε has the following behaviour represented in Fig. 3 and characterised by the following sign properties:

$$\varepsilon(\infty, \bar{S}) = 0$$

$$\varepsilon(\alpha, \bar{S}_{max}) = 0 \tag{2.13}$$

$$\varepsilon > 0 \quad \text{in all other cases}$$

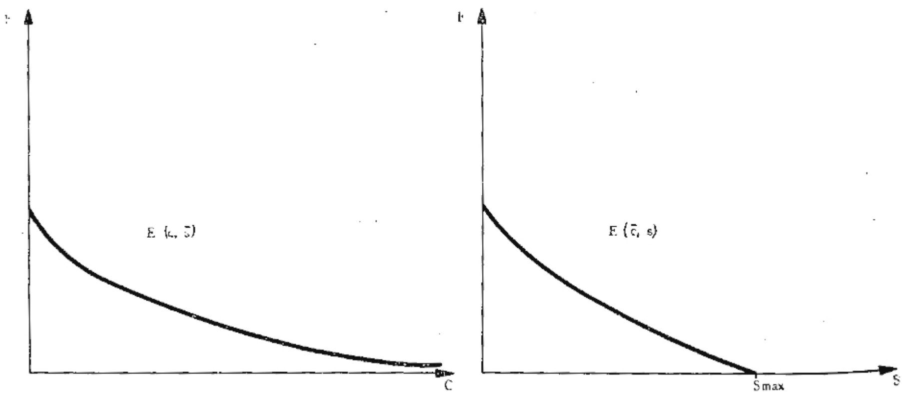

Fig. 3. Behaviour of the function E

The optimisation problem which must be solved for the identification of the optimal (most economical) system structure is the following

$$\max_{n,\nu,m,c} \varepsilon(\alpha, \bar{S}) \tag{2.14}$$

subject to the constraints

$$n \geqslant 0,\ \nu \geqslant 0,\ m \geqslant 0,\ c \geqslant 0,\ \mu \geqslant \mu_0,\ \gamma \geqslant \gamma_0 \tag{2.15}$$

The first question which arises is that of the existance of one or more solutions to the problem 2.14—2.15. We shall try next to give some qualitative answer to this problem with some assumptions of the qualitative behaviour of the efficiency function.

3. SOME RESULTS ON THE EXISTENCE OF OPTIMAL SOLUTION

In this section we shall prove the existence of one point which solves the maximisation problem 2.14—2.15.

In order to prove this result we must first obtain some information on the properties of the efficiency function ε.

Consider first the explicit expression of the efficiency function; we have

$$\varepsilon = \varepsilon\Big(\sum_{i=1}^{k} n_i v_i a_i(c_i m_i),\ \sum_{i=1}^{k} \sum_{j=1}^{h} n_i s_{ij} p_{ij}\Big) \tag{3.1}$$

Consider next the derivatives of ε with respect to m_i and c_i

$$\frac{\partial \varepsilon}{\partial m_i} = \frac{\partial \varepsilon}{\partial \alpha}\left(m_i v_i \frac{\partial a_i}{\partial m_i}\right) + \frac{\partial \varepsilon}{\partial S} \sum \left(m_i p_{ij} \frac{\partial s_{ij}}{\partial m_i} + n_i s_{ij} \frac{\partial p_{ij}}{\partial m_i}\right) \tag{3.2}$$

and

$$\frac{\partial \varepsilon}{\partial c_i} = \frac{\partial \varepsilon}{\partial \alpha}\left(n_i v_i \frac{\partial a_i}{\partial c_i}\right) + \frac{\partial \varepsilon}{\partial S} \sum \left(m_i p_{ij} \frac{\partial s_{ij}}{\partial c_i} + n_i s_{ij} \frac{\partial p_{ij}}{\partial c_i}\right) \tag{3.3}$$

Now it is easy to show that the following sign conditions hold

$$\partial a_i / \partial m_i > 0 \tag{3.4}$$

$$\partial s_{ij}\ \partial m_i > 0 \tag{3.5}$$

$$\partial p_{ij} / \partial m_i > 0 \tag{3.6}$$

The inequality 3.4 is justified by the fact that the purchasing cost (in absolute figures) increases with the capacity of the memory, while inequalities 3.5 and 3.6 are due to the fact that by increasing the size of the memory the probability of faults also increases, and therefore the associated costs also increase.

Similar arguments can be applied in showing that also the following three inequalities hold

$$\partial a_i/\partial c_i > 0, \quad \partial s_{ij}/\partial c_i > 0, \quad \partial p_{ij}/\partial c_i > 0 \tag{3.7}$$

In addition since the efficiency of the system decreases as the technical costs and the expected value of the loss due to malfunctions increases, we have:

$$\partial\varepsilon/\partial\alpha < 0, \qquad \partial\varepsilon/\partial S < 0 \tag{3.8}$$

Thus from 3.2—3.8 it follows that

$$\partial\varepsilon/\partial m_i < 0, \qquad \partial\varepsilon/\partial c_i < 0, \tag{3.9}$$

which shows that the efficiency function ε is decreasing with respect to m_i as well as c_i. Thus the maximum of ε will belong to the set of tangency points of the level surfaces $\varepsilon = \text{const}$ with the equality constraints

$$\mu = \mu_0, \qquad \gamma = \gamma_0 \tag{3.10}$$

If we consider next the behaviour of the function ε with respect to the variables n_i and v_i we can make the following remarks. Since α is increasing with respect to v_i and

$$v_i \to \infty \text{ implies } \alpha \to \infty \tag{3.11}$$

from the properties of ε it follows that α

$$v_i \to \infty \text{ implies } \varepsilon \to \infty \tag{3.12}$$

and

$$n_i \to \infty \text{ implies } \varepsilon \neq 0 \tag{3.13}$$

Hence the maximum of ε (2.14) on the admissible set 2.15 cannot be at infinity. Thus there exists one point

$$(n_i, v_i) \quad i = 1, \ldots, k \tag{3.14}$$

in the subspace of the decision variables $\boldsymbol{n}, \boldsymbol{x}$ in which ε reaches its maximum value.

From the consideration above it follows that there exists one point

$$\boldsymbol{n}^*, \boldsymbol{v}^*, \boldsymbol{m}^* (\boldsymbol{n}^*, \boldsymbol{v}^*), \boldsymbol{c}^* (\boldsymbol{n}^*, \boldsymbol{v}^*) \tag{3.15}$$

such that

$$\boldsymbol{e}^T \boldsymbol{m}^* = \mu_0 \tag{3.16}$$

and

$$\boldsymbol{e}^T \boldsymbol{c}^* = \gamma^0 \tag{3.17}$$

where ε reaches its maximum value.

4. CONCLUSIONS

The particular structure of the problem shows that from the numerical point of view it is rather easy for each application to find its optimal solution, after some analytical expression for the efficiency function has been derived. Due to the finite number of systems configurations which are technically available, the procedure suggested must be amended to take into account the discreteness of the problem or incorporated in a simulation model.

SUMMARY

In recent years management information systems based upon an extensive use of computers has been applied to an ever expending range to financial management of firms, insurance companies and banks, to the management of local governments and of income tax offices.

The shift of areas of application has increased dramatically the cost associated with the possible malfunction or breakdown of the system. In the design of the information systems one must try to find ways of ıeducing the damage due to same fault in the system.

A second component which effcts the overall cost of the management information system is its structure i.e. the allocation of the memory and of the arithmetical capabilities among the various levels arranged in a hierarchical order versus a centralized solution.

The aim of this paper is to formalise an optimization problem which for a given problem and the relative costs associated to a fault in the system will enable the optimal structure of a multi-level management information system to be found.

From the numerical point of view it is rather easy for each application to find its optimal solution, after some analytical expression for the efficiency function has been derived.

Instytut Badań Systemowych PAN
Nakład 300 egz. Ark. wyd. 25,0. Ark. druk. 23,75. Papier druk. sat. kl. III 80 g 61×86. Oddano do składania 8 X 1976 Podpisano do druku w sierpniu 1978 r. Druk ukończono w sierpniu 1978 roku

CDW — Zakład nr 5 w Bielsku-Białej zam. 62/K/77 J-124