New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

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Principal Component Analysis and Factor Analysis for IF data sets

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Abstract

In this paper we compare two method for reduction of dimensionality. First method is Principal component analysis and second method is Factor analysis. We present this methods for data from Atanassov's intuitionistic fuzzy sets. We construct an example for usage of these methods. The calculations are realized in the program R.

Keywords: Principal component analysis, Factor analysis, Atanassov intuitionistic fuzzy sets.

1 Introduction

The motivation to write this paper was the article by E.Szmidt, J.Kacprzyk, P.Bujnowski (2012) - Advances in Principal Component Analysis for Intuitionistic Fuzzy Data Sets. In this article I saw practical use Atanassov intuitionistic fuzzy sets for to solve the problem of reduction of dimensionality data. I think that data from the Atanassov intuitionistic fuzzy set better describes character of the studied compounds. In the classical case we examine a sample of the one-sided point of view, but if sample is from IF sets then this sample is examined from three perspectives (membership function, non-membership function and hesitation margin of IF set).

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2 Basic definitions about IF-sets

Let $X \neq 0$. By an Intuitionistic Fuzzy set (IF set) we consider a pair $A = (\mu_A, \nu_A)$ of functions

$$\mu_A : X \to \langle 0, 1\rangle$$

$$\nu_A : X \to \langle 0, 1\rangle$$

such that $\mu_A + \nu_A \leq 1$. The function μ_A is called a membeship function of A and ν_A a non-membership function of A.
For each IF set in X, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

the intuitionistic fuzzy index (or hesitation margin). The $\pi_A(x)$ expresses a lack of knowledge of whether x belongs to A or not. It is obvious, that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$ (Atanassov [1]).

3 Correlation between the Atanassov IF sets

Correlation between the Atanassov IF sets (denote A-IFSs) was introduced by Szmidt and Kacprzyk [2] in 2010.

Let A, B be A-IFSs defined on $X = \{x_1, x_2, ..., x_n\}$. The sets A, B are characterized by a sequence of pairs:
$[(\mu_A(x_1), \nu_A(x_1), \pi_A(x_1)), (\mu_B(x_1), \nu_B(x_1), \pi_B(x_1))]$,
$[(\mu_A(x_2), \nu_A(x_2), \pi_A(x_2)), (\mu_B(x_2), \nu_B(x_2), \pi_B(x_2))]$,
...
$[(\mu_A(x_n), \nu_A(x_n), \pi_A(x_n)), (\mu_B(x_n), \nu_B(x_n), \pi_B(x_n))]$
which correspond to the membership values, non-membership values and hesitation margins of A and B.

Definition 1 (Szmidt, Kacprzyk, Bujnowski [3]) *The correlation coefficient $r_{A-IFS}(A, B)$ between two A-IFSs A and B in X is:*

$$r_{A-IFS}(A, B) = \frac{1}{3}\left(r_1(A, B) + r_2(A, B) + r_3(A, B)\right) \tag{1}$$

where

$$r_1(A, B) = \frac{\sum_{i=1}^{n} \left(\mu_A(x_i) - \bar{\mu_A}\right)\left(\mu_B(x_i) - \bar{\mu_B}\right)}{\left(\sum_{i=1}^{n} \left(\mu_A(x_i) - \bar{\mu_A}\right)^2\right)^{0.5} \left(\sum_{i=1}^{n} \left(\mu_B(x_i) - \bar{\mu_B}\right)^2\right)^{0.5}} \tag{2}$$

$$r_2(A,B) = \frac{\sum_{i=1}^{n}\left(\nu_A\left(x_i\right) - \bar{\nu_A}\right)\left(\nu_B\left(x_i\right) - \bar{\nu_B}\right)}{\left(\sum_{i=1}^{n}\left(\nu_A\left(x_i\right) - \bar{\nu_A}\right)^2\right)^{0.5}\left(\sum_{i=1}^{n}\left(\nu_B\left(x_i\right) - \bar{\nu_B}\right)^2\right)^{0.5}} \quad (3)$$

$$r_3(A,B) = \frac{\sum_{i=1}^{n}\left(\pi_A\left(x_i\right) - \bar{\pi_A}\right)\left(\pi_B\left(x_i\right) - \bar{\pi_B}\right)}{\left(\sum_{i=1}^{n}\left(\pi_A\left(x_i\right) - \bar{\pi_A}\right)^2\right)^{0.5}\left(\sum_{i=1}^{n}\left(\pi_B\left(x_i\right) - \bar{\pi_B}\right)^2\right)^{0.5}} \quad (4)$$

where
$\bar{\mu_A} = \frac{1}{n}\sum_{i=1}^{n}\mu_A\left(x_i\right)$, $\bar{\nu_A} = \frac{1}{n}\sum_{i=1}^{n}\nu_A\left(x_i\right)$, $\bar{\pi_A} = \frac{1}{n}\sum_{i=1}^{n}\pi_A\left(x_i\right)$,

$\bar{\mu_B} = \frac{1}{n}\sum_{i=1}^{n}\mu_B\left(x_i\right)$, $\bar{\nu_B} = \frac{1}{n}\sum_{i=1}^{n}\nu_B\left(x_i\right)$, $\bar{\pi_B} = \frac{1}{n}\sum_{i=1}^{n}\pi_B\left(x_i\right)$

The correlation coefficient (1) depends on the two factors:

- the amount of information expressed by the membership and nonmembership degrees (2), (3)
- the reliability of information expressed by the hesitation margins (4).

The correlation coefficient (1) has the following properties [3]:

1. $r_{A-IFS}(A,B) = r_{A-IFS}(B,A)$
2. If $A = B$ them $r_{A-IFS}(A,B) = 1$
3. $|r_{A-IFS}(A,B)| \leq 1$

This properties are fulfilled by its every component (2) - (4).

4 Principal Component Analysis and Factor Analysis for the A-IFS data

Principal component analysis (PCA) was invented in 1901 by Karl Pearson. It is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it be orthogonal to (i.e., uncorrelated with) the preceding components.

Basic steps of PCA:

- construct a correlation matrix of the source data
- find a eigenvalues of the correlation matrix and rearrange eigenvalues from largest to smallest($\lambda_1 > ... > \lambda_n$)
- find a eigenvectors of the correlation matrix corresponding eigenvalues $(v_1, ..., v_n)$
- calculate the variability of the source data $\left(\sigma^2\right)$
- select a subset of the eigenvectors as the basic vectors
- convert the source data into new basis

Factor analysis (FA) was invented in 1904 by Charles Edward Spearman. It is a statistical method used to describe variability among observed, correlated variables in terms of a potentially lower number of unobserved variables called factors. In other words, it is possible, for example, that variations in three or four observed variables mainly reflect the variations in fewer unobserved variables. Factor analysis searches for such joint variations in response to unobserved latent variables. The observed variables are modeled as linear combinations of the potential factors, plus ”error” terms. The information gained about the interdependencies between observed variables can be used later to reduce the set of variables in a dataset.
Factor loadings reflect the effect of the k-th common factor for j-th random variable.
To **estimate the factor loadings** is used several methods are called methods of extraction factors. We will to use *Principal Component Analysis*.

Determining of the number of common factors

- **The criterion of the eigenvalues** - the factors, which have their eigenvalues $\lambda > 1$ are significant. If the number of variables from 20 to 50, then the rule is reliable.
- **Variance explained criteria**

- **Scree plot** - graph of the eigenvalues. We use a number of factors that are before the breaking point in the graph.

Communality: The sum of the squared factor loadings for all factors for a given variable is the variance in that variable accounted for by all the factors, and this is called the communality. The communality measures the percent of variance in a given variable explained by all the factors jointly and may be interpreted as the reliability of the indicator.

Basic steps of FA:

- selection of data
- determining the number of common factors
- estimation of the parameters
- rotation of factors (*Varimax Method*=orthogonal rotation)
- estimate of elements of the factor matrix (=matrix of factor loadings)
- convert the source data into new basis

The result of the Factor analysis is matrix of factor loadings. If factor loading is high (0.5), than this factor is statistically significant.

Factor analysis is related to principal component analysis, but the two are not identical. Latent variable models, including factor analysis, use regression modelling techniques to test hypotheses producing error terms, while PCA is a descriptive statistical technique.

The main idea of both methods: FA and PCA are trying to reduce the dimensionality of the data group.

5 Example

Job position
We have 20 candidates to job. In the selection of candidates were rated four criteria:

A - Qualifying
B - Communication
C - Independent
D - Skill

Each criterion was evaluated 2 times. How many percent is the criterion is met for each participant and how many percent is the criterion is not met. The results are in following Table 1.

	A (%)		B (%)		C (%)		D (%)	
	m	nm	m	nm	m	nm	m	nm
1	32	50	64	21	67	15	70	10
2	61	20	37	55	65	20	65	25
3	59	25	40	50	43	22	40	30
4	36	50	62	32	35	40	40	40
5	62	20	46	50	40	38	20	75
6	52	35	84	10	87	5	80	5
7	76	15	52	35	80	12	75	10
8	89	5	70	15	73	10	70	15
9	59	37	85	5	58	21	64	22
10	53	47	40	54	52	37	52	37
11	78	12	62	37	60	18	62	28
12	90	5	54	38	78	5	82	10
13	65	24	30	51	82	2	75	13
14	58	40	15	65	62	15	59	24
15	75	12	65	17	40	45	80	12
16	32	60	85	8	54	38	30	56
17	11	70	49	40	20	65	5	70
18	65	18	85	10	20	58	68	22
19	74	10	95	2	63	13	74	10
20	55	40	42	45	52	23	52	33

Table 1: designation: m = meets, nm = not meets

We assign the membership functions and non-membership functions to dates A, B, C and D from Table 1. Following conditions must be met, that $\mu, \nu \in \langle 0, 1\rangle$ and $\mu + \nu \leq 1$ for A, B, C and D. Then there are A-IFS data. (Tabuka 2). We calculate the hesitation margins for dates A,B,C a D from following formula $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

	A		B		C		D	
	μ_A	ν_A	μ_B	ν_B	μ_C	ν_C	μ_D	ν_D
1	0.32	0.50	0.64	0.21	0.67	0.15	0.70	0.10
2	0.61	0.20	0.37	0.55	0.65	0.20	0.65	0.25
3	0.59	0.25	0.40	0.50	0.43	0.22	0.40	0.30
4	0.36	0.50	0.62	0.32	0.35	0.40	0.40	0.40
5	0.62	0.20	0.46	0.50	0.40	0.38	0.20	0.75
6	0.52	0.35	0.84	0.10	0.87	0.05	0.80	0.05
7	0.76	0.15	0.52	0.35	0.80	0.12	0.75	0.10
8	0.89	0.05	0.70	0.15	0.73	0.10	0.70	0.15
9	0.59	0.37	0.85	0.05	0.58	0.21	0.64	0.22
10	0.53	0.47	0.40	0.54	0.52	0.37	0.52	0.37
11	0.78	0.12	0.62	0.37	0.60	0.18	0.62	0.28
12	0.90	0.05	0.54	0.38	0.78	0.05	0.82	0.10
13	0.65	0.24	0.30	0.51	0.82	0.02	0.75	0.13
14	0.58	0.40	0.15	0.65	0.62	0.15	0.59	0.24
15	0.75	0.12	0.65	0.17	0.40	0.45	0.80	0.12
16	0.32	0.60	0.85	0.08	0.54	0.38	0.30	0.56
17	0.11	0.70	0.49	0.40	0.20	0.65	0.05	0.70
18	0.65	0.18	0.85	0.10	0.20	0.58	0.68	0.22
19	0.74	0.10	0.95	0.02	0.63	0.13	0.74	0.10
20	0.55	0.40	0.42	0.45	0.52	0.23	0.52	0.33

Table 2:

Principal component analysis

We calculate the correlation matrices for membership $\mathbf{R}_\mu$, non-membership values $\mathbf{R}_\nu$ and hesitation margins $\mathbf{R}_\pi$ (correlation components (2) - (4) and their eigenvalues and eigenvectors.

$$\mathbf{R}_\mu = \begin{pmatrix} 1.00000000 & 0.03399483 & 0.44397730 & 0.6749821 \\ 0.03399483 & 1.00000000 & -0.04322014 & 0.2015552 \\ 0.44397730 & -0.04322014 & 1.00000000 & 0.6576481 \\ 0.67498208 & 0.20155524 & 0.65764810 & 1.0000000 \end{pmatrix}$$

$$\mathbf{R}_\nu = \begin{pmatrix} 1.00000000 & 0.08211306 & 0.43986483 & 0.5309616 \\ 0.08211306 & 1.00000000 & -0.06139627 & 0.2920429 \\ 0.43986483 & -0.06139627 & 1.00000000 & 0.6663423 \\ 0.53096160 & 0.29204286 & 0.66634231 & 1.0000000 \end{pmatrix}$$

$$\mathbf{R}_\pi = \begin{pmatrix} 1.0000000 & -0.23705667 & 0.17778691 & 0.2057912 \\ -0.2370567 & 1.00000000 & -0.07366807 & 0.1886569 \\ 0.1777869 & -0.07366807 & 1.00000000 & 0.3668935 \\ 0.2057912 & 0.18865693 & 0.36689347 & 1.0000000 \end{pmatrix}$$

The eigenvalues for the correlation matrix $\mathbf{R}_\mu$ with respect to the membership values are :

```
2.2023149, 1.0281328, 0.5496674, 0.2198849.
```

The total variation $\sigma^2 = 4$.

We present Results PCA obtained through the program R:

```
Importance of components:
                         Comp.1   Comp.2   Comp.3    Comp.4
Standard deviation     1.484019 1.013968 0.741395 0.4689188
Proportion of Variance 0.550578 0.257033 0.137416 0.0549712
Cumulative Proportion  0.550578 0.807611 0.945028 1.0000000
```

In line "Standard deviation" are values of the variance of principal components $\left(\sqrt{\lambda_i},\ i = 1, 2, 3, 4\right)$.
In line "Proportion of Variance" are proportions $\frac{\lambda_i}{\sigma^2},\ i = 1, 2, 3, 4$.
In line "Cumulative Proportion" are cumulative proportions of the variability. We can see that three first components explain $94,5\%$ of the overall variation.

Similar we proceed to non-membership values and hesitation margins.

The eigenvalues for the correlation matrix $\mathbf{R}_\mu$ with respect to the non-membership values are :

```
2.1286980, 1.0511994, 0.5800064, 0.2400963.
```

The total variation $\sigma^2 = 4$.

```
Importance of components:
                         Comp.1   Comp.2   Comp.3    Comp.4
Standard deviation     1.459005 1.025280 0.761581 0.4899961
Proportion of Variance 0.532174 0.262799 0.145001 0.0600240
Cumulative Proportion  0.532174 0.794974 0.939975 1.0000000
```

We can see that three first components explain 94% of the overall variation.

The eigenvalues for the correlation matrix $\mathbf{R}_\mu$ with respect to the hesitation margins are :

```
1.5146290, 1.2168545, 0.7610724, 0.5074441.
```

The total variation $\sigma^2 = 4$.

```
Importance of components:
                         Comp.1   Comp.2   Comp.3   Comp.4
Standard deviation     1.230702 1.103111 0.872394 0.712351
Proportion of Variance 0.378657 0.304213 0.190268 0.126861
Cumulative Proportion  0.378657 0.682870 0.873139 1.000000
```

We can see that three first components explain $87,31\%$ of the overall variation.

- We calculate the total correlation matrices $\mathbf{R}$ (correlation components (1)) and their eigenvalues and eigenvectors.

$$\mathbf{R} = \begin{pmatrix} 1.00000000 & -0.04031626 & 0.35387635 & 0.4705783 \\ -0.04031626 & 1.00000000 & -0.05942816 & 0.2274183 \\ 0.35387635 & -0.05942816 & 1.00000000 & 0.5636280 \\ 0.47057831 & 0.22741834 & 0.56362796 & 1.0000000 \end{pmatrix}$$

The eigenvalues for the total correlation matrix $\mathbf{R}$ are :

```
1.9385953, 1.0683501, 0.6544004, 0.3386543
```

and three first components explain $91,65\%$ of the overall variation.

Now, we can determine the number of principal components which is sufficient for presentation of original four variables. Original four components can be replaced by three principal components while maintaining $91,65\%$ of the variability of original dates.

The result of PCA procedure is in the Table 3. Columns in the table are three first eigenvectors of the covariance matrices $\mathbf{R}_\mu$, $\mathbf{R}_\nu$, $\mathbf{R}_\pi$. Principal components

	"membership"			"non-membership"			"hesitation"		
	Principal Components								
	1.	2.	3.	1.	2.	3.	1.	2.	3.
A	0.55	0.06	0.72	-0.52	0.09	0.84	0.49	-0.43	0.66
B	0.10	-0.97	-0.11	-0.17	-0.93	-0.06	-0.09	0.80	0.33
C	0.54	0.23	-0.69	-0.56	0.33	-0.48	0.61	0.07	-0.65
D	0.62	-0.09	-0.02	-0.62	-0.12	-0.25	0.60	0.41	0.17

Table 3:

is obtained by multiplying the eigenvectors with the original dates.
In this way we obtain a reduced description of the problem in 3 instead of 4 dimensions.

Factor analysis

We will deal with cases of the factor analysis based on the method PCA. The input data are in Table 2. The correlation matrices and their eigenvalues we have calculated in the previous method PCA. Since that the set of source set have 20 variables, we have at least 2 criteria to determine the number of factors.
Only the first two eigenvalues of the correlation matrices $\mathbf{R}_\mu$, $\mathbf{R}_\nu$, $\mathbf{R}_\pi$ are larger than 1, in any case.
Further, we examine the overall variation of source set. If we consider two factors then is the overall variation of data equal 75%. Such percentage of variation is sufficient in the case, that we consider data set from the social sciences. Further analysis, we will consider two factors.

At first, we solve the case of FA for data of the membership function $\mu_A, \mu_B, \mu_C, \mu_D$. First two factors represent 80,8% overall variation. The matrix of factor loadings is:

$$\mathbf{A}^*_\mu = \begin{pmatrix} 0.8213470 & 0.06529165 \\ 0.1495805 & -0.98058653 \\ 0.8045785 & 0.22929373 \\ 0.9262738 & -0.09871300 \end{pmatrix}$$

We can see from the matrix, that first factor (first column of matrix) have high loadings in the 1., 3. and 4. variable. Second factor (second column of matrix) have high loadings in the 2. variable. We can say that the matrix have simple structure because the matrix have high factor loadings in only one factor. Then the matrix is not necessary rotate. Values of communalities are:

```
0.6788739, 0.9839243, 0.6999222, 0.8677274
```

The values are sufficiently great. Then we can use two factors instead four original variables.

Next, we solve the case of FA for data of the nonmembership function $\nu_A, \nu_B, \nu_C, \nu_D$. First two factors represent 79,5% overall variation. The matrix of factor loadings is:

$$\mathbf{A}^*_\nu = \begin{pmatrix} -0.7622182 & 0.0912679 \\ -0.2450798 & -0.9562097 \\ -0.8178901 & 0.3369507 \\ -0.9048277 & -0.1224619 \end{pmatrix}$$

We can say that the matrix $\mathbf{A}^*_\nu$ have simple structure. Values of communalities are:

```
0.5893063, 0.974401, 0.78248, 0.83371
```

The values are sufficiently great. Then we can use two factors instead four original variables.

Next, we solve the case of FA for data of the nonmembership function $\pi_A, \pi_B, \pi_C, \pi_D$. First two factors represent 68,3% overall variation. The matrix of factor loadings is:

$$\mathbf{A}^*_\pi = \begin{pmatrix} 0.6132218 & -0.47283838 \\ -0.1190048 & 0.88433688 \\ 0.7577326 & 0.07817622 \\ 0.7417999 & 0.45289640 \end{pmatrix}$$

We can see from the matrix, that first factor have high loadings in the 3. and 4. variable. Second factor have high loadings in the 2. variable. In the first variable the factor loadings is not sufficiently high for any factor. We have to rotate the matrix. The rotate matrix of factor loadings is:

$$\mathbf{A}^{*\mathbf{rot}}_\pi = \begin{pmatrix} 0.411 & -0.656 \\ 0.195 & 0.871 \\ 0.738 & -0.190 \\ 0.853 & 0.167 \end{pmatrix}$$

In the first variable the factor loadings is not sufficiently high for any factor. We will calculate values of communalities:

```
0.5996172, 0.7962139 , 0.5802703, 0.7553822
```

The values are sufficiently great. Then we can use two factors instead four original variables.
We remove first variable from data set and again we calculate FA without this variable.
In this case, first two factors represent 81,5% overall variation. The matrix of factor loadings is:

$$\mathbf{AA}_{\pi}^{*} = \begin{pmatrix} 0.2729327 & 0.9171304 \\ 0.7609551 & -0.4536824 \\ 0.8560128 & 0.1108828 \end{pmatrix}$$

The rotate matrix of factor loadings is:

$$\mathbf{AA}_{\pi}^{*\mathbf{rot}} = \begin{pmatrix} & 0.957 \\ 0.856 & -0.227 \\ 0.793 & 0.342 \end{pmatrix}$$

We can say that the matrix $\mathbf{A}_{\nu}^{*}$ have simple structure. A missing value in the matrix is number close to zero.
Values of communalities are:

```
0.9156203 , 0.7848804, 0.7450528
```

The values are sufficiently great. We have confirmed the theory that we can use two factors instead four original variables.

In this way we obtain a reduced description of the problem in 2 instead of 4 dimensions.

6 Conclusions

Both methods allow dimensionality reduction of original dataset while maintaining the sufficient variability of the original data. In the case method PCA is dimensionality reduction from 4 to 3 and overall variation is 91,65%. In the case method FA is dimensionality reduction from 4 to 2 and overall variation is 81,5%. If we consider with two components in the method PCA, then overall variation will be 75,17%. In our example, we worked with a small data set and a small number of variables. We assume that a gradual increase in the number of variables, the difference in the dimensionally reduction of the data set for these methods may lapse.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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