New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

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ISBN 83-894-7546-4

Intelligent Counting – Connections with Classification Issues

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Abstract

The subject of this paper is intelligent counting under imprecision possibly combined with incompleteness of information. Formally, it collapses to counting in fuzzy sets and their extensions. We show that there is a strong connection between intelligent counting and classification issues whenever counting by thresholding and FECounts are involved.

Keywords: intelligent counting, fuzzy set, Atanassov's intuitionistic fuzzy set, classification.

1 Preliminaries

This paper is about counting. It is a procedure forming one of the most elementary and frequent mental activities of human beings. Its results are a basis for coming to a decision in a lot of situations and dimensions of our life. Speaking about counting, however, one should distinguish between two essentially different cases.

- The objects of counting are *precisely specified*, e.g.

"How many apples are there in the basket?".

This is a trivial task collapsing to counting in a set. It can be summarized as "Do not think, just count the apples".

● The objects of counting are *imprecisely specified*, e.g.

"How many *big* apples are there in the basket?".

Each apple is big to a degree. So, what and how to count? Also only to a degree? This counting thus requires intelligence and, consequently, what we deal with can be termed intelligent counting. Formally, it collapses to counting in a fuzzy set.

There are two general classes of tools for intelligent counting in a fuzzy set $A\colon \boldsymbol{U} \to [0, 1]$.

● *Scalar tools*: the result of counting in A then forms a nonnegative real number (see e.g. [2-4]). The basic case is here *counting by thresholding* in which that results is

$$\sigma_t(A) = |A_t|,$$

where $t \in (0, 1]$ and A_t denotes the t-cut set of A, $A_t = \{x \in \boldsymbol{U}\colon A(x) \geq t\}$. So, one counts up those elements whose quality is high enough, is at least equal to t with a given threshold t.

● *Fuzzy tools*: the result of counting in A is now a fuzzy set of nonnegative integers. An important particular case seems to be $\mathrm{FE}(A)$, the *FECount of A* for which ([4])

$$\mathrm{FE}(A)(k) = \min(a_k, 1 - a_{k+1}), \quad k = 0, 1, \ldots, \tag{1}$$

where a_k denotes the kth greatest membership degree in A, including possible repetitions, and one puts $a_0 = 1$, $a_k = 0$ for $k > |\mathrm{supp}(A)|$.

Both scalar and fuzzy cardinalities of fuzzy sets and their extensions are formalizations and reflections of real, human counting procedures performed under imprecision and possibly incompleteness of information (see [2, 3] for details).

2 Intelligent Counting and Classification

We like to focus on intelligent counting in A via counting by thresholding and FECounts. Let us try to divide the universe $\boldsymbol{U}$ into two disjoint classes ***class1*** and ***class2***, where t is a fixed number from $(0, 0.5]$:

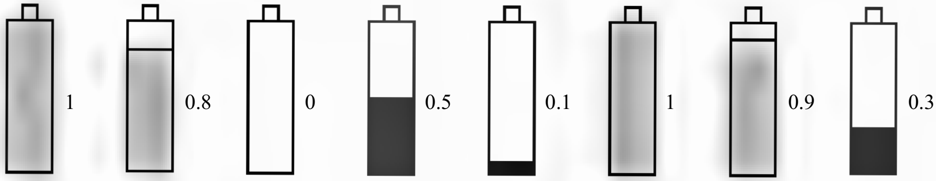
● $x \in \boldsymbol{U}$ is assigned to ***class1*** whenever $A(x) > 1 - t$,
● x is assigned to ***class2*** if $A(x) < t$,
● x remains unclassifiable whenever $A(x) \in [t, 1 - t]$.

We will refer to this way of classification as the *t-classification*. One shows that the number of unclassifiable elements is then equal to

$$|\{x\colon A(x)\in[t, 1-t]\}| = \sigma_t(A\cap A') = \max(0, |(\mathrm{FE}(A))_t| - 1) \qquad (2)$$

with $\cap$ and $'$ denoting the standard operations of intersection and complementation for fuzzy sets, respectively. FE(A) can thus be viewed as an encoded piece of information about the number of unclassifiable elements in t-classifications.

Example 2.1 (The eight-bottle example). Let us look at the following instance in which A is a fuzzy set of full bottles of water.



Clearly, this model situation can be understood as a fuzzy set of some resources, preferences, etc. The water level in bottle b_i is the membership degree of b_i in A, $i = 1, \dots, 8$. How many bottles of water do we have? Various counting procedures and, consequently, various answers are possible (see [2, 3] for details). For instance, by (1),

$$\mathrm{FE}(A) = 0.1/2 + 0.2/3 + 0.5/4 + 0.5/5 + 0.3/6 + 0.1/7$$

as

$$a_1 = a_2 = 1,\ a_3 = 0.9,\ a_4 = 0.8,\ a_5 = 0.5,\ a_6 = 0.3,\ a_7 = 0.1.$$

Applying the concept of a t-classification with $t\in(0, 0.5]$, ***class1*** = ***full*** and ***class2*** = ***empty***, we get

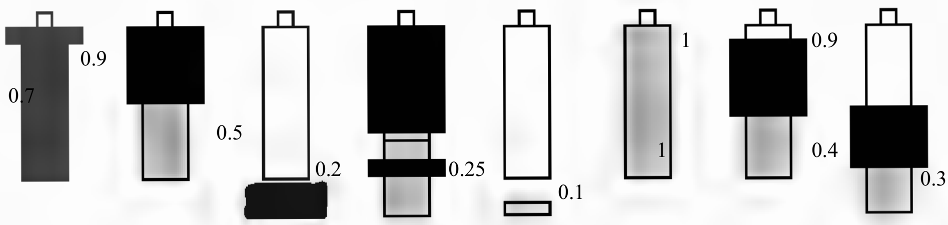
- b_i is classified as ***full*** if $A(b_i) > 1-t$, and is classified as ***empty*** if $A(b_i) < t$,
- b_i is viewed as unclassifiable whenever $A(b_i)\in[t, 1-t]$; b_i then contains too little water to be classified as ***full*** and too much water to be treated as ***empty***.

Notice that b_i always remains unclassifiable if $A(b_i) = 0.5$. Using (2), say, with any $t\in(0.2, 0.3]$, we have $|(\mathrm{FE}(A))_t| - 1 = 2$. This means that two bottles are then unclassifiable (b_4, b_8). Moreover, $b_1, b_2, b_6, b_7 \in$ ***full*** and $b_3, b_5 \in$ ***empty***. ■

3 The Case of Incompletely Known Fuzzy Sets

Let us make the situation more sophisticated. Assume A: $\boldsymbol{U} \rightarrow [0, 1]$ we deal with is an incompletely known fuzzy set, i.e. incompletely known are (at least some) membership degrees in A. This kind of a vague collection of elements from $\boldsymbol{U}$ can be modeled as an interval-valued fuzzy set, a pair (A_l, A_u) of fuzzy sets in which the first component is contained in the second one. Another option is modeling through an I-fuzzy set, Atanassov's intuitionistic fuzzy set (IFS); see e.g. [1]. These two ideas are formally (but not practically) equivalent. Recollect that an IFS representing incompletely known A is a pair $\mathcal{E} = (A^+, A^-)$ of fuzzy sets such that the first component is contained in $(A^-)'$. We will focus on the case of modeling by means of IFSs.

Example 3.1 (The eight-bottle example modified). Look at the following fuzzy set A of full bottles of water.



The content of some bottles is now partially covered up. We thus deal with an incompletely known fuzzy set. Again, how many bottles of water do we have? This collapses to counting under imprecision combined with incompleteness of information. Let us model A as an IFS $\mathcal{E} = (A^+, A^-)$. Then, practically, $A^+(b_i)$ is a visible degree to which b_i is full, whereas $A^-(b_i)$ forms a visible degree to which b_i is empty, $i = 1, \ldots, 8$. For instance, $A^+(b_7) = 0.4$ and $A^-(b_7) = 0.1$. There is no direct generalization of (2) to IFSs and their FECounts. However, generally,

$$\sigma_t(\mathcal{E} \cap \mathcal{E}') = [\,\sigma_t(A^+ \cap A^-), \sigma_t((A^+)' \cap (A^-)')]. \qquad (3)$$

The first component of the integer interval in (3) is the number of surely unclassifiable elements whenever the t-classification is involved. The second one forms the number of surely or possibly unclassifiable elements. These two endpoints, in other words, are the minimum and maximum possible number of unclassifiable elements, respectively. Referring to the above figure, we get

$$\sigma_t(\mathcal{E} \cap \mathcal{E}') = [1, 4] \quad \text{for each } t \in (0.2, 0.3].$$

This means that exactly one bottle, b_8, is then surely unclassifiable. Four bottles (b_2, b_4, b_7, b_8) are surely or possibly unclassifiable. Furthermore, b_1, $b_6 \in$ ***full*** and b_3, $b_5 \in$ ***empty***. ■

4 Conclusions

This paper has been devoted to intelligent counting. We mean counting under imprecision and possibly incompleteness of information about the objects of counting. Formally, this collapses to counting in fuzzy sets and their extensions. We have shown that there is a strong and natural connection between intelligent counting and some classification issues. The interested reader is referred to [3] for further discussion and details.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The Workshop has also been in part technically supported by COST Action IC0806 ``Intelligent Monitoring, Control and Security of Critical Infrastructure Systems" (INTELLICIS).

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475466

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