Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

SRI PAS **IBS PAN**

Modern Approaches in Fuzzy Sets,
Intuitionistic Fuzzy Sets,
Generalized Nets and Related Topics.
Volume I: Foundations

Systems Research Institute
Polish Academy of Sciences

Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Editors

Krassimir Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

IBS PAN **SRI PAS**

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 83-894-7553-7

An intuitionistic fuzzy estimation of the area of 2D-figures based on the Pick's formula

Evgeniy Marinov
Dept. of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences,
105 Acad. G. Bonchev Str, Sofia 1113, Bulgaria
e-mail: evgeniy.marinov@biomed.bas.bg
Emilia Velizarovav
Institute of Forest Research
Bulgarian Academy of Sciences
132 St. Kliment Ohridski Blvd, Sofia 1756, Bulgaria
e-mail: velizars@abv.bg
Krassimir Atanassov
Dept. of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences,
105 Acad. G. Bonchev Str, Sofia 1113, Bulgaria
e-mail: krat@bas.bg

Abstract

An iterative procedure for estimation of the area surrounded by a simple closed curve in the real 2D space is proposed. We employ the Pick's Formula for calculating the area surrounded by a special types of polygons. Starting from an initial grid-step and ending up with a smaller grid-step - satisfactory to be able to build the inner and outer polygons. We propose in this paper also a formula for intuitionistic fuzzy estimation for the area surrounded the curve. The proposal is a numerical method allowing to program the algorithm in any procedural language. The iterative process stops when

Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations *(K.T. Atanassow, M. Baczyński, J. Drewniak, J. Kacprzyk, M. Krawczak, E. Szmidt, M. Wygralak, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2014*

a satisfactory small enough limit between the upper and lower estimation has been reached.

Keywords: Square hull, inner and outer polygon, intuitionistic fuzzy estimation.

1 Introduction

We propose in this paper a formula for intuitionistic fuzzy estimation for the area surrounded by a continuous simple closed curve in the real 2D space, i.e. the area of its interior. By a *simple* curve we mean that it has no self intersections and if, moreover, the curve is continuous, this implies that its interior is a *simply connected* domain (cf. Munkres [6], Ch. 9). In contrast with Marinov et al. [5], the algorithm introduced in this paper employs the *Pick's formula*. *Pick's theorem* provides a simple formula for calculating the area S surrounded by this polygon in terms of the number I of grid-points in the interior of the polygon, i.e. not touching any of the sides, and the number B of grid-points on the boundary, i.e. placed on the polygon's perimeter. Assuming that we have a grid with grid-step equal to one, Pick's formula provides the number of unit squares through the following expression:

$$S = I + \frac{B}{2} - 1 \tag{1}$$

Therefore, assuming a grid-step equal to l_0, the formula provides

$$S(l_0) = l_0\Big(I + \frac{B}{2} - 1\Big) \tag{2}$$

for the area surrounded by the given polygon.

Given a 2D Cartesian coordinate system Oxy and a simple curve parametrized by

$$\vec{r}(t) = (r_1(t), r_2(t)) : [0,1] \to \mathbb{R} \times \mathbb{R} \tag{3}$$

we are going to split the underlying space to a grid with lines parallel to the two axes (see Fig 1.). This mesh has to be fine enough for a good estimation of the given curve. To be explained later what exactly "*fine enough grid*" means. We will, moreover, at the end consider only the smallest part of the grid with borders, consisting of lines parallel to the two axes, which enclose the curve. In order to define the intuitionistic fuzzy measure of the area enclosed by the initial curve, we are going to introduce an algorithm that provides a special type of polygon surrounding the initial curve $\vec{r}$. We suppose that the given parametrization $\vec{r}$ of the curve provides a positive orientation, which means anti-clockwise orientation.

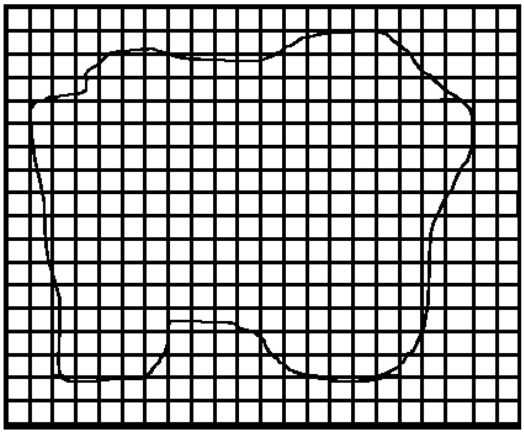

Figure 1: Simple curve and a grid with lines parallel to the two axes.

That is, following the path of $\vec{r}$ on the figure, the outer part will be on the right-hand side, while the inner one will lie on the left-hand side. Applying the same procedure for the curve $\overrightarrow{r}^{-1}(t) = \vec{r}(1-t)$, for which the inner part of $\overrightarrow{r}^{-1}$ coincides with the outer one of $\vec{r}$, while its outer part coincides with the inner part of $\vec{r}$. Therefore, in order to build an inner and outer polygon enclosing the given initial curve, it suffices to describe only the outer (surrounding) polygon of $\vec{r}$. These two polygons will be of use to build a hull of the curve, which is analogous of the *square hull* introduced in [5]. We will afterwards present an intuitionistic fuzzy estimation through the produced hull of polygons introduced in the following section.

2 Building the outer polygon of $\vec{r}$

Taking initially the grid with a step of length l_0, supposed to be sufficiently small, we pass along the curve with $\vec{r}(t)$ letting the parameter t vary from 0 to 1 and, as assumed in the previous section, the curve is positively oriented. Along its path, $\vec{r}(t)$ intersects the lines of the grid in different points and may pass through various squares of the grid leaving/entering them through their vertices or edges.

2.1 Basic notions and definitions

Considering squares, we mean by default the square taken with its boundary, i.e. together with the four corresponding edges and vertices as we did in [5]. In [5], we

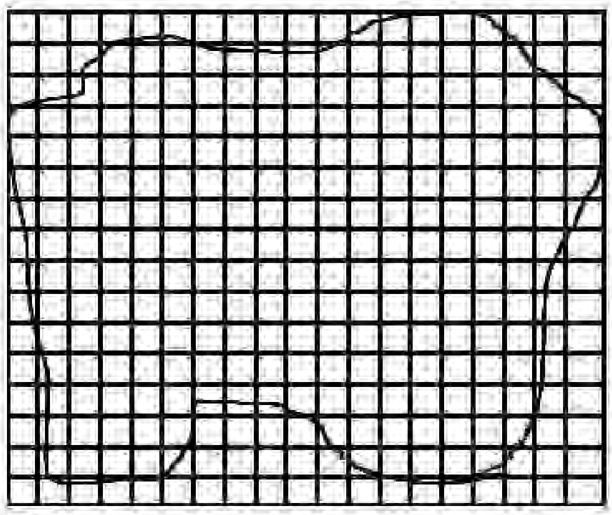

Figure 2: Minimal rectangle with edges parallel to the axis part of the grid containing the curve. The area of this rectangle is $\mathcal{S}(\vec{r}, l_0) = 20 \times 16 \times l_0$.

took the approximated square hull to consist of squares only, while now through the inner and outer polygons we introduce a better approximation considering the diagonals of the squares as well. The main *atom-figure*, i.e. particle of the grid which can not be further split, building the hull will be taken to be the family of all triangles of the grid build up by the squares split by their two diagonals.

Let us take all the nodes of the grid to be marked as white in the beginning. Then, passing through some nodes while the procedure is running, they will be marked as "black" or "not-allowed", i.e. nodes already lying in the $P - stack$ (program stack). For a more detailed definition of the notion $P-stack$, the reader may consult [5]. Briefly, the procedure passes through some nodes building the outer polygon by edges and diagonals of squares of the grid putting them in the so called $P - stack$. Let us mention that every node N of the grid has exactly eight neighbors because the node belongs to exactly four squares. These four squares have $4 \times 4 = 16$ nodes (vertices), four of them taken two times in the sum and the node in consideration is counted four times as well. Therefore, the neighbor nodes of the node in consideration are exactly $16 - 4 - 4 = 8$. They can be *diagonal* or *edge* nodes in respect of the current vertex, i.e. four diagonal neighbors and four edge neighbors as well – to be denoted $DiagN(N)$ and $EdgeN(N)$, respectively (see Fig. 3). Let us denote $NeighborN(N) = DiagN(N) \cup EdgeN(N)$ and define the square consisting of all this neighbor nodes as the *neighbor square* of

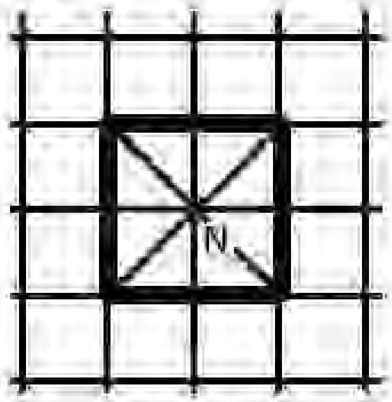

Figure 3: The vertex N with the neighbor square $NeighborS(N)$ and the border $\partial NeighborS(N)$ in bold.

the current node and denote it by $NeighborS(N)$. The border of the neighbor square consists of eight sides, to be denoted by $\partial NeighborS(N)$. There are also four inner sides of length the grid-step l_0 and for diagonal sides (edges) of length $\sqrt{2}l_0$, denoted by $StraightE(N)$ and $DiagE(N)$, respectively. Let us denote by $InnerE(N) = StraightE(N) \cup DiagE(N)$ all the inner sides of $NeighborS(N)$. We can also consider $NeighborS(N)$ as the union of eight right isosceles triangles, i.e. triangles with a right angle ($\frac{1}{2}\pi$), and also two equal angles (sides). The family of these eight triangles will be denoted by $NeighborT(N)$.

The grid is principally infinite but for the sake of simplicity we will concentrate ourselves, as already done in [5], on the minimal part of the grid, which contains the curve as shown on Fig 2. This constraint is important because on the basis of this picture we are going to introduce intuitionistic fuzzy estimation in the next section.

Just after a node (vertex) has been colored "black" and put into the $P-stack$, consider only the "allowed" ones of its neighbors as candidates to be taken as "next node" when building the oriented polygon. The orientation of the closed polygon enclosing $\vec{r}$ will be the same as the one of $\vec{r}$, i.e. anti-clockwise. Starting by the first vertex lying near the curve in the outer part of $\vec{r}$, we are going to describe a procedure which will add new nodes to the $P - stack$ in a sequence building correctly the outer polygon. If, according to the rules of the algorithm, we come to a step where the "next" node, say N'', which should be added to the stack, would be such one that the atomic particle $N'N''$ of the polygon crosses the curve, we stop the procedure and start again with a twice shorter grid-step $\frac{l_0}{2^n}$ if the current grid-step is $\frac{l_0}{2^{(n-1)}}$. In what follows, with N' we will always denote current node, i.e. the last node in the $P - stack$. In case that in the current step of the procedure we cross some part of the polygon already built and lying within the $P - stack$, we break and start the procedure again with a twice shorter grid-step than the current one.

Let us suppose that at the beginning of the procedure we have a list of the vertices (nodes), which lie in the inner part of the initial curve - $InnerN(\vec{r})$. In what follows, we will denote the nodes lying outside of or on $\vec{r}$ by $\overline{OuterN}(\vec{r})$, where $OuterN(\vec{r})$ are the points lying outside of the closed curve.

And therefore, we can check if a node lies inside the curve, outside or on it. For example, there are many fast and efficient algorithms employing the theory of the winding number of a point with respect to a closed curve.

2.2 Algorithm producing the outer polygon of $\vec{r}$

Let us now describe the procedure in a meta-programming style, as done in [5]. We take as initial grid-step a sufficiently small number l_0 and start from a point $N \in \overline{OuterN}(\vec{r})$ as shown on Fig. 4. Supposing that the procedure has already accomplished few steps and N is the last node in the $P - stack$, let us describe the next step. In what follows, by $R(t)$ we will denote the point from the curve $\vec{r}$ at $t \in [0, 1]$, i.e. $\vec{r}(t) = OR(t)$ where O is the origin of the coordinate system. For the point N (see Fig. 4) we want that for some $t_0 \in [0, 1]$ the point $R(t_0)$ lies on some edge from $StraightE(N)$ and therefore we have that $d(R(t_0), N) \leq l_0$. Let for $t_1 \geq t_0$ the point $R(t)$ enters the inside of some of the four neighbor triangles $T \in NeighborT(N)$. Suppose that $R(t)$ then goes outside of the triangle T and enters T again, i.e. there are $t_3 > t_2 > t_1$ such that $R(t_1), R(t_3) \in T$ while $R(t_2) \notin T$. If this happens, then we break and start the procedure again with a grid-step $\frac{l_0}{2}$. Therefore, we can suppose that the grid-step l_0 is small enough so that the above situation can not happen, i.e. for any right angle triangle T with vertices - three neighbor nodes on the grid, it is not possible that

$$(\exists t_1, t_2, t_3 \in [0, 1])(R(t_1), R(t_3) \in T \,\&\, R(t_2) \notin T).$$

Let us go back to the current last point N in the $P - stack$ with $R(t_0)$ lying on one of the $StraightE(N)$. Then, we have two cases

- IF $R(t_0, 1)$ lies in the neighbor square of N, i.e. $NeighborS(N)$. That means that the first point from the $P - stack$, N_1, belongs to

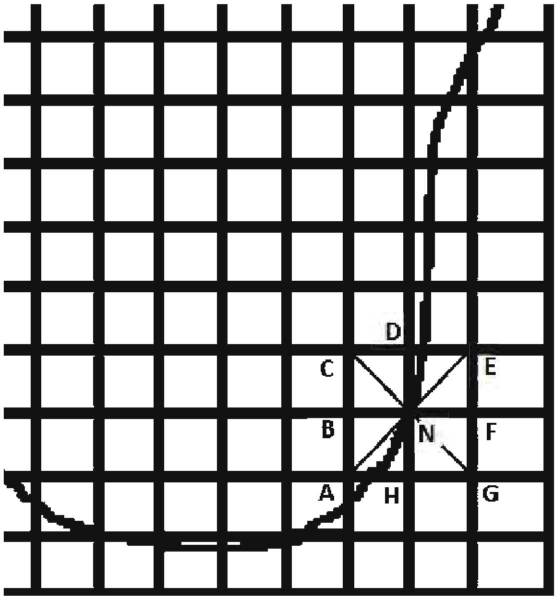

Figure 4: The vertex N with the neighbor square and all the neighbor nodes A, B, C, D, E, F, G and H. This is the right lower corner from the main figure.

$NeighborS(N)$, i.e. $N_1 \in NeighborN(N)$. If all the points of NN_0 lie in the outside of $\vec{r}$, then the **procedure finishes successfully** in letting N be the last node in the $P - stack$. Otherwise, if NN_0 has a point belonging to the inside of $\vec{r}$, then **break and restart the procedure with a grid-step $\frac{l_0}{2}$.**

- ELSE there is $t' \in (t_0, 1]$ for which $R(t') \in \partial NeighborS(N)$. If all the points of $NR(t')$ lie in the outside part of $\vec{r}$, then the **procedure continues** in letting $R(t')$ be the current last node in the $P - stack$. Otherwise, if $NR(t')$ has a point belonging to the inside of $\vec{r}$, then **break and restart the procedure with a grid-step $\frac{l_0}{2}$.**

Under the assumptions made in this section and all the denotations from the last section, the whole procedure of finding an outer polygon square has been described very easily. As we have already mentioned, we may apply the same procedure for the curve $\overrightarrow{r}^{-1}(t) = \vec{r}(1-t)$, for which the inner part of $\overrightarrow{r}^{-1}$ coincides with the outer one of $\vec{r}$, while its outer part coincides with the inner part of $\vec{r}$. Let us suppose that the procedure has stopped successfully with a grid-step

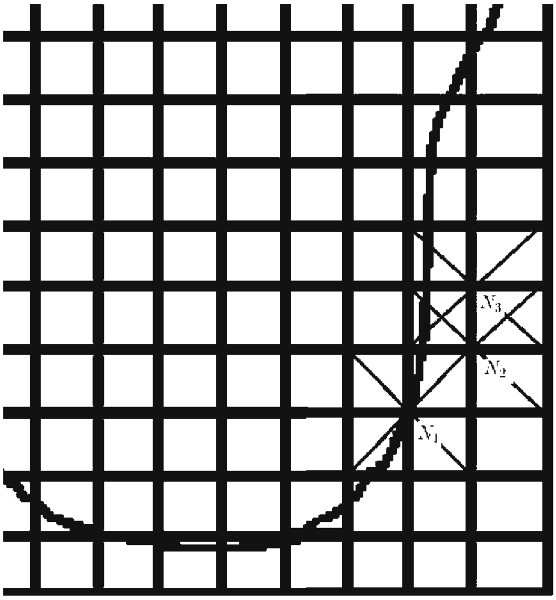

Figure 5: The vertices $N_1, N_2, N3$, which are the first three nodes from the $P-stack$. This is the right lower corner from the main figure.

$\frac{l_0}{2^{n_1}}$ in finding the outer polygon and $\frac{l_0}{2^{n_2}}$ for the inner polygon. We then take the minimum of the two grid-steps to be $\frac{l_0}{2^{j_0}}$ ($j_0 = \max(n_1, n_2)$) and start the procedure again to find a new outer polygon (if $n_1 > n_2$) or to find a new inner polygon (if $n_1 < n_2$). Therefore, we get the outer (in blue) and inner polygons (in green) in the same grid with grid-step $\frac{l_0}{2^{j_0}}$ as shown on Fig. 6 and Fig. 7.

3 Intuitionistic fuzzy estimation of the inner area

Let us now give an instuitionistic fuzzy estimation of the initial curve based on the inner and outer polygons produced by the algorithm proposed in the last section.

The notion of intuitionistic fuzzy set (or abbreviated as IFS) provides a very intuitive and natural tool for an adequate estimation of the area enclosed by a simple continuous curve. As an application of this method we give an estimation for the area of a forest fire spread.

A fuzzy set in X (cf. Zadeh [8]) is given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{4}$$

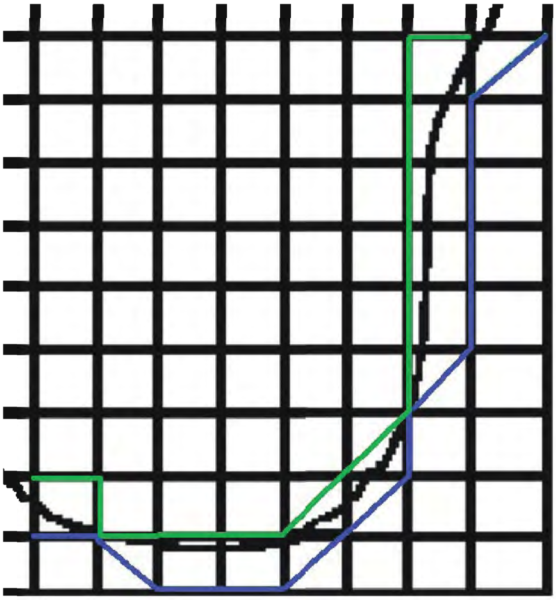

Figure 6: The inner and outer polygons with the end grid-step $\frac{l_0}{2^{j_0}}$. This is the right lower corner from the main figure.

where $\mu_{A'}(x) \in [0,1]$ is the *membership function* of the fuzzy set A'. As opposed to the Zadeh's fuzzy set (abbreviated FS), Atanassov extended its definition to an intuitionistic fuzzy set (IFS) (cf. [2] and [3]) A, given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{5}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{6}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a *degree of membership* and a *degree of non-membership* of x to A, respectively. An additional concept for each IFS in X, that is an obvious result of (5) and (6), is called

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{7}$$

the *degree of uncertainty*, expressing a lack of knowledge of whether x belongs to A or not (cf. [2]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$. Hesitation margins turn out to be relevant for both applications and the development of theory

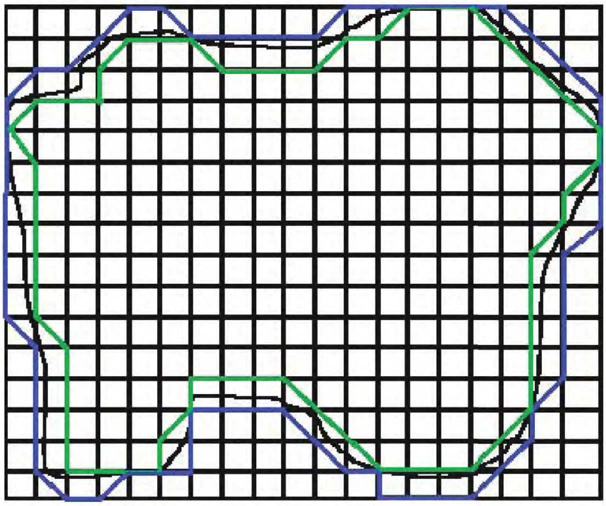

Figure 7: The whole picture of the inner and outer polygons with the end grid-step $l_1 = \frac{l_0}{2^{j_0}}$ of the first iteration. The inside area of the inner polygon is 205.5 while the area of the outer one is 252.5.

of IFSs. For instance, distances between IFSs are calculated in the literature in two ways, using two parameters only (cf. Atanassov [2]) or all three parameters (cf. Szmidt and Kacprzyk [7], Atanassov et al. [4]). Both ways are proper from the point of view of pure mathematical conditions concerning distances, but one cannot say that both ways are equal when assessing the results obtained by the two approaches.

As described in the previous section, the iterative procedure starts with inputs - the simple continuous curve $\vec{r}(t)$, an initial sufficiently small grid-step l_0. The algorithm ends up at an iteration, say $j(\vec{r}, l_0) = j_0$, where we may assume that j_0 is the maximum of the numbers of iterations producing the inner and outer polygons at first approximation of the procedure. And therefore, the end grid step becomes $l_1 = \frac{l_0}{2^{j_0}}$. In what follows, we denote the already defined $\mathcal{S}_{j_0} = \mathcal{S}(\vec{r}, \frac{l_0}{2^{j_0}})$, for the fixed curve and initial step l_0. Let us give, on the basis of the already defined $\mathcal{A}^o$ and $\mathcal{A}^i$, an intuitionistic fuzzy estimation.

The area enclosed by the inner and outer polygons for the given grid-step will be denoted by $\mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0}})$ and $\mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0}})$, respectively. The areas $\mathcal{A}^i$ and $\mathcal{A}^o$ can be calculated by Pick's formula (2). Moreover, the numbers I and B of the interior and boundary vertices for any of the polygons can be determined, for instance, with the algorithm given by D. Alciatore and R. Miranda in [1].

Definition 1 *In the above notations let us define*

$$\mu_{(\vec{r},l_0)}(0) = \frac{\mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0}})}{\mathcal{S}_{j_0}} \textit{ and } \nu_{(\vec{r},l_0)}(0) = \frac{\mathcal{S}_{j_0} - \mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0}})}{\mathcal{S}_{j_0}},$$

$$\mu_{(\vec{r},l_0)}(1) = \frac{\mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0+1}}))}{\mathcal{S}_{j_0}} \textit{ and } \nu_{(\vec{r},l_0)}(1) = \frac{\mathcal{S}_{j_0} - \mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0+1}})}{\mathcal{S}_{j_0}}.$$

More generally, let us inductively define for any positive integer k,

$$\mu_{(\vec{r},l_0)}(k) = \frac{\mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0+k}}))}{\mathcal{S}_{j_0}} \textit{ and } \nu_{(\vec{r},l_0)}(k) = \frac{\mathcal{S}_{j_0} - \mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0+k}})}{\mathcal{S}_{j_0}}.$$

Taking in consideration the last definition we may write down the degree of uncertainty for the kth step as $\pi_{(\vec{r},l_0)}(k) = 1 - \mu_{(\vec{r},l_0)}(k) - \nu_{(\vec{r},l_0)}(k)$. Therefore, we have that

$$\pi_{(\vec{r},l_0)}(k) = \frac{\mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0+k}}) - \mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0+k}})}{\mathcal{S}_{j_0}},$$

which is exactly the intuitionistic fuzzy estimation of the degree of uncertainty for the corresponding grid-step $\frac{l_0}{2^{j_0+k}}$. It can be proved that for $k_1 < k_2$ we have that $\pi_{(\vec{r},l_0)}(k_1) > \pi_{(\vec{r},l_0)}(k_2)$, which means exactly that $\pi_{(\vec{r},l_0)}$ is a decreasing function on the set of positive integer numbers $\mathbb{N}$. Therefore, we may suppose that we are given a small enough positive real number ϵ_0 based on the curve $\vec{r}$, which inside is supposed to be estimated. Through the described algorithm, we are computing then iteratively upper and lower estimations (through outer and inner polygons) until a positive integer k has been reached for which $0 \leq \pi_{(\vec{r},l_0)}(k) \leq \epsilon_0$. The k-th iteration provides then a satisfactory intuitionistic fuzzy estimation of the curve. This also means that $\mathcal{A}^o(\vec{r}, \frac{l_0}{2^{j_0+k}})$ and $\mathcal{A}^i(\vec{r}, \frac{l_0}{2^{j_0+k}})$ provide a corresponding satisfactory upper and inner estimations of the area surrounded by the curve.

Example 1 *An example of a curve $\vec{r}$ has been provided through the pictures of this paper. (see Fig. 7). According to the Pick's formula, for the corresponding areas we have, (say after the end of the procedure started for first time with a grid-step l_0 and finishing with end grid-step $l_1 = \frac{l_0}{2^{j_0}}$), that*

- $\mathcal{S}(\vec{r}, l_1) = \mathcal{S}_{j_0} = 320 \times l_1,$
- $\mathcal{A}^i(\vec{r}, l_1) = \left(178 + \frac{57}{2} - 1\right) \times l_1 = 205.5 \times l_1,$
- $\mathcal{A}^o(\vec{r}, l_1) = \left(223 + \frac{61}{2} - 1\right) \times l_1 = 252.5 \times l_1.$

Therefore, for the intuitionistic fuzzy estimations after the first application of the procedure, we have that

- $\mu_{(\vec{r},g,l_0)}(1) = \frac{205.5}{320}$,
- $\nu_{(\vec{r},g,l_0)}(1) = \frac{320-252.5}{320} = \frac{67.5}{320}$,
- $\pi_{(\vec{r},g,l_0)}(1) = \frac{252.5-205.5}{320} = \frac{47}{320}$.

Let us remark that, as done in [5], for the second, third etc. time repeating the initial procedure producing the polygons, we use the same $\mathcal{S}_0$ which is the output from the first start of the procedure with grid-step l_0.

4 Conclusion

We see now that the described procedure and the intuitionistic fuzzy estimation give an iterative numerical algorithm, which can be implemented in any procedural programming language.

The method described in this paper can be adequately applied to the estimation of the area of a forest fire spread. The reader may compare the proposed algorithm with the one described in [5]. In a next research, we will discuss a modification of the Pick's formula for 3D-figures.

Acknowledgements

This work has been partially supported by the Bulgarian National Science Fund under the grant "Simulation of wild-land fire behavior" - I01/0006

References

[1] Alciatore, D. and Miranda, R. (1995), "A Winding Number and Point-in-Polygon Algorithm," Glaxo Virtual Anatomy Project research report, Department of Mechanical Engineering, Colorado State University.

[2] Atanassov K. (1999), Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag, Heidelberg.

[3] Atanassov K. (2012), On Intuitionistic Fuzzy Sets Theory. Springer-Verlag, Berlin.

[4] Atanassov K., Tasseva V, Szmidt E. and Kacprzyk J. (2005) On the geometrical interpretations of the intuitionistic fuzzy sets. In: Issues in the Representation and Processing of Uncertain and Imprecise Information. Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets, and Related Topics. (Eds. Atanassov K., Kacprzyk J., Krawczak M., Szmidt E.), EXIT, Warsaw 2005.

[5] Marinov E., Velizarova E., Atanassov K. (2013), An intuitionistic fuzzy estimation of the area of 2D-figures, Notes on Intuitionistic Fuzzy Sets, Vol. 19, No. 2, 57-70

[6] Munkres J. (2000) Topology, 2nd ed., Prentice Hall

[7] Szmidt E. and Kacprzyk J. (2000) Distances between intuitionistic fuzzy sets. Fuzzy Sets and Systems, Vol. 114, No. 3, 505–518.

[8] Zadeh L.A. (1965), Fuzzy sets. Information and Control, Vol. 8, 338–353.

The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2013

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN 838947553-7

9 788389 475534