Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

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IBS PAN SRI PAS

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

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Connection between balanced fuzzy set and Atanassov intuitionistic fuzzy set

Paweł Drygaś
Institute of Mathematics, University of Rzeszów
ul. Rejtana 16A, 35-310 Rzeszów, Poland
paweldr@univ.rzeszow.pl

Abstract

There are many generalizations of fuzzy set, e.g. balanced fuzzy set, Atanassov intuitionistic fuzzy set, interval valued fuzzy set, L-fuzzy set and it's generalization. In this paper we compare two of them, i.e. balanced fuzzy set and Atanassov intuitionistic fuzzy set.

Keywords: fuzzy set, balanced fuzzy set, Atanassov intuitionistic fuzzy set, L-fuzzy set.

1 Introduction

Since it has been introduced by L. Zadeh the concept of fuzzy set in 1965 ([16]) it has appeared many generalizations of fuzzy set, e.g. balanced fuzzy set, Atanassov intuitionistic fuzzy set, interval valued fuzzy set, L-fuzzy set and it's generalization. Fuzzy set describe the degree to which a certain point belongs to a set. In real life a person may assume that a point belongs to a set A to a certain degree, but it is possible that he is not so sure about it, i.e., there may be a uncertainty about the membership degree of x in A.

In fuzzy sets theory, there is no any information about the hesitation of the membership degrees. The generalizations of fuzzy sets give us the possibility to model

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hesitation by generalize the concept of membership function. One of them use an additional degree (Atanassov intuitionistic fuzzy set) and another enlarge the set of value of the membership function (balanced fuzzy set).

In this paper we compare two of a generalizations of fuzzy set, i.e. balanced fuzzy set and Atanassov intuitionistic fuzzy set.

2 Basic notions

Arbitrary set A on the universe X may be represented by its characteristic function $\chi_A : X \to \{0, 1\}$ yielding the value 1 for element belonging to the set A and the value 0 for element not belonging to the set A (see Fig. 1). Through such presentation we can easy define the sets operations (intersection, union and complement) as functions $\min$, $\max$ and $N(x) = 1 - x$.

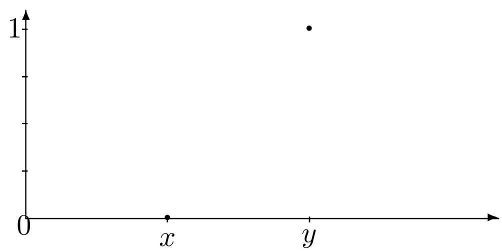

Figure 1: Representation of the crisp set

In real life a person may not be sure that an element x belong to the set A. For example "The element is in the set A", "Probably the element is in the set A", "There is possible that the element is in the set A", "The element is not in the set A". The fuzzy set allow to describe the unprecise information. This concept generalize the characteristic function by allowing images of elements to be in the interval $[0, 1]$.

Definition 1 ([16]). *A fuzzy set A in a universe X is a mapping $A : X \to [0, 1]$ (see Fig. 2).*

The definitions of the sets operations are expressed, as in the crisp sets.

3 Balanced fuzzy set

We can notice that the positive information is graduated by interval $(0, 1]$ whereas the whole negative information is concentrated in the value 0. So, we can observe the asymmetry of the set of values of the membership function. The concept of

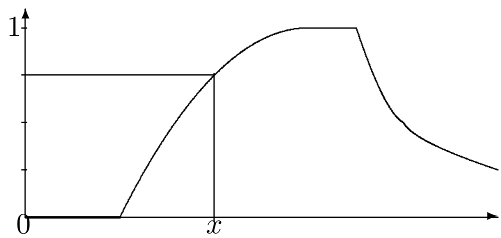

Figure 2: Representation of the fuzzy set

balanced fuzzy set expands negative information from the point 0 into the interval $[-1, 0]$.

Definition 2 ([12]). *A balanced fuzzy set A in a universe X is a mapping $A : X \to [-1, 1]$ (see Fig. 3).*

Thus, the scale of information is symmetrical. Of course both scales of information are indistinguishable in the meaning of the linear mapping $h(x) = 2x - 1$. Difference two this scale results from psychological approach of person. However, e.g. in economy the psychological attempt to decision-making process with uncertain premises overheads traditional models of customer behavior. It refers to peoples tendency to make risk-averse choices if the expected outcome is positive, but risk-seeking choices to avoid negative outcomes.

Aggregation of positive and negative premises leads to implementation of a crisp decision. Modeling of such an attempt requires processing of positive, neutral and negative information.

4 Atanassov intuitionistic fuzzy sets

Different approach from above-mentioned to generalize the concept of membership function is realized by using an additional degree. This approach give us the possibility to model hesitation of the membership degrees.

Definition 3 ([2], [3]). *An Atanassov intuitionistic fuzzy set A in a universe X is a triple*

$$A = \{(x, \mu(x), \nu(x)) : x \in X\}$$

$$\textit{where } \mu, \nu : X \to [0, 1], \quad \mu(x) + \nu(x) \leq 1.$$

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation degree of x (see Fig. 4).

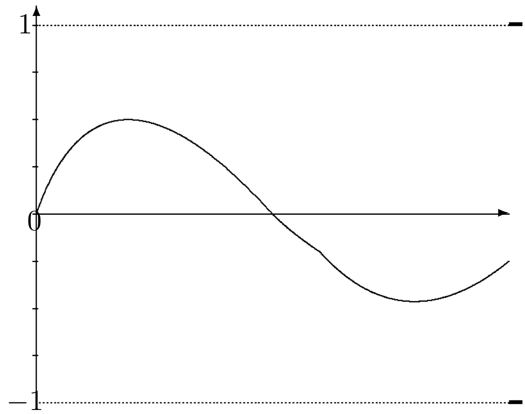

Figure 3: Representation of the balanced fuzzy set

In fuzzy set theory the nonmembership degree of an element x of the universe X is defined as one minus the membership degree and it is fixed. Moreover the hesitation degree is equal zero. In Atanassov intuitionistic fuzzy set theory the membership degree and nonmembership degree are more or less independent. The only condition is that its sum is smaller than or equal to one.

If we consider the μ and ν as a fuzzy sets, then for each element $x \in X$ the length of the interval $[\mu_A(x), 1 - \nu_A(x)]$ denote the hesitation degree.

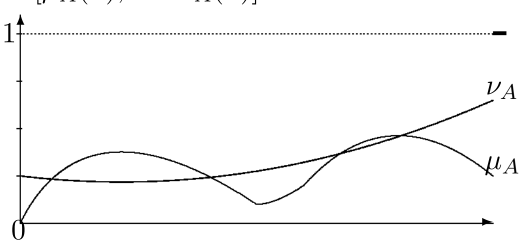

Figure 4: Representation of the Atanassov intuitionistic fuzzy set

The Atanassov intuitionistic fuzzy sets one can consider as L-fuzzy set with a special lattice L^*

Definition 4 ([10]). *An L-fuzzy set A in a universe X is a function $A : X \to L$ where L is a lattice.*

The lattice L^* (see Fig. 5) is defined as follows

$$L^* = \{(x_1, x_2) \in [0,1]^2 : x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq (y_1, y_2) \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$ for all $(x_1, x_2), (y_1, y_2) \in L^*$.

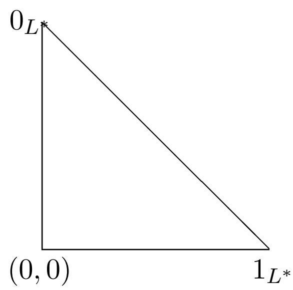

Figure 5: Lattice L^*

The isomorphism assign the Atanassov intuitionistic fuzzy set A and the L^*-fuzzy set A_{L^*} as follows:
$A = (x, \mu_A(x), \nu_A(x)) \mapsto (\mu_A(x), \nu_A(x)) \in L^*$.
So, we can use notions of these sets equivalently.

Atanassov intuitionistic fuzzy set on a universe X is a generalization of fuzzy set. Because of this we put the connection between the fuzzy set and Atanassov intuitionistic fuzzy set. Fuzzy set is interpreted as Atanassov intuitionistic fuzzy set such that the nonmembership function is equal one minus membership function. So, these set are in the form $(x, \mu_A(x), 1 - \mu_A(x))$ (see Fig. 6) and the values are on the diagonal in the lattice L^*.

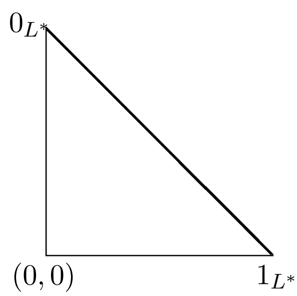

Figure 6: Atanassov intuitionistic fuzzy set in the form $(x, \mu_A(x), 1 - \mu_A(x))$

5 Balanced fuzzy set and Atanassov intuitionistic fuzzy set

If we consider the functions $\mu_A(x)$ and $-\nu_A(x)$ in Atanassov intuitionistic fuzzy set then we obtain positive and negative information about belonging the point x to the set A (see Fig. 7).

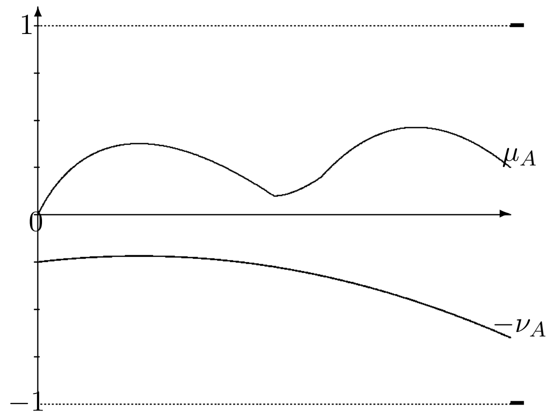

Figure 7: Positive and negative information for Atanassov intuitionistic fuzzy set

This information allow as to obtain the following connection between balanced fuzzy set and Atanassov intuitionistic fuzzy set.

Theorem 1. *The balanced fuzzy set A is isomorphic with some Atanassov intuitionistic fuzzy set.*

The isomorphism is given by the following formula

$$\Phi : [-1, 1] \to L^*$$
$$\Phi(x) = \begin{cases} (x, 0) & if\ x \geq 0, \\ (0, -x) & if\ x < 0. \end{cases}$$

We can easy see, that the set of value of the function Φ is given by

$$\{(x, 0),\ (0, -x) :\ x \in [0, 1]\}.$$

So, we can ask, if each set of such form leads to the balanced fuzzy set (see Fig. 8).

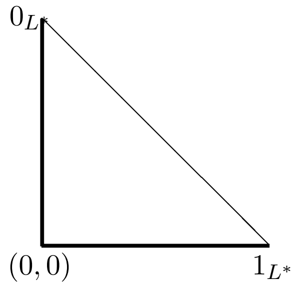

Figure 8: Part of lattice L^* which leads to Balanced fuzzy set

Theorem 2. *The balanced fuzzy set A is isomorphic with some Atanassov intuitionistic fuzzy set if and only if $\mu_A(x) \cdot \nu_A(x) = 0$ for all x.*

The isomorphism is again given by

$$\Phi : [-1, 1] \to [0, 1]^2$$
$$\Phi(x) = \begin{cases} (x, 0) & if\ x \geq 0, \\ (0, -x) & if\ x < 0. \end{cases}$$

Here we can see that both fuzzy sets and balanced fuzzy set are isomorphic, but leads to different Atanassov intuitionistic fuzzy set.

6 Conclusions

In this paper we compare two of a generalizations of fuzzy set, i.e. balanced fuzzy set and Atanassov intuitionistic fuzzy set. If we known the relation between the Atanassov intuitionistic fuzzy set, balanced fuzzy set and fuzzy set we may ask about the connection between operations defined on these sets.

Here we put some basic definitions of operations

Definition 5 ([13])**.** *A triangular norm T is an increasing, commutative, associative operation $T : [0, 1]^2 \to [0, 1]$ with neutral element* 1.
A triangular conorm S is an increasing, commutative, associative operation $S : [0, 1]^2 \to [0, 1]$ with neutral element 0.

Definition 6 ([15])**.** *An operation $U : [0, 1]^2 \to [0, 1]$ is called a uninorm if it is commutative, associative, increasing and has the neutral element $e \in [0, 1]$.*

If in above definitions we replace the interval $[0, 1]$ by lattice L^* we obtain definitions of t-norm, t-conorm and uninorm on L^*.

Definition 7 ([1]). *Let $U : [0,1]^2 \to [0,1]$ be increasing, associative, monotonic operation. If for some $z \in (0,1)$ there exist $e_1 \leq z \leq e_2$ such that, $U(e_1, x) = x$ for all $x \leq z$ and $U(e_2, x) = x$ for all $x \geq z$ then $\{e1, e2\}_z$ is called a 2-neutral element of U and U is called as 2-uninorm.*

Definition 8 ([1]). *The element $\{e_1, e_2, ..., e_n\}_{z_1, z_2, ..., z_n}$ where $0 = z_0 < z_1 < z_2 < ... < z_n = 1$ is called an n-neutral element if $e_i \in [z_{i-1}, z_i]$ such that, $U_n(e_i, x) = x$ for all $x \in [z_{i-1}, z_i]$ for $i = 1, 2, ..., n$. Operation U with n-neutral element is called an n-uninorm.*

Definition 9 ([12]). *The balanced t-norm is an increasing, commutative, associative operation $T : [-1,1]^2 \to [-1,1]$ such that*

$$T(1, x) = x \text{ for } x \in [0,1] \text{ and}$$

$$T(x, y) = N(T(N(x), N(y))).$$

The balanced t-conorm is an increasing, commutative, associative operation $S : [-1,1]^2 \to [-1,1]$ such that

$$S(0, x) = x \text{ for } x \in [0,1] \text{ and}$$

$$S(x, y) = N(S(N(x), N(y))).$$

where $N(x) = -x$

Definition 10 ([12]). *The balanced uninorm is an increasing, commutative, associative operation $U : [-1,1]^2 \to [-1,1]$ such that*

$$\exists e \in [0,1] \ \ U(e, x) = x \text{ for } x \in [0,1] \text{ and}$$

$$U(x, y) = N(U(N(x), N(y))),$$

where $N(x) = -x$.

Definition 11 ([12]). *The balanced nullnorm is an increasing, commutative, associative operation $V : [-1,1]^2 \to [-1,1]$ such that there exists $z \in [0,1]$ such that*

$$V(0, x) = x \quad \text{for } x \in [0, z],$$

$$V(1, x) = x \quad \text{for } x \in [z, 1].$$

$$V(x, y) = N(V(N(x), N(y))),$$

where $N(x) = -x$.

Definition 12 ([12]). *For a given balanced t-conorm S the iterative t-conorm is a function $S_{it} : R \times R \to R$*

$$S_{it}(x,y) = \begin{cases} S(x-2k-2l, y+2k-2l)+2l & (x-2k-2l, y+2k-2l) \\ & \in [-1,1]\times[-1,1] \\ & \textit{and } k,l - integer \\ 1+2l & (x-2k-2l, y+2k-2l) \\ & \in [1,3]\times[-1,1] \\ & \textit{and } k,l - integer \end{cases}$$

Open Problems

1. Find the relation between operations on balanced fuzzy sets and n-uninorms.
2. Find the relation between operations on balanced fuzzy sets and operation on L^*-fuzzy sets.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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